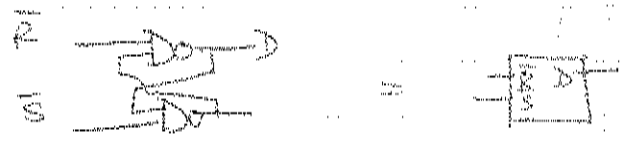


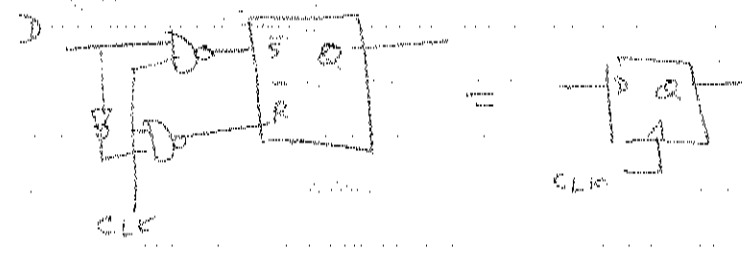
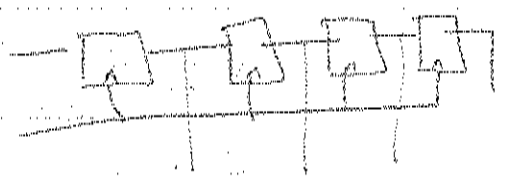
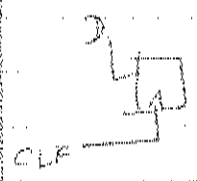
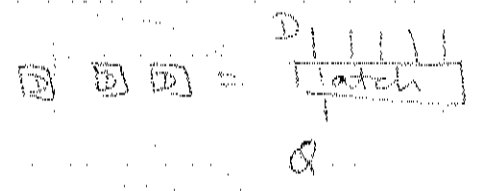
compute from NAND gates

Build

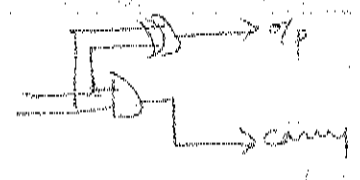
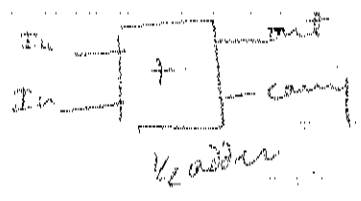
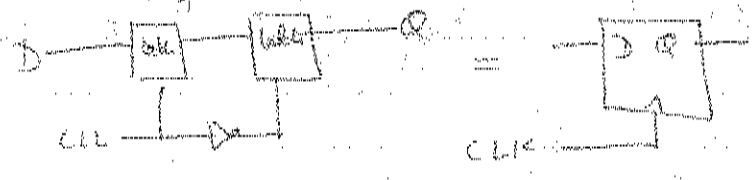


↳ make D FF

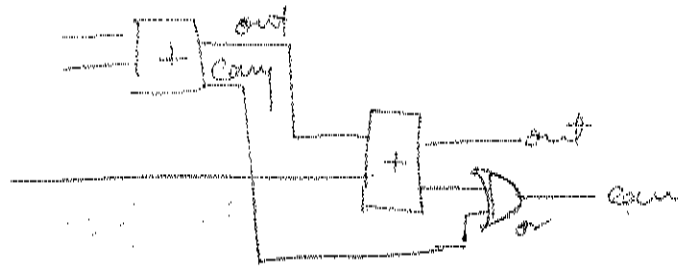
↳ make latch



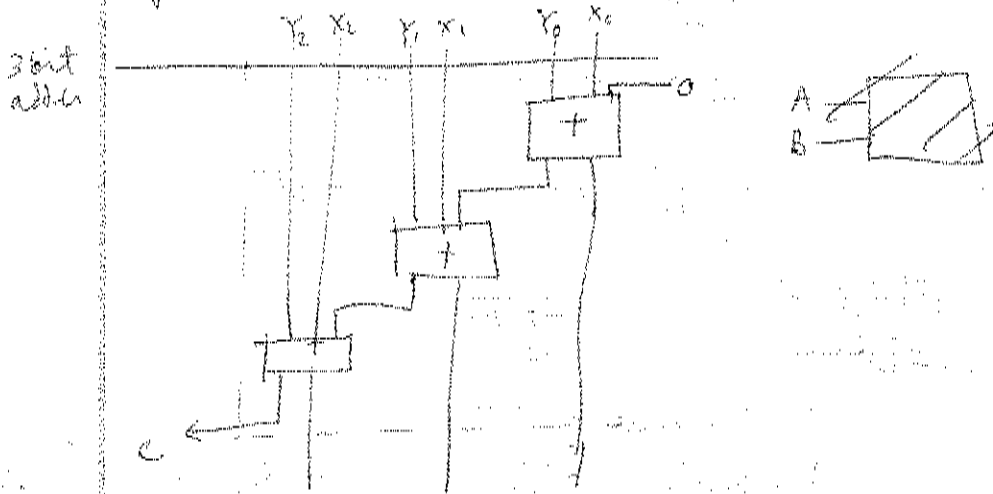
↳ make M.S. without many extra components



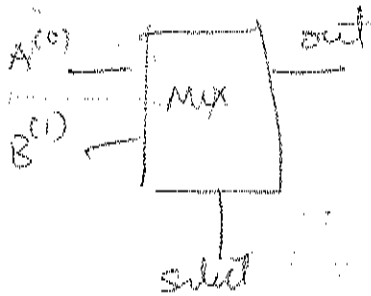
# Full Adder



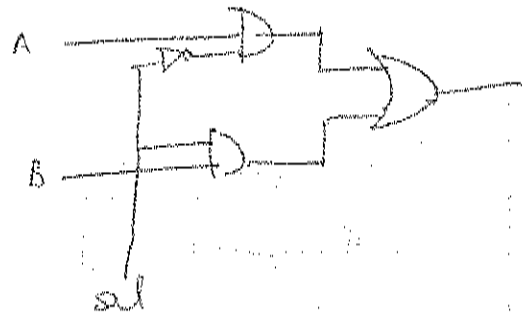
n full adders make the n bit adder



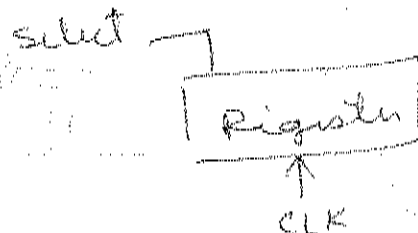
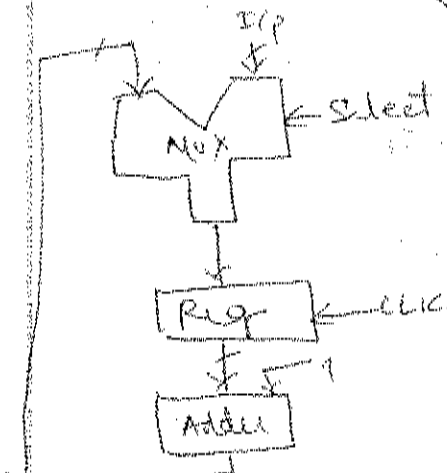
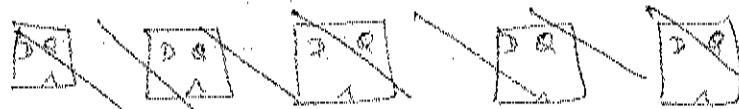
# MUXES



=



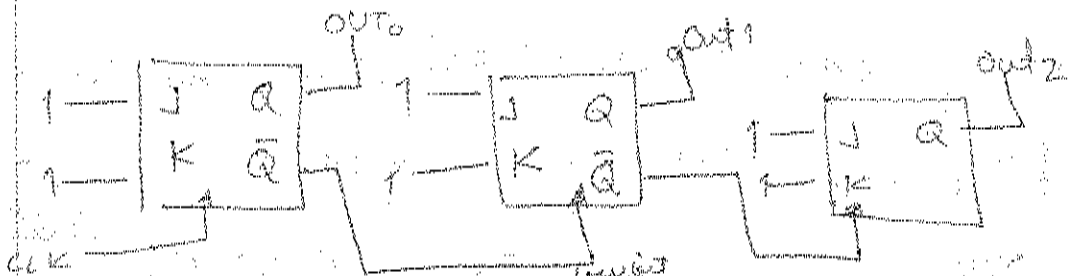
# counter



select = 0 : load

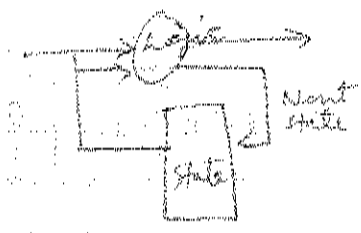
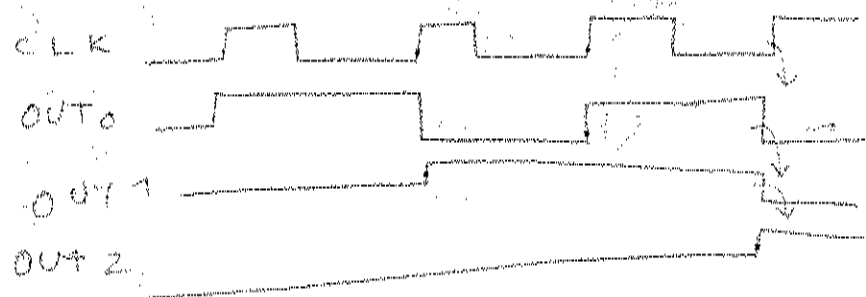
select = 1 : increment

# 3 bit JK Ripple counter



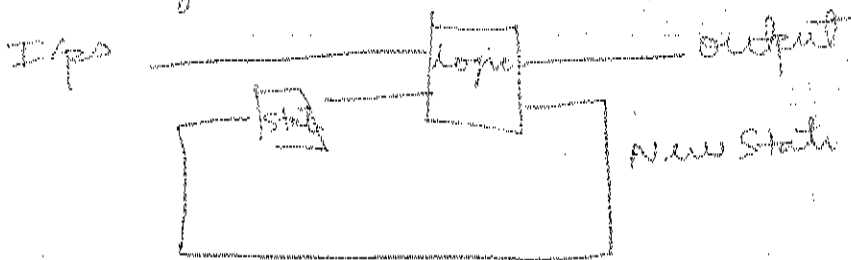
toggles when 1 → 0  
 toggles on each clock

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



## State M/cs

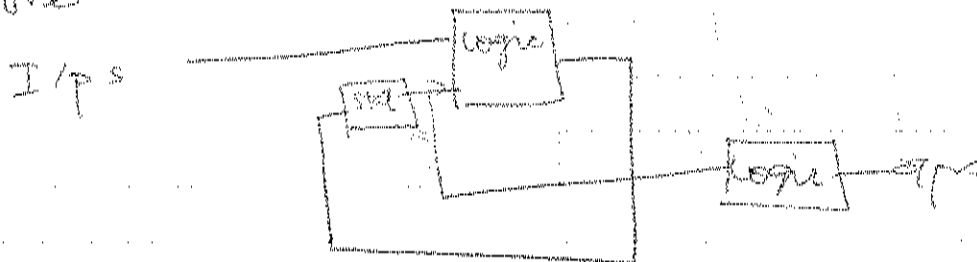
### Mealy



$$O/P = f(I/P, state)$$

$$O/P \text{ on edges}$$

### Moore



$$O/P = f(STATE)$$

$$O/P \text{ in states}$$

Moore is synchronous

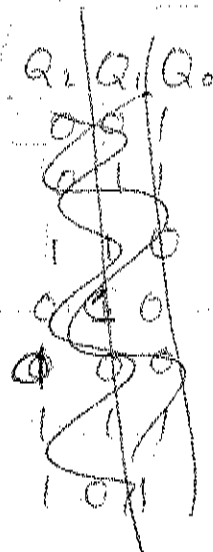
Grey code : only 1 bit changes each step

Reflected grey code (Minor method)

000	000
001	001
010	011
011	010
111	110
110	111
101	101
100	100

w

$Q_2$	$Q_1$	$Q_0$
0	0	0
0	0	1
0	1	0
1	0	0
1	0	1
1	1	0
1	1	1



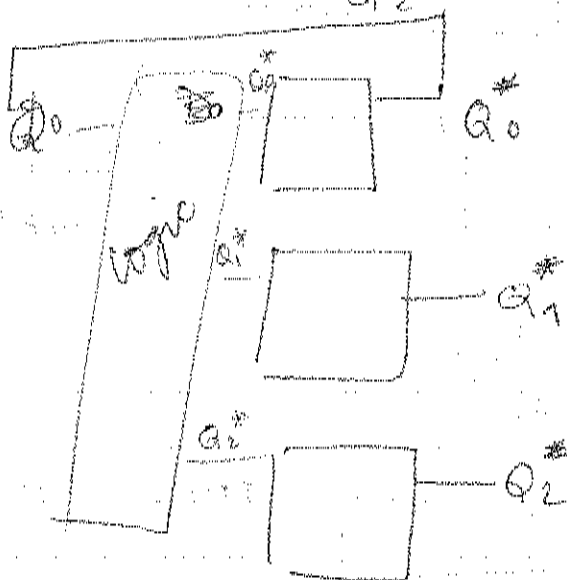
$Q_2^*$	$Q_1^*$	$Q_0^*$
0	0	1
0	1	1
1	1	0
0	1	0
0	0	0
1	0	0
1	1	1
1	0	1

	$D_2$	$D_1$	$D_0$	01	11	10
$Q_2^*$	0	1	0	1	0	0
$Q_1^*$	0	0	1	1	1	1

$$Q_0^* = \bar{D}_2 \bar{D}_1 + D_2 D_1$$

$$Q_1 =$$

$$Q_2 =$$



J	K	Q	Q'	have	wrong	J	K
0	0	0	0	0	0	0	x
0	0	1	1	0	0	0	x
0	1	0	0	0	1	1	x
0	1	1	0	1	0	x	1
1	0	0	1	1	1	x	0
1	0	1	1				
1	1	0	1				
1	1	1	0				

Back to Gray counter:

Q <sub>2</sub>	Q <sub>1</sub>	Q <sub>0</sub>	Q <sub>0</sub> '	J	K
0	0	0	0	1	x
0	0	1	1	x	0
0	1	0	0	0	x
0	1	1	0	x	1
1	0	0	0	0	x
1	0	1	0	x	1
1	1	0	1	1	x
1	1	1	1	x	0

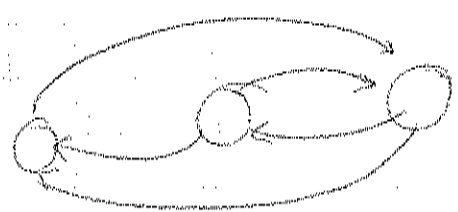
⇒ make Kmap

Q <sub>1</sub> Q <sub>0</sub>	00	01	11	10
Q <sub>0</sub> = 0	x	0	1	x
Q <sub>0</sub> = 1	x	1	0	x

$$K_0 = \bar{Q}_2 Q_1 + Q_2 \bar{Q}_1$$

State Assignment: counters ⇒ trivial

But



assign states to max of logic sum  
max 0-0 initial state.

Also adjacent states should differ by as little as possible

Given 5 i/p 3 F ~~1000~~ 0/p

# Mealy M/c

- how many states? 1 to  $2^5 \cdot 2^3$
- how many distinct o/p patterns configurations
- how many lines from each state
- how many ~~bits~~ <sup>lines</sup> to each state 0 to  $2^5 \cdot 2^3$

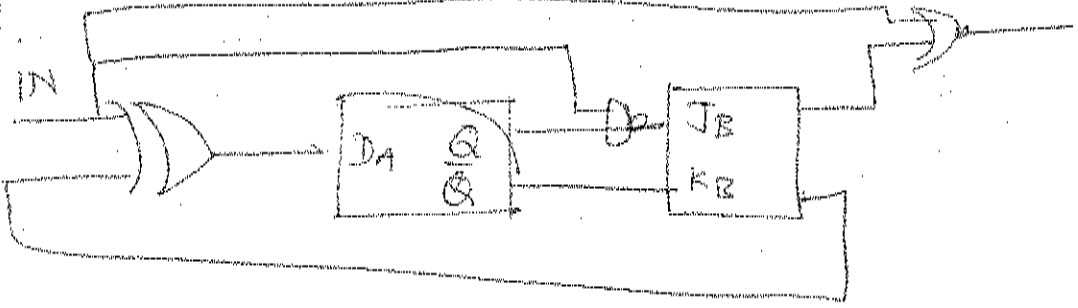
→ 5

→ straight forward

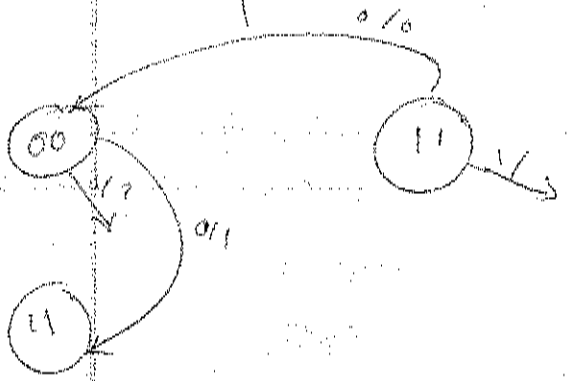
Mealy - each edge has distinct o/p pattern

more  $2^3$  o/p configurations

remember to have a start state



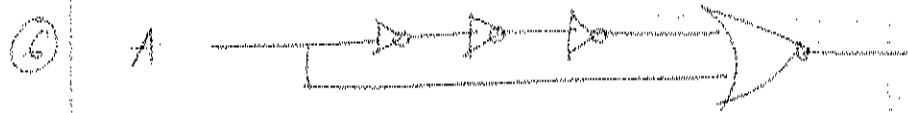
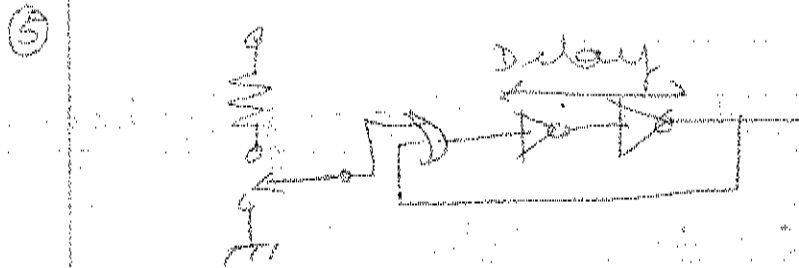
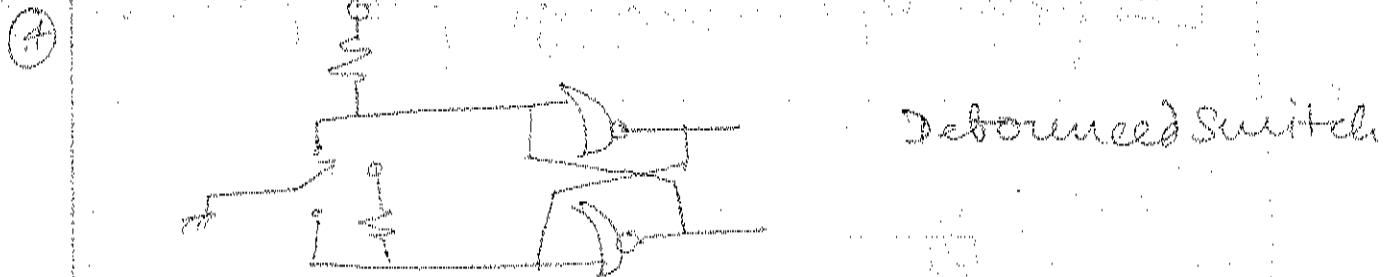
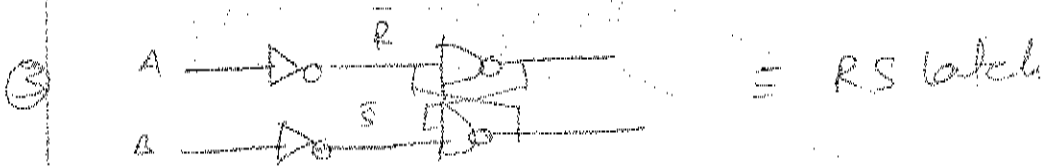
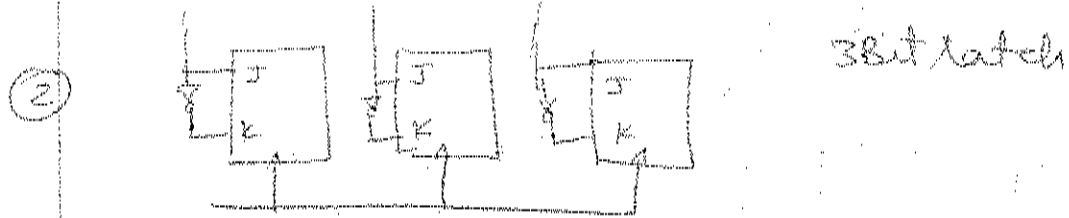
Mealy



i/p	state		D <sub>A</sub>	J <sub>B</sub>	K <sub>B</sub>	Out	Q <sub>1</sub>
	Q <sub>A</sub>	Q <sub>B</sub>					
0	0	0	1	1	1	1	1
0	0	1	0	1	1	0	0
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

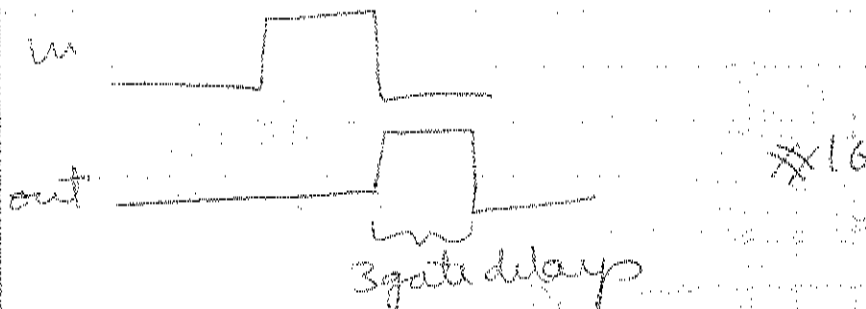
# Ccf identification

① XNOR



0 in  $\Rightarrow$  0 out

1 in  $\Rightarrow$  0 out, then goes to 1  $\Rightarrow$  3 gate delays, later zero

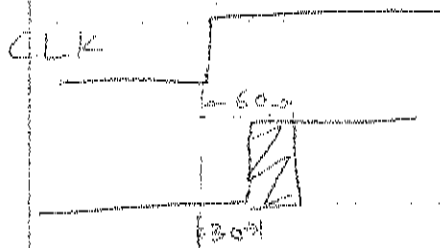


# Timing Analysis

Flip Flops

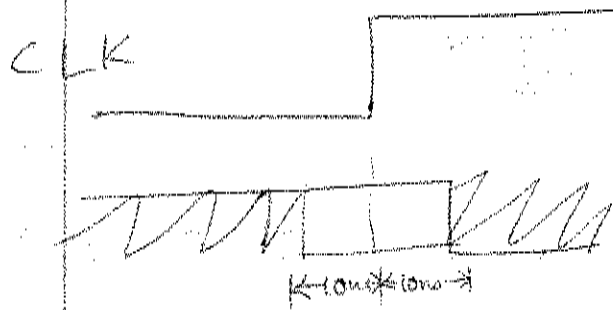
- max delay 60 ns
- min delay 30 ns
- set up time 10 ns
- hold time 10 ns

→ from i/p CLK changing to o/p = changing

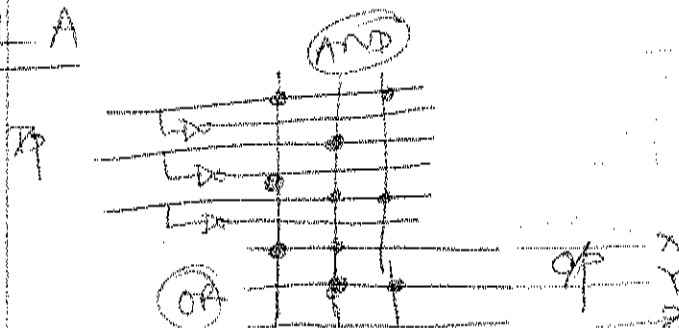


→ time i/p's must be unchanging before CLK

→ time unchanging after CLK



PLA



$$X = \overline{A}B + BC$$

$$Y = AC + BC$$





# Mathematical Induction

**Principle of Mathematical Induction**

Let  $P(n)$  be a statement involving the natural number  $n$ . If

- $P(1)$  is true, and
- $P(k) \Rightarrow P(k+1)$  for every natural number  $k$ ,

then  $P(n)$  is true for every natural number  $n$ .

**Example 1:** Prove that  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$  for all  $n \in \mathbb{N}$ .

*Proof:* Let  $P(n)$  be the statement  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ .

Step 1:  $P(1)$  is true because  $1 = \frac{1(1+1)}{2} = 1$ .

Step 2: Assume  $P(k)$  is true, i.e.,  $1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2}$ . We need to show  $P(k+1)$  is true, i.e.,  $1 + 2 + 3 + \dots + k + (k+1) = \frac{(k+1)(k+1+1)}{2}$ .

From the assumption,  $1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2}$ . Adding  $(k+1)$  to both sides, we get  $1 + 2 + 3 + \dots + k + (k+1) = \frac{k(k+1)}{2} + (k+1) = \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$ . Thus,  $P(k+1)$  is true.

**Example 2:** Prove that  $2^n > n$  for all  $n \in \mathbb{N}$ .

*Proof:* Let  $P(n)$  be the statement  $2^n > n$ .

Step 1:  $P(1)$  is true because  $2^1 = 2 > 1$ .

Step 2: Assume  $P(k)$  is true, i.e.,  $2^k > k$ . We need to show  $P(k+1)$  is true, i.e.,  $2^{k+1} > k+1$ .

From the assumption,  $2^k > k$ . Multiplying both sides by 2, we get  $2^{k+1} > 2k$ . Since  $2k > k+1$  for all  $k \in \mathbb{N}$ , we have  $2^{k+1} > 2k > k+1$ . Thus,  $P(k+1)$  is true.

**Example 3:** Prove that  $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$  for all  $n \in \mathbb{N}$ .

*Proof:* Let  $P(n)$  be the statement  $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$ .

Step 1:  $P(1)$  is true because  $1^2 = \frac{1(1+1)(2 \cdot 1 + 1)}{6} = 1$ .

Step 2: Assume  $P(k)$  is true, i.e.,  $1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$ . We need to show  $P(k+1)$  is true, i.e.,  $1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$ .

From the assumption,  $1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$ . Adding  $(k+1)^2$  to both sides, we get  $1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{k(k+1)(2k+1) + 6(k+1)^2}{6} = \frac{(k+1)(k(2k+1) + 6(k+1))}{6} = \frac{(k+1)(2k^2 + k + 6k + 6)}{6} = \frac{(k+1)(2k^2 + 7k + 6)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$ . Thus,  $P(k+1)$  is true.

**Example 4:** Prove that  $1 + 3 + 5 + \dots + (2n-1) = n^2$  for all  $n \in \mathbb{N}$ .

*Proof:* Let  $P(n)$  be the statement  $1 + 3 + 5 + \dots + (2n-1) = n^2$ .

Step 1:  $P(1)$  is true because  $1 = 1^2$ .

Step 2: Assume  $P(k)$  is true, i.e.,  $1 + 3 + 5 + \dots + (2k-1) = k^2$ . We need to show  $P(k+1)$  is true, i.e.,  $1 + 3 + 5 + \dots + (2k-1) + (2(k+1)-1) = (k+1)^2$ .

From the assumption,  $1 + 3 + 5 + \dots + (2k-1) = k^2$ . Adding  $(2(k+1)-1)$  to both sides, we get  $1 + 3 + 5 + \dots + (2k-1) + (2(k+1)-1) = k^2 + (2(k+1)-1) = k^2 + 2k + 1 = (k+1)^2$ . Thus,  $P(k+1)$  is true.

**Example 5:** Prove that  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$  for all  $n \in \mathbb{N}$ .

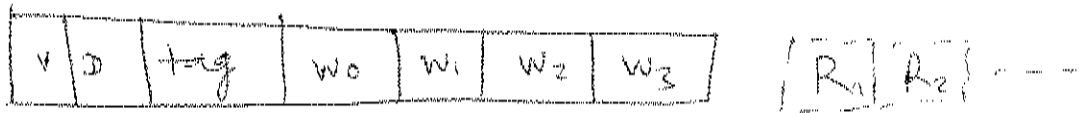
*Proof:* Let  $P(n)$  be the statement  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ .

Step 1:  $P(1)$  is true because  $1 = \frac{1(1+1)}{2} = 1$ .

Step 2: Assume  $P(k)$  is true, i.e.,  $1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2}$ . We need to show  $P(k+1)$  is true, i.e.,  $1 + 2 + 3 + \dots + k + (k+1) = \frac{(k+1)(k+1+1)}{2}$ .

From the assumption,  $1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2}$ . Adding  $(k+1)$  to both sides, we get  $1 + 2 + 3 + \dots + k + (k+1) = \frac{k(k+1)}{2} + (k+1) = \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$ . Thus,  $P(k+1)$  is true.





1K  $\Rightarrow$  1K of data (not incl. the tag & dirty bits)

v  $\rightarrow$  valid bit (not valid)  
 D  $\rightarrow$  dirty bit (eg switching processes, virtual memory)

write through  $\rightarrow$  goes to memory (wired writes)  
 (don't need the extra bit)  
 faster on a cache miss

write back ( $\equiv$  copy back)  
 cache doesn't write to memory  
 slower on misses!  
 adv. don't need to write each time

How do you decide which to throw away?

- (1) Random (use counters)
- (2) R  $\leftarrow$  replace dep on R

Random - for HW caches almost as good  
 < R0 - don't want to do each access  
 # of bus errors

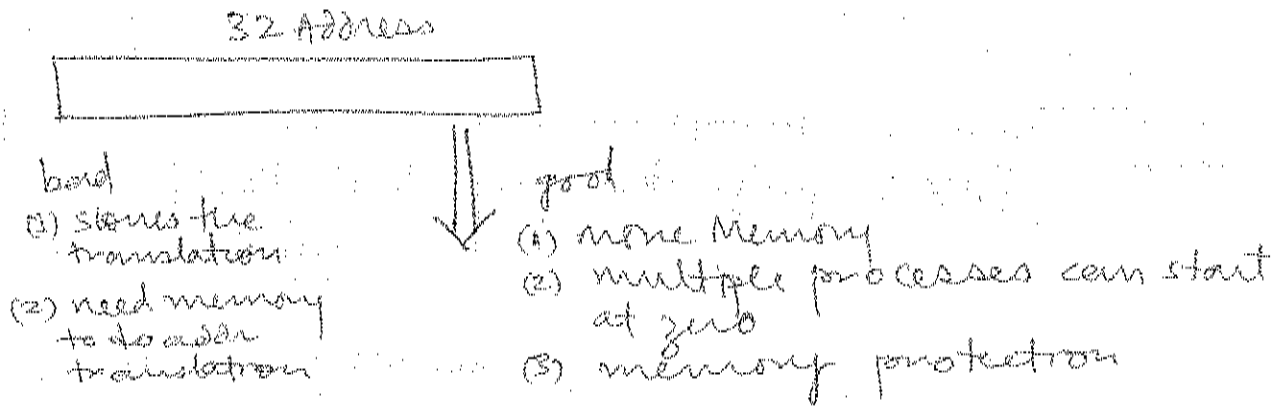
no D bit in write through cache  
 Dirty direct mapped  $\rightarrow$  write back yes  
 for other valid no

write through  $\rightarrow$  maybe yes/no  
 direct mapped  $\Rightarrow$  no

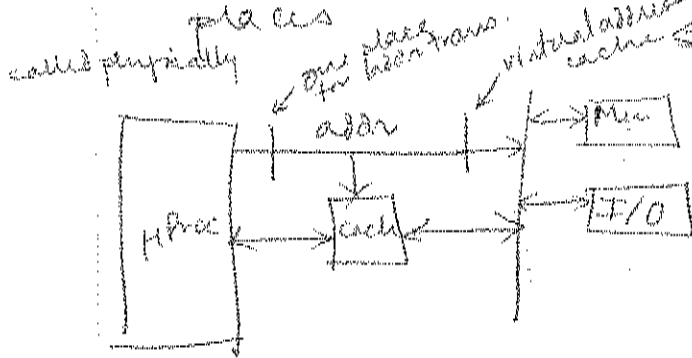
p. 25

valid bit	always	always
dirty bit	never	sometimes
replacement bit	sometimes	never

# Virtual Memory: ( $\equiv$ addr translation)

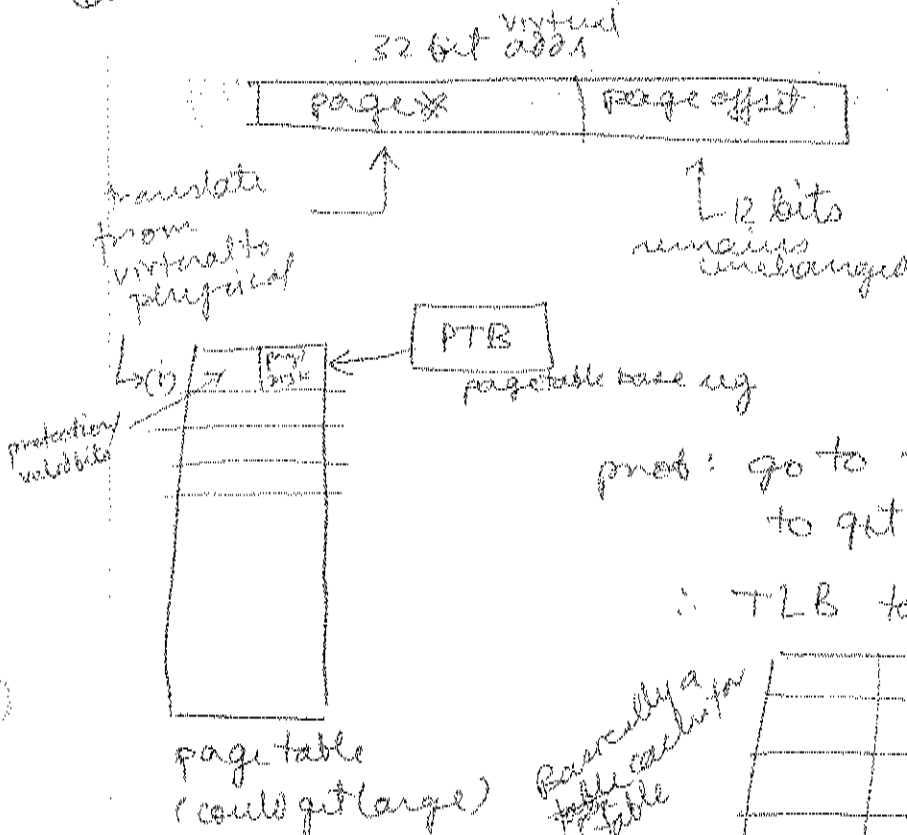


Virtual Addr. translation can occur in one of two places



Task, don't need to do the Addr. translation for cache

Can't do virtual mem if no addr translation!

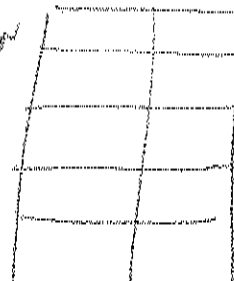


BSD 4.3  
4K byte page size

prob: go to memory encryption to get access

: TLB table to ~~lookup~~ <sup>lookup</sup> buffer

← read only cache changes only when page table



page fault;

many processes  
each needs own  
page table

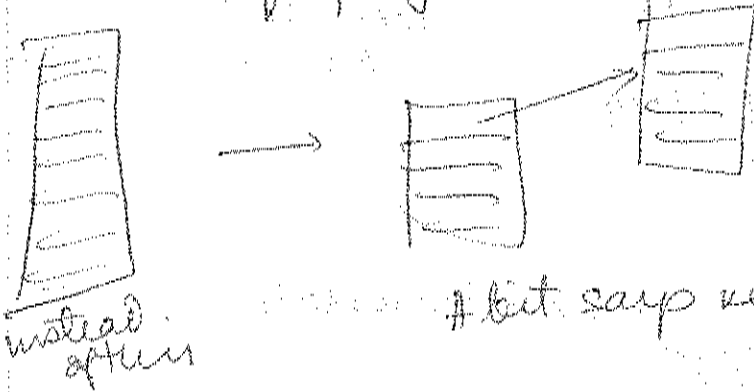
now no program ever needs entire page table

VAX: page the page table itself  
bits in virtual memory

Now you need two memory accesses

page table hangs out in system space  
→ protection

Tree of pages



# bit say keep going or stop!

segment: variable length pages

I/O: now memory mapped

eg. any address in  $ffff8000 - ffff8010$  will go to  
Disk Drive

↑ could be virtual addresses

How do you tell the device is ready

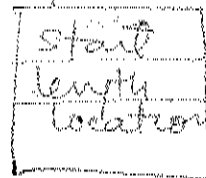
(i) poll: check each place every now and then  
status bit

(ii) interrupts

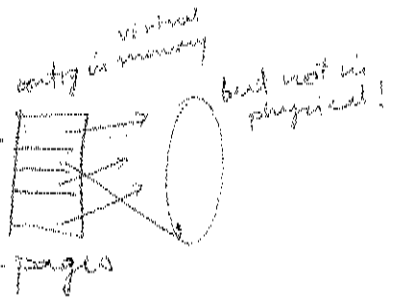
3 ways to actually comm<sup>n</sup>

(i) Programmed I/O CPU actually goes out  
for slow devices

(ii) DMA disks, video, high speed stuff  
CPU builds table



adv. of virtual mem address for table



(iii) data channel  ~~bunch of processes share an I/O processor~~

bunch of I/O devices hang off a single DMA processor



(i) do one fully

(ii) round robin  
(bit of each)

inst set Arch

# ISA

Wish to design an instruction set

32

Word/Address

How many addresses in each w/c instr

0

1

2

3

↳ ADD  $\frac{blob}{\text{source}}, \frac{blob}{\text{source}}, \frac{blob}{\text{destination}}$  2 sources & 1 destination

mistaken  $\Rightarrow$  3 instr

ADD  $\frac{\text{blob}}{\text{source}}, \frac{\text{blob}}{\text{source}}$   $\rightarrow$  2

ADD  $\frac{\text{blob}}{\text{source}}$   $\Rightarrow$  1  
implicitly using Accumulator

0  $\Rightarrow$  stack based

Addressing modes: out these days  
is its a subset of instr  
(general by shift & add)

now  $\rightarrow$  Load Store

Register file size  $\rightarrow$  32 is a popular #

Floating pt vs int registers  
Addr vs Data registers

Before: wanted to encode instr cheaply (i.e. min. expansion)  
now not a real issue  
so don't need separate Addr & Data reg

P. 11



Most imp instr.

- MOV
- ADD
- SUB
- MUL
- DIV/REM
- AND
- OR
- XOR

ADD  
SUB  
MUL  
DIV

- branch / jump
- bcc → branch on cond.
- src → set reg on cond

VAX



1 byte opcode

some specify immediate

MIPS

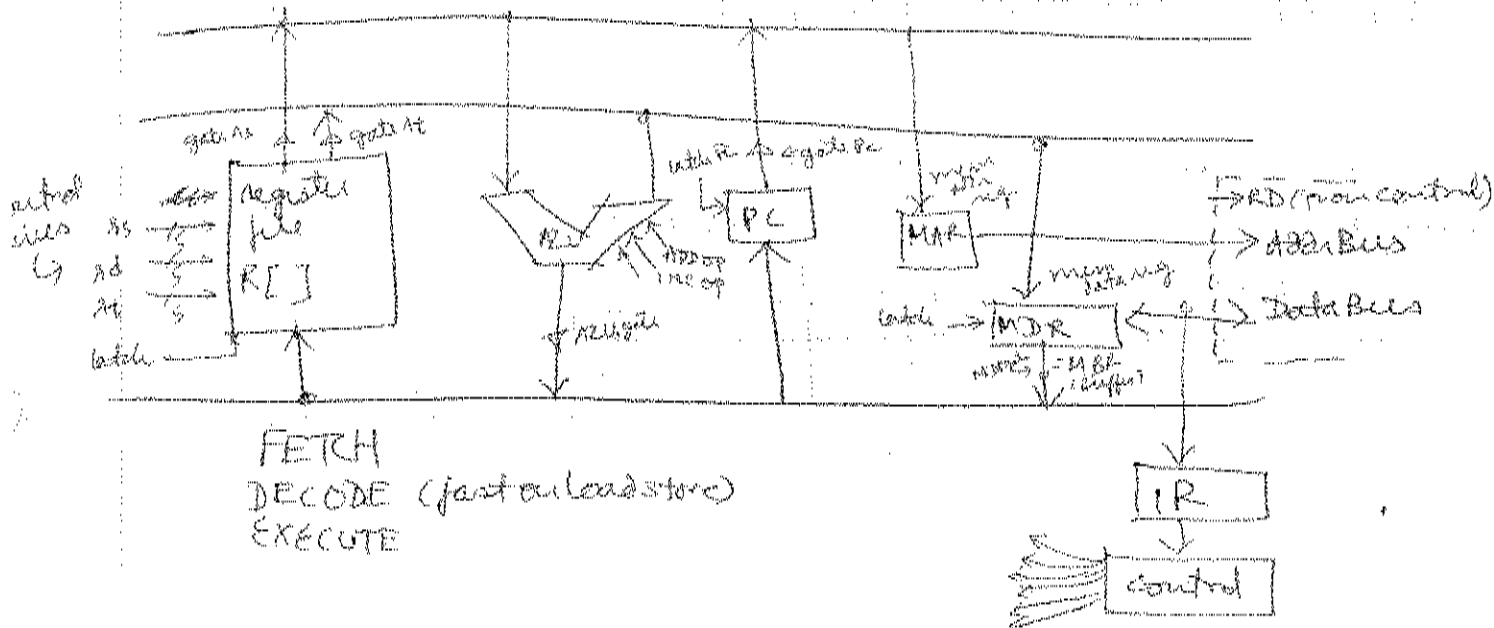


VERY LITTLE space, but slow to decode

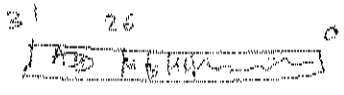
every instr is 32 bits wide!  
VERY Fast

one of 4 instr formats

## IMPLEMENTATION



FETCH  
DECODE (fast on lead store)  
EXECUTE



Spang '00 - Katz & Cullis  
I/O bus solution

for MIPS

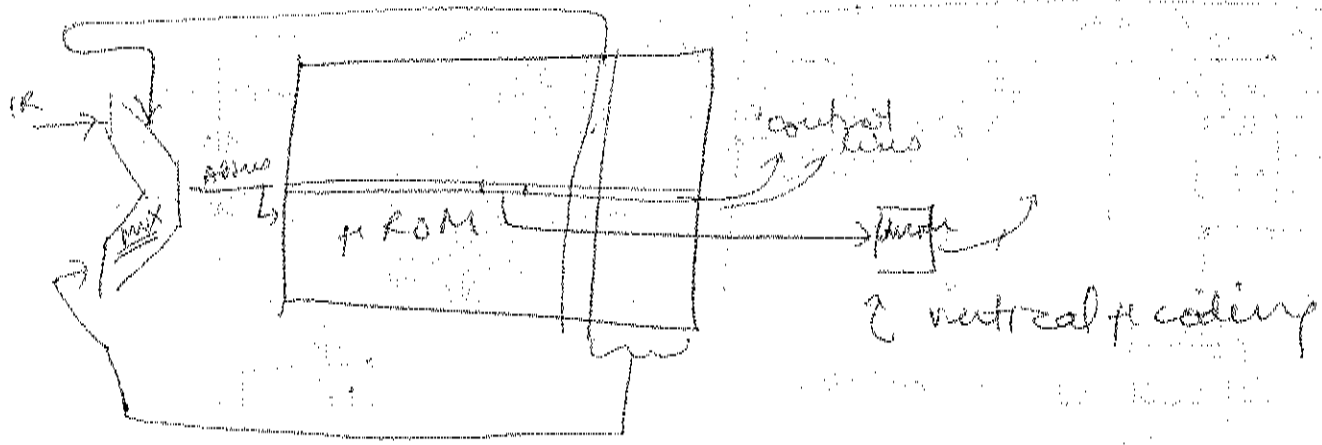
- RTL:
- ①  $MAR \leftarrow PC$   
 $PC \leftarrow PC + 1$   
assert  $R_0$
  - ②  $IR \leftarrow data\ bus$
  - ③ branch ( $IR_{31..26}$ )
  - ④  $R[Li] \leftarrow R[Rs] + R[Rd]$   
branch (?)
- Notes:   
 - Addr (in early MIPS had two phase clocks so could be both in 1 clock cycle)  
 - basically a problem of

can't merge  
stages  
events

# USE LOTS OF REGISTERS

## TO LATCH EVERYTHING!

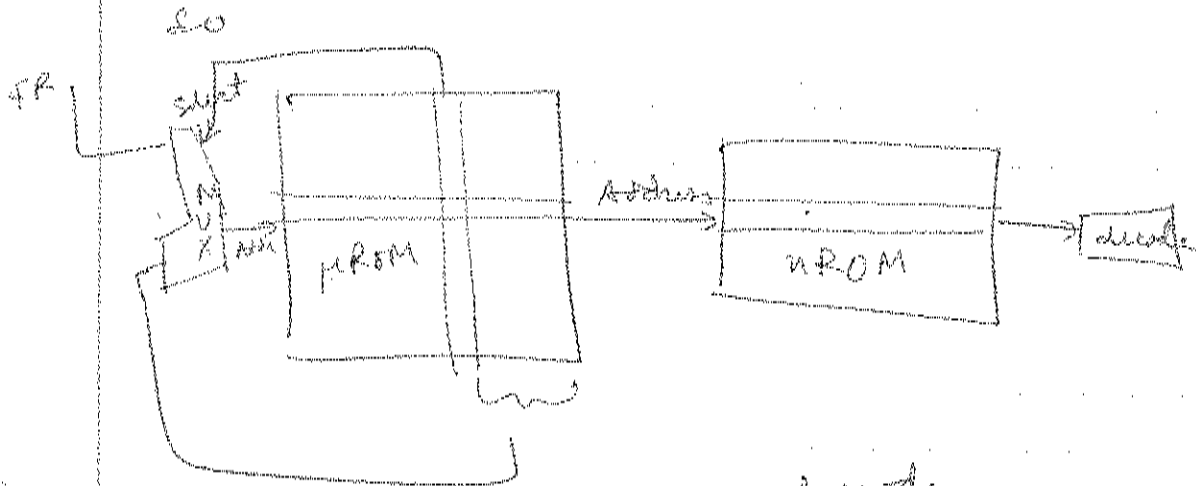
RTL	gate PC	gate RS	gate R	control	latch MAR	OP RD	OP RD	RD latch PC	latch IR	latch MAR	next address from DR	next pinout
① $MAR \leftarrow PC$ $PC \leftarrow PC + 1$ assert $R_0$	1				1	1		1				000010
② branch $IR \leftarrow data\ bus$									1			000011
③ branch ( $IR_{31..26}$ )											1	~
④ $R[Li] \leftarrow R[Rs] + R[Rd]$ branch (?)		1	1			1						000001



code: wasteful, lot of zeroes

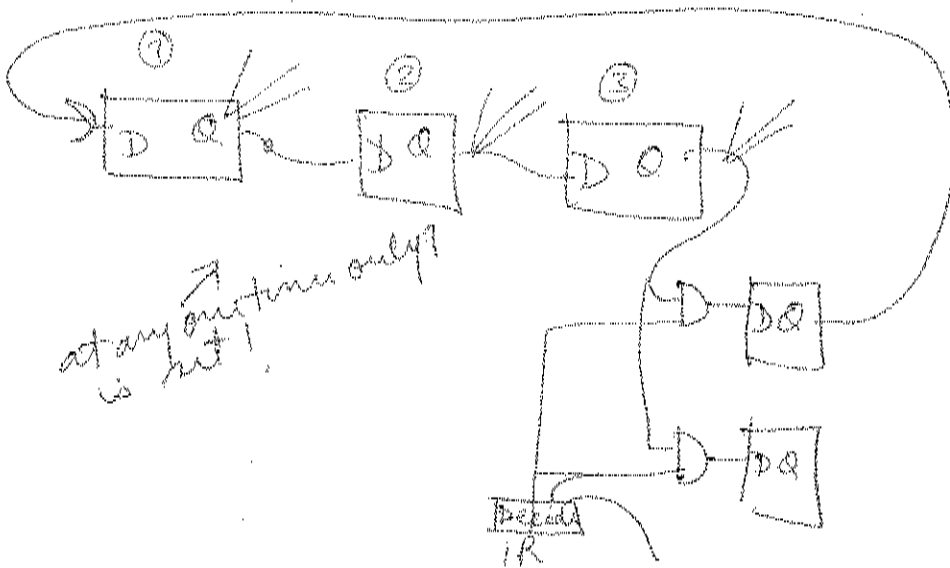
Makes sense to collapse ALU ops down  $\rightarrow$  decoder cctry

can narrow further  
 have a separate line of code for <sup>encoded</sup> AND, +, OR etc  
 even though ~~everything~~ everything same



2 unit mode

Hardwired control



at any one time only?  
 is hit!

for same above arch.

