

# EE382m: Homework 3

## Design Verification

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1. Suppose the netlist of Figure 1 was combinational. Describe how would you use BDDs to verify that for each output  $\mathbf{x}$  that has a majority of 1s, there exists some input  $\mathbf{u}$  whose parity is even, such that  $N$  outputs  $\mathbf{x}$  on input  $\mathbf{u}$ .

**8 marks**

2. Two CTL formulas  $f, g$  are said to be *logically equivalent* if for every netlist  $\eta$  and every state  $s$  in  $\eta$ , it is the case that  $\eta, s \models f$  if and only if  $\eta, s \models g$ .

Rewrite the following CTL formulas as logically equivalent formulas not involving the operators  $EF, AX, AG, AF, AU$ ; justify your transformations.

- (a)  $EFAGf$
- (b)  $AXAXf$
- (c)  $AGf$

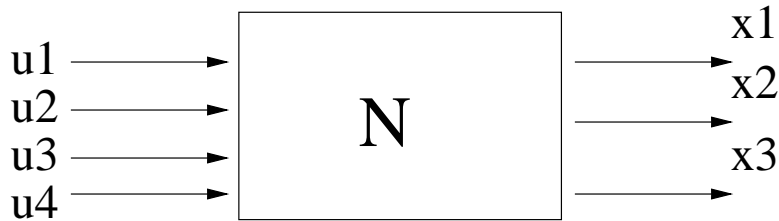


Figure 1: A netlist.

- (d)  $AfUExg$
- (e)  $AFAGf$

**10 marks**

3. Write CTL formula that express the following properties for the netlist in Figure 1.
- (a) the outputs  $x_1$  and  $x_2$  are always complementary
  - (b) whenever  $x_1$  high, sometime in the strict future  $x_2$  becomes high
  - (c) it is impossible for  $x_1$  to go high and remain high forever
  - (d) there is no path on which  $x_1$  eventually becomes low and stays low
  - (e) whenever  $x_1$  is high within two or three time steps it becomes low

**10 marks**

4. Let  $\phi$  be an arbitrary CTL formula. Prove that, in general, there does not exist a formula  $\psi$  such that, in any netlist, the set of states satisfying  $\psi$  is exactly the set of states which can be reached in one step from the states satisfying  $\phi$ . (In essence, this shows the “image” operation can not be expressed using CTL.)

**12 marks**

5. Let  $S$  be the set of states in a given a Moore netlist (i.e., one in which every output is driven only by latches). A relation  $\mathcal{E} \subset S \times S$  is said to be a *bisimulation* if the following conditions are true for all states  $s$  and  $t$  in  $S$ :
- (a) the output at state  $s$  is equal to the output at state  $t$ , and
  - (b) for every state  $s'$  that  $s$  can make a transition to, there is a state  $t'$  that  $t$  can make a transition to such that  $(s', t') \in \mathcal{E}$ , and
  - (c) for every state  $t'$  that  $t$  can make a transition to, there is a state  $s'$  that  $s$  can make a transition to such that  $(t', s') \in \mathcal{E}$ .

Two states  $s$  and  $t$  are defined to be *bisimilar* if there is a bisimulation relation  $\mathcal{D}$  such that  $(s, t) \in \mathcal{D}$ .

Prove that if states  $s$  and  $t$  are bisimilar, then for any CTL formula  $\phi$ , state  $s \models \phi$  if and only if  $t \models \phi$ .<sup>1</sup>

*Hints:* Use induction — take as your induction hypothesis that  $s$  and  $t$  agree on all CTL formula of length less than or equal to  $n$ . Also, the fact that  $EF, AX, AU, AF, AG$  operations can be expressed in terms of  $EX, EG, EU$  (cf. Problem 2 in this homework) can be used to simplify the proof.

**15 marks**

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<sup>1</sup>The converse is also true, but the proof is much more involved. You may want to compare the notion of bisimulation with state equivalence, as defined in Homework 1.