Problem Set 1
Due: September 15, 2003 in class

Instructions:

The homework you turn in should be your own work. You may discuss problems
with your colleagues. But though you may share ideas and possible approaches to
solving problems, you may not share actual solutions.

Do not forget to write your **TA's name and your discussion session time** on
the homework you turn in.

Please **staple** your homework.

Note: Numbers in parentheses are problem numbers from text.

1. (2.3)
   a. Assume that there are about 400 students in your class. If every student is to
      be assigned a unique bit pattern, what is the minimum number of bits
      required to do this?
   b. How many more students can be admitted to the class without requiring
      additional bits for each student's unique bit pattern?

2. (2.5)
   Write the representations of 7 and -7 in 4-bit 1's complement, 4-bit signed
   magnitude and 4-bit 2's complement integers.

3. (2.8)
   a. What is the largest positive number one can represent in an 8-bit 2's
      complement? Write your result in binary and decimal.
   b. What is the greatest magnitude negative number one can represent in 8-bit
      2's complement? Write your result in binary and decimal.
c. What is the largest positive number one can represent in \( n \)-bit 2's complement?

d. What is the greatest magnitude negative number one can represent in \( n \)-bit 2's complement?

4. (2.9)
How many bits are needed to represent Avogadro's number \( (6.02 \times 10^{23}) \) in 2's complement binary representation?
\[
(6.02 \times 10^{23} = 602,000,000,000,000,000,000,000,000)
\]

5. (2.11)
Convert these decimal numbers to 8-bit 2's complement binary numbers.

a. 102
b. -128

6. (2.12)
If the last digit of a 2’s complement binary number is 0, then the number is even. If the last two digits of a 2’s complement binary number are 00 (e.g., the binary number \( 01100 \)), what does that tell you about that number?

7. (2.24)
Give two 16-bit 2’s complement integers such that their sum causes an overflow.

8. (2.34)
Compute the following.

a. NOT (1011) OR NOT (1100)

b. NOT (1000 AND (1100 OR 0101))

9. (2.42)
A computer programmer wrote a program that adds two numbers. He/she ran the program and observed that when 5 is added to 8, the result is the character ‘m’. Explain why the program is behaving erroneously.

10. (2.49)
Perform the following additions. The corresponding 16-bit numbers are in 2's complement notation. Write your answers in hexadecimal.

a. \( \text{x025B} + \text{x26DE} \)
b. \( \text{xA397} + \text{xA35D} \)
c. What else can you say about the answer to part b?

11. (2.50)
Fill in the truth table for the equations given below. The first line is done as an example.
Q₁ = NOT(A AND B)
Q₂ = NOT(NOT(A) AND NOT(B))

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Q₁</th>
<th>Q₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Express Q₂ another way.

12. (2.55)
Consider the base-4 number system. We will refer to unsigned base-4 numbers as quad numbers. Quad digits can be 0, 1, 2 and 3.
a. What is the largest unsigned decimal value one can represent with 3 quad digits?
b. What is the binary representation of the quad number 123.3?
c. Add the two quad numbers:

    123
+ 123