Course notes for EE394V Restructured Electricity Markets: Market Power

Ross Baldick

1

Background

- This review of background material is based, in part, on:
  - Part 1 of *Power System Economics*, by Steven Stoft, and
  - EE394V, Locational Marginal Pricing.

Outline

(i) Restructured electricity markets,
(ii) Variable operating costs,
(iii) Marginal costs,
(iv) Price-taking assumption,
(v) Offer-based economic dispatch,
(vi) Example,
(vii) Fixed demand and pivotal offers,
(viii) Computational issues,
(ix) Forward contracts,
(x) Summary,
(xi) Homework exercises:
   - Due Tuesday, February 9 by 10pm, via email, and
   - each Tuesday thereafter.

1.1 Restructured electricity markets

- Until the 1990s, most electricity in North America was supplied by *regulated utilities* that each had a franchise service area:
  - utility had right to sell electricity at retail in franchise service area,
  - prices for delivered energy to retail customers set by state body and were chosen to allow an opportunity to earn a mandated level of profit,
  - some wholesale trade between utilities and other entities, but competition for sales to retail customers essentially prohibited,
  - "cost-of-service" regulation of monopoly.
- Rationale for such arrangements was that a single firm could provide services more cheaply than multiple competing firms because of *economies of scale*:
  - cost of one firm providing all services lower than
  - sum of costs of multiple firms providing services.
- An industry where it is cheapest for a single firm to provide all services is called a "natural monopoly."

Restructured electricity markets, continued

- Typical criticisms of such arrangements include lack of incentives to innovate and lack of incentives to keep total capital and operating costs at minimum.
- Large-scale transmission networks, among other things, have allowed for a different arrangement where multiple owners of generators compete to sell their energy in a restructured electric market:
  - geographical scope of restructured market, (for example ERCOT,) covers what were the franchise service areas of multiple regulated utilities (TXU, Houston Lighting and Power, etc),
  - transmission and distribution functions remain regulated under assumption that building and operating transmission and distribution are still natural monopolies, typically consolidating the transmission operations of multiple utilities under an Independent System Operator (ISO, for example ERCOT),
  - fierce competition amongst generation firms provides stronger incentives to minimize costs and (arguably) incentives for innovation,
  - but, not clear that competition amongst firms is fierce!

Restructured electricity markets, continued

- Purpose of this course is to understand competitive interactions between generation firms in restructured electricity markets:
  - interaction between several, but not a huge number of, firms that are assumed to maximize their profits over the choices they can make,
  - emphasize unique aspects of electricity, such as transmission and the need to balance supply and demand collectively, which significantly affect operation of electricity markets.
- For reasons that will become clearer, we will usually neglect issues related to limited competition between *purchasers* of energy.

1.2 Variable operating costs

- To produce electricity, generators incur "operating costs," which include fuel and other costs, such as maintenance:
  - these costs would be avoided if the generator were out of service.
- To emphasize the distinction between:
  - those operating costs that depend on the *level* of production, and
  - "fixed" costs such as construction costs, "fixed" maintenance costs, and operating costs such as start-up costs that are not directly related to production level,

  we will say "variable operating costs."
- To emphasize the distinction between:
  - the *total* variable operating costs of producing at a particular level,
  - the *derivative* of the variable operating costs, and
  - the *average* of the variable operating costs (variable operating costs divided by production),

  we sometimes say "total variable operating costs" for "variable operating costs."
- Phrases used to describe these concepts are not completely uniform.

Variable operating costs, continued

- Assume each generator i has total variable operating costs $c_i : \mathbb{R} \rightarrow \mathbb{R}$ that specify the cost of operating (in dollars per hour) versus generation level.
  - Recall that "$c_i : \mathbb{R} \rightarrow \mathbb{R}$" is shorthand for saying that c_i is a function that takes a real-valued argument (specified by the set $\mathbb{R}$) and returns a real value (also specified by the set $\mathbb{R}$).
  - In particular, $c_i(Q_i)$ is the cost per hour of operating at the production level Q_i.
  - We will follow the "economics" convention of writing q or Q for quantity and p or P for price (not reactive and real power–we will stay away from reactive power prices!)
  - We will also follow the economics convention of graphing prices on the vertical axis and quantity on the horizontal axis, whichever is the "independent" variable in a particular formulation.
- c_i may only be useful in some operating range, such as $[\underline{q}_i, \overline{q}_i]$.
  - Recall that the notation $[\underline{q}_i, \overline{q}_i]$ means the closed interval between some lower capacity limit $\underline{q}_i$ and some upper capacity limit $\overline{q}_i$.

Variable operating costs, continued

- We typically assume that this variable operating cost function is *convex* or "bowl-shaped" on $[\underline{q}_i, \overline{q}_i]$, although this often simplifies reality.

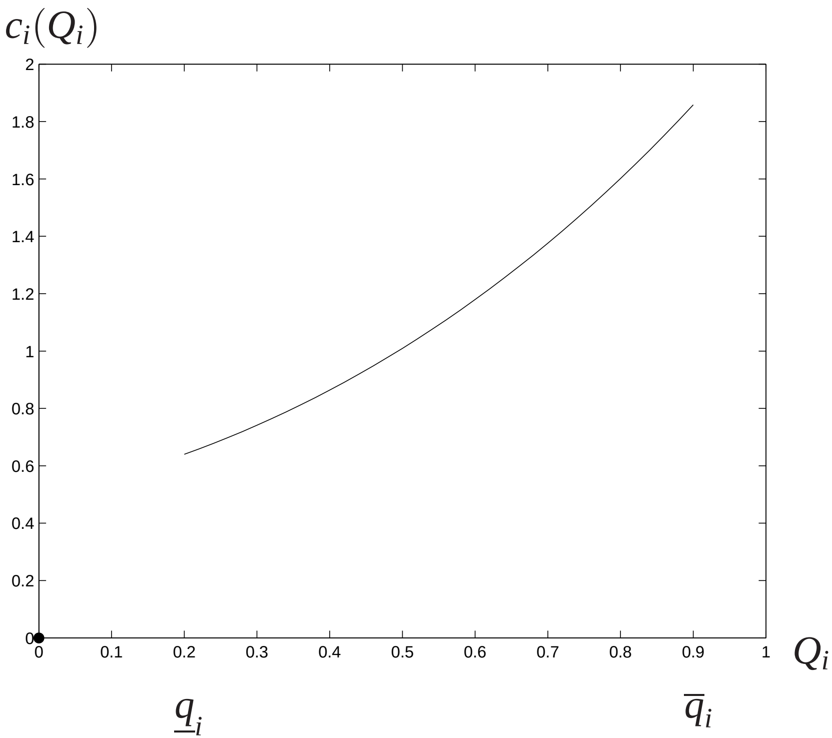

Fig. 1.1. Example of convex total variable operating costs $c_i(Q_i)$ versus production Q_i for a generator.

Variable operating costs, continued

- Convex functions have a number of desirable properties.
- For example, convex functions are differentiable *almost everywhere*; that is, except at a finite number or countably infinite number of points.

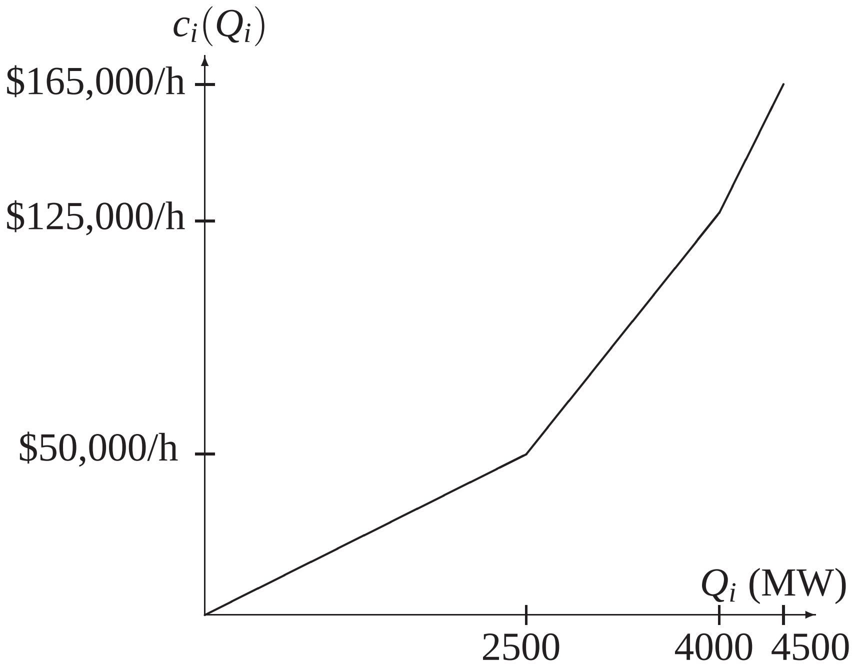

Fig. 1.2. Second example of convex total variable operating costs $c_i(Q_i)$ versus production Q_i for a generator.

Variable operating costs, continued

- Typically, convex functions are piece-wise continuously differentiable.
- The derivative of the total variable costs function from Figure 1.2 is well-defined over the open interval $(0,4500)$ except at $Q_i = 2500, 4000$.

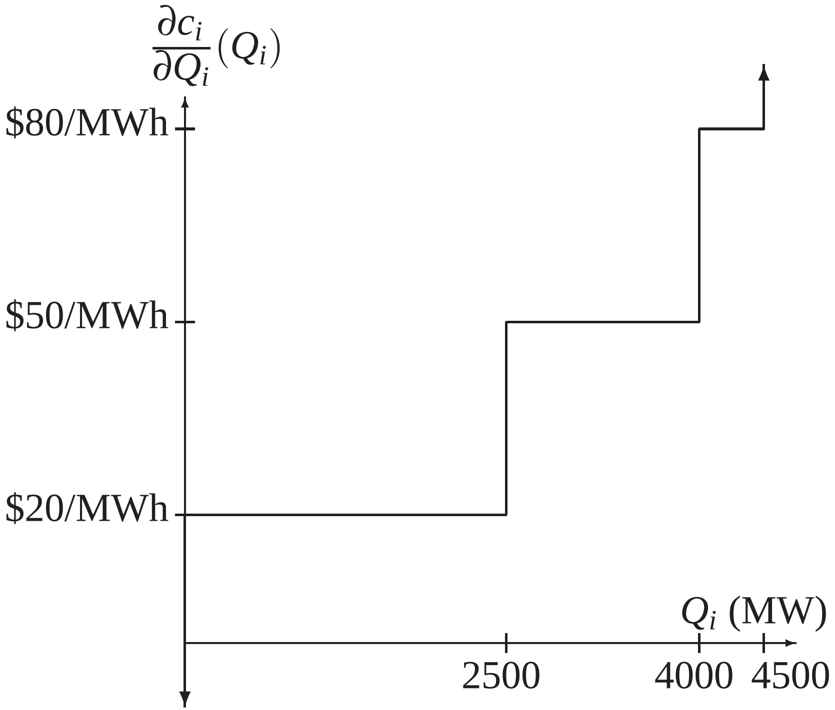

Fig. 1.3. Derivative of convex variable operating costs from Figure 1.2.

1.3 Marginal costs

- The derivative of total variable operating costs is called the *marginal costs*, in dollars per megawatt-hour.
- At the points of discontinuity of the marginal costs, the value jumps up from the *left-hand marginal cost* to the *right-hand marginal cost.*
- We also define the *marginal cost range* to be the interval between the left-hand and right-hand marginal cost.
- A property of a convex function is that its derivative is a *non-decreasing* function:
  - marginal costs are non-decreasing,
  - as shown in Figure 1.3.

Marginal costs, continued

- At each point of differentiability of total variable operating costs:
  - the left-hand and right-hand marginal costs are the same,
  - the marginal costs are continuous, and
  - the marginal cost range is a single point, namely equal to the derivative.
- The left-hand marginal cost, evaluated at production Q_i, shows the savings from reducing production below Q_i.
- The right-hand marginal cost, evaluated at production Q_i, shows the extra cost of increasing production above Q_i.
- At maximum production $\overline{q}_i$, we can think of the right-hand marginal cost as being infinite:
  - as suggested by the upward arrow at quantity 4500 MW in the marginal costs in Figure 1.3.
- At minimum production $\underline{q}_i$, we can think of the left-hand marginal cost as being negative infinite:
  - as suggested by the downward arrow at quantity 0 MW in the marginal costs in Figure 1.3.

Marginal costs, continued

- In essentially all cases, from a practical perspective, we can imagine "smoothing out" the jumps in the marginal cost curve by assuming a thin interval of production over which marginal cost raises rapidly:
  - Over a few megawatts, say, the marginal cost increases from the left-hand to the right-hand marginal cost.
  - At maximum production, the marginal cost rises from the left-hand marginal cost towards infinity.
- Results with the smoothed marginal cost curve will almost always be essentially the same as with the discontinuous marginal cost curve and are often easier to interpret.

1.4 Price-taking or competitive behavior

- A generator is a *price-taker* in the economics sense or *behaves competitively* if, given a price, it sets its production (or, arranges that its production is set) so that the marginal cost range contains the price:
  - this use of the phrase “price-taker” in an economics sense to behave competitively is *different* to a typical use of this phrase in electricity markets where, for example, a generator schedules its production and “takes” whatever the price happens to be.
  - We will see that a price-taker in the “electricity markets” sense may not be behaving competitively!
  - We will be careful to distinguish these two uses of the phrase “price-taker”!

Price-taking behavior, continued

- If the marginal costs are continuous then a price-taker in the economics sense (that is, a generator behaving competitively) chooses production so that marginal cost equals price.
- With the smoothed-out marginal cost curve interpretation, a price-taker in the economics sense always chooses production so that marginal cost equals price:
  - However, note that at the maximum production level, the smoothed marginal cost rises above the left-hand marginal cost.
- With piece-wise continuously differentiable marginal costs, a price-taker in the economics sense always chooses production so that the marginal cost range contains the price.

Price-taking behavior, continued

- An analogous definition applies to load:
  - the focus is on the *benefits* of consumption as a function of demand $b_k(D_k)$ and on the derivative of benefits, which is called the *willingness-to-pay*.
  - A price taking load chooses demand such that willingness-to-pay equals price (or willingness-to-pay range contains the price).
- In the context of benefits of consumption, the notion of a "fixed" demand (that is, not associated with any finite willingness-to-pay) is meaningless since it implies an infinite desire for consumption!
- We will usually think of demands as having either:
  - an explicit willingness-to-pay as specified by its "bid," or
  - an implicit proxy to the *dis*-benefit of involuntary curtailment, which is called the "value of lost load" (VOLL), and which is usually specified in practice by a price or offer "cap."
- We will see that a fixed demand independent of price, or having a very high willingness-to-pay, is also problematic in the context of market power!

1.5 Offer-based economic dispatch

- We assume an electricity market where owners of generators make offers to sell electricity and representatives of demand make bids to buy electricity to an independent system operator (ISO):
  - an offer for generator i is a function $p_i : \mathbb{R} \to \mathbb{R}$ from quantity to price that specifies, for each quantity Q_i produced, the minimum price $p_i(Q_i)$ to produce at that quantity;
  - a bid for demand k is a function $w_k : \mathbb{R} \to \mathbb{R}$ from quantity to price that specifies, for each quantity D_k consumed, the maximum willingness-to-pay $w_k(D_k)$ for that quantity,
  - an offer or bid cap limits the highest allowable value of p_i or w_k:
    - currently $2250/MWh in ERCOT,
  - an offer or bid floor limits the lowest allowable value of p_i or w_k.

Offer-based economic dispatch, continued

- The ISO seeks a price $P^\star$ to "match" supply and demand.
- That is, the ISO finds quantities $Q_i^\star$ of production for each generator and quantities $D_k^\star$ for each demand such that:
  - total generation equals total demand: $\sum_{\text{generators } i} Q_i^\star = \sum_{\text{demands } k} D_k^\star$,
  - for each generator i:
    - $Q_i^\star$ satisfies $p_i(Q_i^\star) \leq P^\star \leq p_i(Q_i^+)$, where Q_i^+ is any quantity larger than $Q_i^\star$,
    - so that generator i is willing to produce $Q_i^\star$ for price $P^\star$, and
    - would not prefer (based on its offer) to be producing more at this price than $Q_i^\star$,
  - for each demand k:
    - $D_k^\star$ satisfies $w_k(D_k^\star) \geq P^\star \geq w_k(D_k^+)$, where D_k^+ is any quantity larger than $D_k^\star$,
    - so that demand k is willing to pay $P^\star$ to consume $D_k^\star$, and
    - would not prefer (based on its bid) to be consuming more at this price than $D_k^\star$.

Offer-based economic dispatch, continued

- If we assume that the bids and the offers are continuous then these conditions simplify:
  - total generation equals total demand: $\sum_{\text{generators } i} Q_i^\star = \sum_{\text{demands } k} D_k^\star$,
  - for each generator i, $Q_i^\star$ satisfies $p_i(Q_i^\star) = P^\star$, and
  - for each demand k $D_k^\star$ satisfies $w_k(D_k^\star) = P^\star$.
- The quantities $Q_i^\star$ and $D_k^\star$ and price $P^\star$ are calculated by the offer-based economic dispatch algorithm:
  - other issues such as reserves can also be incorporated, resulting in prices and quantities for reserves etc.
  - other issues such as start-up costs can also be incorporated, but this greatly complicates the economic dispatch problem to being a "unit commitment" problem, which is in general "non-convex."
- We will focus on issues related to market power in energy and will not discuss in detail market power in ancillary services markets nor in the context of start-up costs.

Offer-based economic dispatch, continued

- A price that matches supply and demand is called *market clearing* or *clears the market*:
  - that is, supply equals demand, given the offers and bids.
- The process can be described as a type of "auction:"
  - the ISO is an "auctioneer," and
  - many results from "auction theory" in economics can be used to help analyze electricity markets.
- Not all ISO rules result in market clearing prices:
  - For example, the ISO may always use the highest offer price (the "marginal offer price,") even if demand is curtailed or if a higher demand bid willingness-to-pay is actually the market clearing price.
  - ISO rules that deviate from market clearing prices are usually based on a desire to "keep prices low."
  - We will, however, usually assume that the ISO sets a market clearing price.

Offer-based economic dispatch, continued

- Suppose that a generator is a price-taker in the economics sense (that is, behaves competitively) in a market where market clearing prices are used.
- What offer should it make into such a market?
- By definition, being a price-taker in the economics sense (that is, behaving competitively) means arranging for generation such that the marginal cost range contains the price.

Offer-based economic dispatch, continued

- Suppose that, for each quantity Q_i, the generator sets its offer $p_i(Q_i)$ equal to its marginal cost $\frac{\partial c_i}{\partial Q_i}(Q_i)$ at Q_i:
  - if there is a jump in the marginal cost at quantity Q_i, then set $p_i(Q_i)$ equal to the left-hand marginal cost at Q_i.
- Then, since the price $P^\star$ satisfies $p_i(Q_i^\star) \leq P^\star \leq p_i(Q_i^{\star +})$ at the quantity $Q_i^\star$ desired by the ISO, the generator will be acting a price-taker in the economics sense by generating at this level (that is, behaving competitively) since the marginal cost range will contain the price.
- We call this a price-taking or competitive offer.
- Similarly, a price-taking or competitive bid is where the bid equals the (left-hand) "marginal benefit" or willingness-to-pay of consumption:
  - we will usually assume that demand is acting as a price-taker in the economics sense, with bid equal to the marginal benefit.

Offer-based economic dispatch, continued

- What is special about price-taking (that is, competitive) behavior?
- Benefits of consumption minus costs of production is called the "net surplus" or sometimes the "total surplus."
- With price-taking (that is, competitive) offers and bids, the matching of supply and demand corresponds to maximizing the net surplus.
- To see this, consider the problem of maximizing the net surplus:

$$\max_{Q_i, D_k} \left\{ \sum b_k(D_k) - \sum c_i(Q_i) \middle| \sum Q_i = \sum D_k \right\}.$$

- Assuming the c_i and b_k are differentiable and ignoring capacity constraints, the first-order necessary conditions for optimizing this problem are that there is a price $P^\star$ such that:

$$\begin{aligned} \forall i, k, \frac{\partial c_i}{\partial Q_i}(Q_i^\star) &= \frac{\partial b_k}{\partial D_k}(D_k^\star) = P^\star, \\ \sum Q_i^\star &= \sum D_k^\star. \end{aligned}$$

- That is, matching of supply and demand with competitive offers and bids results in maximizing net surplus.

Offer-based economic dispatch, continued

- Similarly, with piece-wise continuously differentiable costs and benefits, maximizing net surplus involves a price $P^\star$ such that:
  - the marginal cost range of each generator contains the price $P^\star$,
  - the marginal benefit range of each demand contains the price $P^\star$, and
  - supply and demand is matched.
- That is, with competitive offers, clearing the market results in maximizing the surplus.
- Offer-based economic dispatch can be seen as an auction where a price is *sought* that:
  - clears the market, and
  - maximizes surplus, given that offers reflect marginal costs and bids reflect marginal benefits.
- This price is called the *competitive price* and is where the competitive offers and bids intersect.

Offer-based economic dispatch, continued

- The competitive price is often, but not always, equal to the highest accepted generation offer price, (which is called the “marginal offer.”)
- Sometimes, the competitive price is equal to the willingness-to-pay of demand, or, in the case of curtailment, equals a proxy to the willingness-to-pay, such as “value of lost load.”
- Unless demand actively bids its willingness-to-pay, it may be difficult to determine this willingness-to-pay:
  - difficult to determine the competitive price in the case of a “given” demand when generation capacity is limited.
- Traditional focus of electricity industry and, until recently, electricity markets, has been on meeting a given demand, assuming there is enough generation capacity available:
  - problematic in context of market power (prices may be above competitive), and
  - problematic in context of resource adequacy (prices may be below competitive and therefore not provide enough revenue to cover both operating costs and fixed costs).

Offer-based economic dispatch, continued

- The competitive price and the corresponding supply quantity is called the "competitive equilibrium."
- The basic pricing rule is that all demand pays and all generation is paid this price.
- Given the market clearing price $P^\star$ and competitive offers, the market clearing quantities maximize the profits ("producer surplus") of the generators and maximize the "consumer surplus" of the loads:
  - for each generator i, for the given price $P^\star$, the choice $Q_i^\star$ maximizes the producer surplus $P^\star Q_i - c_i(Q_i)$ over choices of Q_i, and
  - for each demand k, for the given price $P^\star$, the choice $D_k^\star$ maximizes the consumer surplus $b_k(D_k) - P^\star D_k$ over choices of D_k.
- That is, the price aligns the profit-maximizing incentives of demand and generation to be consistent with maximizing the net surplus.

Offer-based economic dispatch, continued

- When transmission constraints limit choices of generation, the prices will vary by bus, leading to *locational marginal pricing*.
  - prices vary by bus,
  - at any particular bus, all demand pays and all generation is paid the same price.

Offer-based economic dispatch, continued

- Why would a generator behave as a price-taker in the economics sense and submit price-taking (that is, competitive) offers?
  - If there are many competitors in the market then being a price-taker in the economics sense (that is, competitive) is profit maximizing!
  - If there are few competitors then price-taking (competitive behavior) is not profit maximizing and the generator has "market power."
- Market power is one reason why offer-based economic dispatch may fail to result in the competitive equilibrium:
  - Other reasons may include errors in market design (such as if the pricing rule does not result in market clearing prices),
  - Errors in market design may exacerbate market power.
- As mentioned, we will generally assume that the number of purchasers of energy is large enough so that they can be assumed to behave competitively:
  - that is, demand bids will be assumed equal to derivative of benefits,
  - but this may be violated when the ISO acts on behalf of demand collectively.

1.6 Example

- Suppose that there are three types of generators:
  - "baseload," ten units each of capacity 250 MW with marginal cost of \$20/MWh, total baseload capacity 2500 MW.
  - "intermediate," ten units each of capacity 150 MW with marginal cost of \$50/MWh, total intermediate capacity 1500 MW.
  - "peaking," ten units each of capacity 50 MW with marginal cost of \$80/MWh, total peaking capacity 500 MW.
- Total capacity is 4500 MW.
- Ignore start-up and min-load costs and variation of marginal cost with production for each type of generator.
- Assume that prices are chosen to clear the market and that the generators offer competitively.

Example, continued

- Competitive offers from the generators mean offers with price:
  - equal to marginal cost for quantities over the range from zero megawatts up to generation capacity, and
  - "infinite" price for quantities higher than capacity (or price equal to the maximum price allowed by the market rules, the "price cap,") and
  - "negative infinite" price for negative quantities.
- "Adding up" the 30 offers "horizontally" yields the "competitive supply" q^{c}.
- The inverse of q^{c} is the corresponding "industry marginal cost function" or "competitive offer" p^{c}:
  - marginal cost is $20/MWh for zero to 2500 MW,
  - marginal cost is $50/MWh for 2500 MW to 4000 MW,
  - marginal cost is $80/MWh for 4000 MW to 4500 MW,
  - "infinite" for higher quantities, and
  - "negative infinite" for negative quantities.
- Note the jumps in offer prices at 0 MW (from negative infinite offer price), 2500 MW, 4000 MW, and 4500 MW (to infinite offer price).

Example, continued

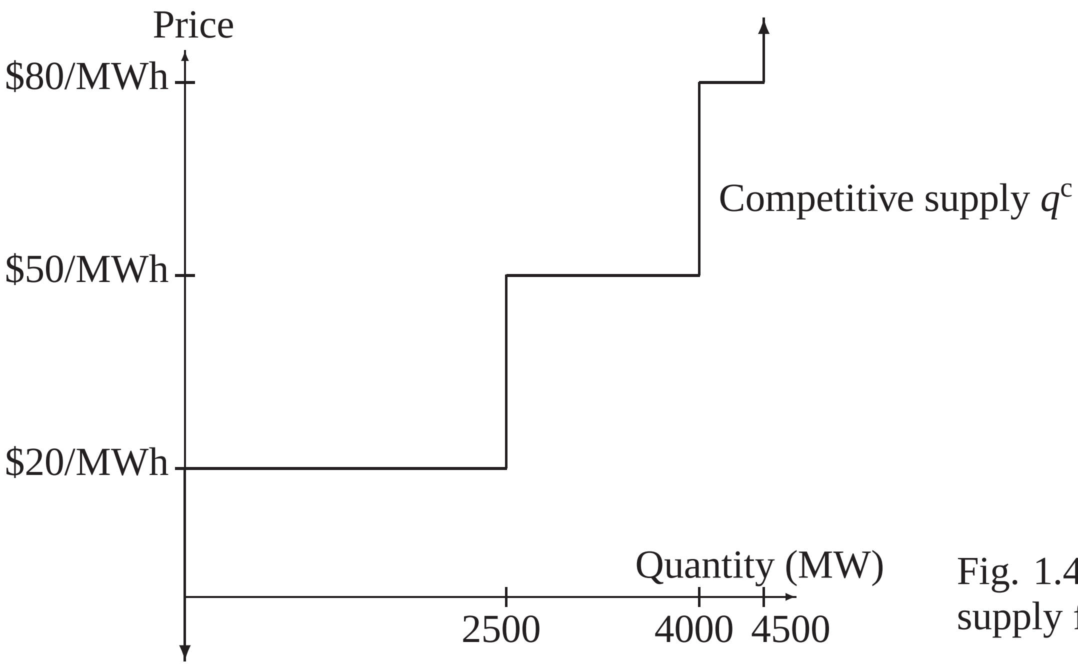

Fig. 1.4. Competitive supply for example.

Example, continued

- Suppose that the demand bid was 2800 MW with a willingness-to-pay of \$500/MWh. The price would be \$50/MWh and all 2800 MW of demand would be served, with 2500 MW generated by baseload and 300 MW by intermediate.

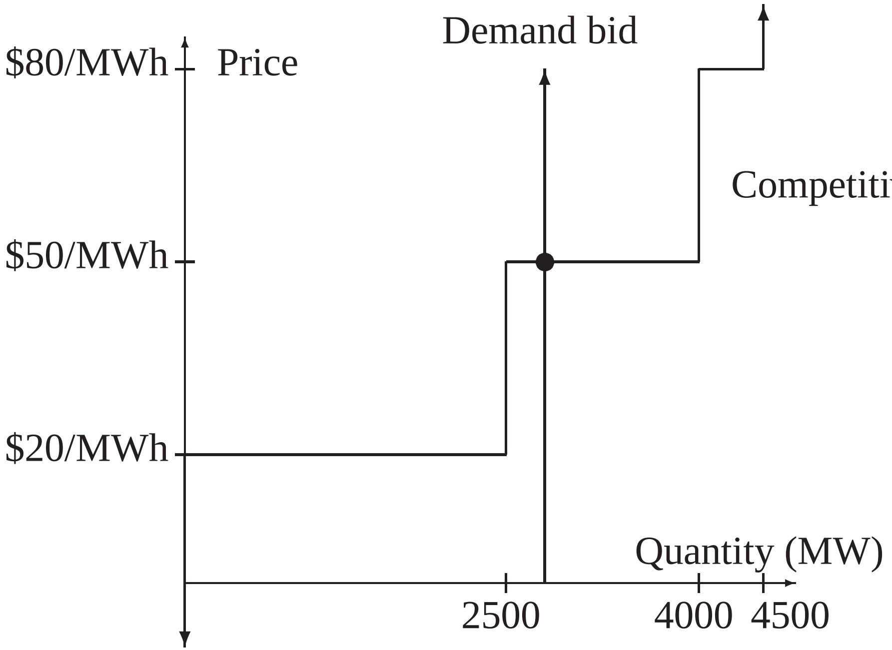

Fig. 1.5. Market clearing for 2800 MW demand for example.

Example, continued

- Suppose that the demand bid was 3500 MW with a willingness-to-pay of \$500/MWh.
  - The price would be \$50/MWh,
  - all 3500 MW of demand would be served,
  - with 2500 MW generated by baseload and 1000 MW by intermediate.
- Suppose that the demand bid was 4200 MW with a willingness-to-pay of \$500/MWh.
  - The price would be \$80/MWh,
  - all 4200 MW of demand would be served,
  - with 2500 MW generated by baseload, 1500 MW by intermediate, and 200 MW by peaking.

Example, continued

- Suppose that the demand bid was 4900 MW with a willingness-to-pay of \$500/MWh. The price would be \$500/MWh and only 4500 MW of demand would be served, with 2500 MW generated by baseload, 1500 MW by intermediate, and 500 MW by peaking.

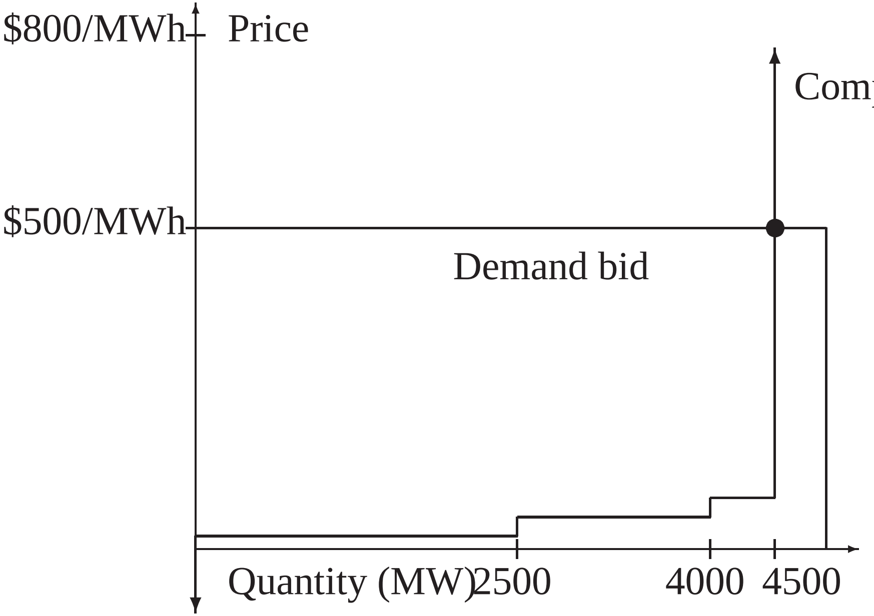

Fig. 1.6. Market clearing for 4900 MW demand for example.

Example, continued

- How about demands of 2500 MW, 4000 MW, 4500 MW, which corresponds to jumps in offers?
- In these cases, the supply and demand intersect in a vertical segment.
- That is, there is a range of market clearing prices corresponding to the segment of overlap:

  2500 MW Any price in the range \$20/MWh to \$50/MWh,
  4000 MW Any price in the range \$50/MWh to \$80/MWh,
  4500 MW Any price in the range \$80/MWh to \$500/MWh.

- In practice:
  - can specify a tie-breaking rule such as lowest price in range, or
  - computational implementation will pick a particular price.
- One tie-breaking rule involves a price based on a thought experiment:
  - suppose that, in addition to actual generators, we are given a costless MW of generation,
  - what is the value of that MW in terms of savings from reducing generation or increased benefits of serving more demand?
  - set price equal to this value, called the "marginal surplus."

1.7 Fixed demand and pivotal offers

1.7.1 Meeting fixed demand

- If we take the "traditional" view of meeting fixed demand, we face the problem that there may be insufficient demand to meet supply.
- Suppose that the demand was fixed at 4900 MW.

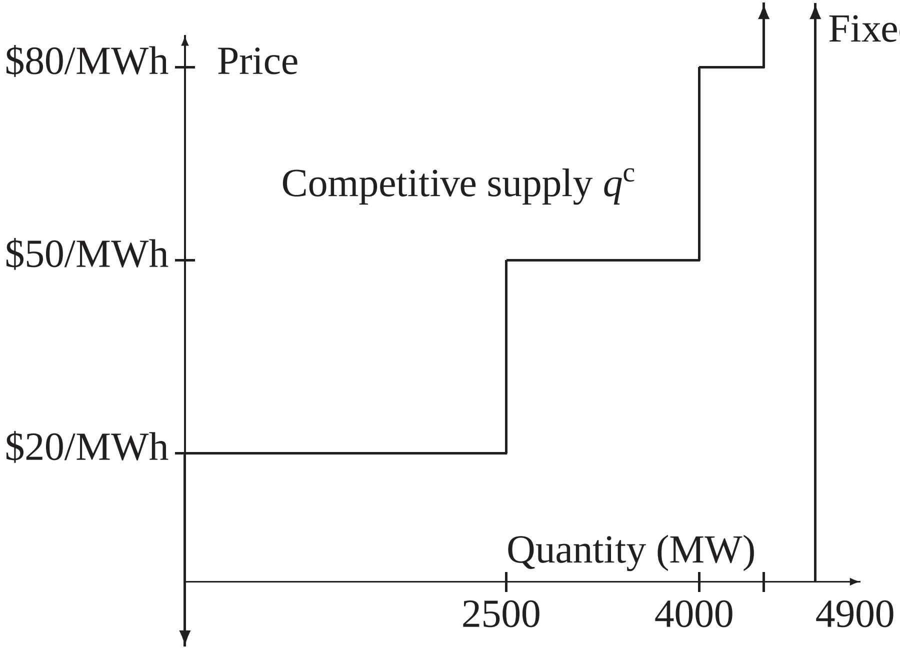

Fig. 1.7. Market does not clear with fixed 4900 MW demand.

Meeting fixed demand, continued

- Taken literally, this situation results in "involuntary curtailment" of 400 MW:
  - in practice, system operator may use deploy "reserves" to avoid curtailment.
- There is no price that equates supply and demand.
- There is no well-defined price in case of curtailment, unless we re-interpret "fixed demand" as really being a demand bid with a high willingness-to-pay, as in the previous example.

1.7.2 Market power implications

- Note that all offers are needed to meet more than 4500 MW of demand.
- Highest priced offer will always be accepted in this case.
- Example assumed competitive offers, but firm owning a peaking unit could increase profits by increasing offer price.
- Actually, any firm could increase profits for a fixed demand of more than 4500 MW by offering above $80/MWh, since some production from every generator is required.
- A firm whose capacity is needed to meet fixed demand is called "pivotal:"
  - in absence of market power mitigation, firm can offer at any price and be sure of being at least partially dispatched.

Market power implications, continued

- Assuming each generator is owned by a different firm:
  - for fixed demand more than 4450 MW, all firms are pivotal,
  - for fixed demand more than 4350 MW up to 4450 MW, all except peaking units are pivotal,
  - for fixed demand more than 4250 MW up to 4350 MW, baseload are pivotal,
  - for fixed demand less than or equal to 4250 MW, no firms are pivotal.
- If firms own multiple generators, then firms can be pivotal for demands below 4250 MW.
- Being pivotal is an extreme form of market power.
- More subtle forms of market power can occur even if no firm is pivotal.
- Defining, understanding, modeling, and other aspects of market power are the topics of this course.

1.8 Computational issues

- The dispatch and prices are solved as an optimization problem.
- The *objective* is the "revealed" benefits minus the "revealed" costs:

$$\begin{aligned}\text{revealed costs: } \tilde{c}_i(Q_i) &= \int_0^{Q_i} p_i(Q)dQ,\\ \text{revealed benefits: } \tilde{b}_k(D_k) &= \int_0^{D_k} w_k(D)dD,\end{aligned}$$

  where p_i and w_k is the corresponding offer and bid.
- The offered capacity and bid demand specifies *upper and lower bound constraints*:

$$\begin{aligned}\text{generation: } 0 \leq Q_i \leq \overline{Q}_i,\\ \text{demand: } 0 \leq D_k \leq \overline{D}_k,\end{aligned}$$

  where $\overline{Q}_i$ is the offered capacity and $\overline{D}_k$ is the quantity where willingness-to-pay falls to zero.
- The power balance *equality constraint* is $\sum_{\text{generators } i} Q_i = \sum_{\text{demands } k} D_k$.

Computational issues, continued

- (Revealed) surplus (or revealed benefits minus costs) is maximized subject to the upper and lower bound constraints and the power balance constraint:
  - with price taking (that is, competitive) offers and bids:
    - $\tilde{c}_k = c_k$, revealed costs equals actual costs, and
    - $\tilde{b}_k = b_k$, revealed benefits equals actual benefits,
    - so that the results of offer-based economic dispatch would maximize benefits minus costs.
- The *Lagrange multiplier* on the equality constraint in the solution is a market clearing price.

1.9 Forward contracts

- Because prices will vary with supply and demand, market participants are exposed to the risk of high or low prices.
- It is possible to "lock-in" an agreed price for an agreed quantity in a "forward contract."
- In an offer-based economic dispatch market, the most natural forward contract takes the form of a "contract for differences."

Forward contracts, continued

- For example, suppose that a generator and demand agree to a forward contract for 10 MW at \$50/MWh:
  - that is, the generator commits to providing 10 MW to demand at a net price paid by demand of \$50/MWh.
- If the market clearing price is actually P then the demand will pay 10 MW $\times$ P to the ISO:
  - To make the *net* payment by the demand equal to 10 MW $\times$ \$50/MWh, the demand should pay to the generator:

$$10 \text{ MW} \times (50 \text{ \$/MWh} - P).$$

  - the forward contract is implemented as an agreement by the demand to pay this amount, called a "contract for differences."
  - General form of payment under contract for differences:

$$(\text{Contract quantity}) \times ((\text{Contract price}) - P).$$

- There are many variations on this arrangement, but this description of contract for differences will provide a useful model of forward contracts.

1.10 Summary

(i) Restructured electricity markets,
(ii) Variable operating costs,
(iii) Marginal costs,
(iv) Price-taking assumption,
(v) Offer-based economic dispatch,
(vi) Example,
(vii) Fixed demand and pivotal offers,
(viii) Computational issues,
(ix) Forward contracts.

Homework exercise: Due Tuesday, February 9, by 10pm

- Break into ten groups, groups $g = 1, 2, 3, 4, 5, \ldots, 10$, of approximately three people each.
- Each group will be assigned a portfolio of three generators.
- The task for each group is to find offers that maximize the operating profit (revenue minus generation costs) for the group over a day's operation.

Types of generators

- Suppose that types of generators are as in the previous example:
  - baseload, capacity 250 MW with marginal cost of $20/MWh.
  - intermediate, capacity 150 MW with marginal cost of $50/MWh.
  - peaking, capacity 50 MW with marginal cost of $80/MWh.
- Ignore start-up and min-load costs and variation of marginal cost with production.
- Groups have different mixes of generation:

  1, 2 three baseload units,
  3, 4 three intermediate units,
  5, 6 three peaking units,
  7, 8, 9, 10 one of each type of unit.
- So there are ten units of each type in total, as in the previous example.

Demand

- There are three pricing intervals $t = 1, 2, 3$, each of eight hours duration, with demand:

  (i) 2800 MW,
  (ii) 3500 MW, and
  (iii) 4200 MW,

  respectively, with willingness-to-pay of \$500/MWh.

Prices with competitive offers

- From the previous example, we know that if the generators were offered competitively at marginal cost then the prices would be:

  (i) \$50/MWh for 2800 MW demand,
  (ii) \$50/MWh for 3500 MW demand, and
  (iii) \$80/MWh for 4200 MW demand.

Profit with competitive offers

- Operating profit *per MWh*, given competitive offers, for the generators:

  2800 MW demand, $50/MWh price

  - baseload ($50/MWh - $20/MWh) = $30/MWh;
  - intermediate ($50/MWh - $50/MWh) = $0/MWh or (not producing) $0/MWh;
  - peaking (not producing) $0/MWh;

  3500 MW demand, $50/MWh price

  - baseload ($50/MWh - $20/MWh) = $30/MWh;
  - intermediate ($50/MWh - $50/MWh) = $0/MWh or (not producing) $0/MWh;
  - peaking (not producing) $0/MWh;

  4200 MW demand, $80/MWh price

  - baseload ($80/MWh - $20/MWh) = $60/MWh;
  - intermediate ($80/MWh - $50/MWh) = $30/MWh;
  - peaking ($80/MWh - $80/MWh) = $0/MWh or (not producing) $0/MWh.

Profit with competitive offers

- To find operating profit over day, multiply profit per MWh times production times eight hours and sum across demand levels:

  2800 MW demand, \$50/MWh price

  – baseload \$30/MWh times 250 MW times 8 hours = \$60,000;
  – intermediate \$0/MWh times production times 8 hours = \$0;
  – peaking \$0/MWh times 0 MW times 8 hours = \$0;

  3500 MW demand, \$50/MWh price

  – baseload \$30/MWh times 250 MW times 8 hours = \$60,000;
  – intermediate \$0/MWh times production times 8 hours = \$0;
  – peaking \$0/MWh times 0 MW times 8 hours = \$0;

  4200 MW demand, \$80/MWh price

  – baseload \$60/MWh times 250 MW times 8 hours = \$120,000;
  – intermediate \$30/MWh times 150 MW times 8 hours = \$36,000;
  – peaking \$0/MWh times production times 8 hours = \$0.

Auction rules for offers

- Submit unit specific offers, one for each of three units $i = 1,2,3$ for each of three pricing intervals $t = 1,2,3$ in each portfolio.
- No "physical withholding," so maximum quantity must equal the capacity.
- For each unit i and pricing interval t specify an *affine* offer with an intercept a_{it} and slope b_{it}:

$$p_{it}(Q_{it}) = a_{it} + b_{it}Q_{it}.$$

- Recall that the interpretation of the offer is that if unit i is asked to produce Q_{it} then the price paid will be at least $p_{it}(Q_{it}) = a_{it} + b_{it}Q_{it}$.
- The offer must be non-decreasing:
  - that is, $b_{it} \geq 0$,
  - this means that the offer is the derivative of a convex function.
- A competitive offer would involve setting the offer equal to the constant marginal cost:
  - set the slope $b_{it} = 0$ for all i and t, and
  - set the intercept a_{it} equal to the marginal cost of production.

What offers would improve your profits?

- Your task is to increase profits over day compared to competitive offers.
- Each group will submit an email to baldick@ece.utexas.edu with subject "EE394V homework group g" where g is the group number $(1,\ldots,10)$, with the specification of parameters in the body of the email as follows:

$$a_{11}, b_{11}, \overline{q}_{11}, a_{12}, b_{12}, \overline{q}_{12}, a_{13}, b_{13}, \overline{q}_{13};$$
$$a_{21}, b_{21}, \overline{q}_{21}, a_{22}, b_{22}, \overline{q}_{22}, a_{23}, b_{23}, \overline{q}_{23};$$
$$a_{31}, b_{31}, \overline{q}_{31}, a_{32}, b_{32}, \overline{q}_{32}, a_{33}, b_{33}, \overline{q}_{33};$$

- That is, data is comma delimited, with a semi-colon at the end of each line.
- Unit 1 is specified on line 1, with the parameters for the three intervals appearing in succession.
- "Physical withholding" is prohibited, so $\overline{q}_{it}$ must match the capacity of the unit you are specifying.
- You are completely free to specify the a_{it} but $b_{it} \geq 0$:
  - No market monitor except for capacity!

Timeline

- Email must be received by 10pm on Tuesday, February 9.
- cc the email to everyone in the group, so that I know who is in each group.
- If you are late or if the format of your email deviates from the required specification, a competitive offer will be submitted instead.
- We will discuss in class on Thursday, February 11.
- Each week, will will follow a similar pattern, with offers due on Tuesday and discussion the following class or in the following week.

Homework exercise: Due Thursday, February 11, at beginning of class

- Assume that the costs c_i and benefits b_k are differentiable and that offers and bids are competitive.
- Assume that the market clearing conditions involve a price $P^\star$, generator quantities $Q_i^\star$, and demand quantities $D_k^\star$.
- Show that the market clearing quantities maximize the profits ("producer surplus") of the generators and maximize the "consumer surplus" of the loads.
- That is, show that:
  (i) for each generator i, for the given price $P^\star$, the choice $Q_i^\star$ maximizes the producer surplus $P^\star Q_i - c_i(Q_i)$ over choices of Q_i, and
  (ii) for each demand k, for the given price $P^\star$, the choice $D_k^\star$ maximizes the consumer surplus $b_k(D_k) - P^\star D_k$ over choices of D_k.
- So, if profits are maximized given the prices $P^\star$, why would any market participant choose to offer non-competitively?