Course notes for EE394V
Restructured Electricity Markets:
Market Power

Ross Baldick

4

Equilibrium analysis of market power

- This material is based on:
  - Ross Baldick, "Computing the Electricity Market Equilibrium: Uses of market equilibrium models." Editor, Xiao-Ping Zhang, *Computing the Electricity Market Equilibrium*, IEEE Press and Wiley, 2010.
  - Paul D. Klemperer and Margaret A. Meyer, "Supply Function Equilibria in Oligopoly Under Uncertainty," *Econometrica*, 57(6):1243–1277, November 1989.

– Richard Green and David M. Newbery, “Competition in the British Electricity Spot Market,” *Journal of Political Economy*, 100(5)929–953, October 1992.
– Richard Green, “Increasing Competition in the British Electricity Spot Market,” *The Journal of Industrial Economics*, XLIV(2):205–216, June 1996.
– Ross Baldick, “Electricity market equilibrium models: The effect of parameterization,” *IEEE Transactions on Power Systems*, 17(4):1170–1176, November 2002.
– Ross Baldick and William Hogan, “Capacity Constrained Supply Function Equilibrium Models of Electricity Markets: Stability, Non-decreasing Constraints, and Function Space Iterations,” University of California Energy Institute POWER Paper PWP-089, www.ucei.berkeley.edu/ucei/ PDF/pwp089.pdf, December 2001, Revised September 2002.
– Ross Baldick, Ryan Grant, and Edward P. Kahn, “Linear Supply Function Equilibrium: Generalizations, Application, and Limitations,” University of California Energy Institute POWER Paper PWP-078, www.ucei.berkeley.edu/ucei/ PDF/pwp078.pdf, August 2000.

– Alex Rudkevich, “Supply function equilibrium in poolco type power markets: Learning all the way,” TCA Technical Report Number 0699-1701, Tabors Caramanis and Associates, June 1999.
– Christopher J. Day and Derek W. Bunn, “Divestiture of Generation Assets in the Electricity Pool of England and Wales: A Computational Approach to Analyzing Market Power,” *Journal of Regulatory Economics*, 19(2):123–141, 2001.
– Edward J. Anderson and Xinmin Xu, “Finding Supply Function Equilibria with Asymmetric Firms,” Australian Graduate School of Management, The University of New South Wales, Sydney, NSW”, 2007.

Outline

(i) Introduction to equilibrium modelling,
(ii) Homework exercises.

4.1 Introduction to equilibrium modelling

(i) Introduction,
(ii) Model formulation,
(iii) Market operation and price formation,
(iv) Equilibrium and solution,
(v) Validity, uses, and limitations of equilibrium models,
(vi) Summary.

4.1.1 Introduction

- We have already seen examples of *economic equilibria*:
  - Cournot equilibrium, and
  - (in principle) equilibrium of group homework.
- These are examples of **Nash equilibrium**:
  - choice of strategic variables by each participant such that no participant can improve its profit by a *unilateral* change to the value of its strategic variables.
- Nash equilibrium is a basic unifying principle in models of interaction.
- We will discuss the formulation of Nash equilibrium models of electricity markets.
- As we will see, there are several difficulties in applying Nash equilibrium to electricity markets, including:

  (i) non-convexity of generator feasible operating region or of operating costs, non-concavity of generator profit function,
  (ii) inelastic demand,
  (iii) complexity of electricity market rules, and
  (iv) representation of regulatory intervention.

4.1.2 Model formulation

Consider the modelling of:

(i) Transmission network,
(ii) Generator cost function and operating characteristics,
(iii) Offer function,
(iv) Demand, and
(v) Uncertainty.

For each, we will distinguish the:

Physical model: a (notionally) exact model of the physical characteristics.
Commercial model: the model used in the actual market.
Economic model: the model used in the equilibrium formulation.

4.1.3 Transmission network

4.1.3.1 Physical model

- Kirchhoff's laws (non-linear equality constraints):
  - for example, at each bus in Figure 4.1 there is a non-linear equation on net power flow and a non-linear equation on net reactive power flow,
  - six equations in total.

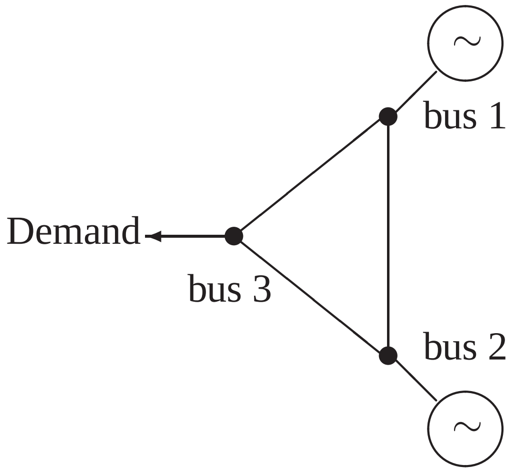

Fig. 4.1. Three bus, three line network.

Physical model, continued

- Thermal, voltage, and stability constraints (linear and non-linear inequality constraints):
  - for example, each line in Figure 4.1 has a thermal limit,
  - thermal limit is an inequality constraint expressed in terms of the voltage magnitudes and angles at the buses joined by the line, often approximated in terms of the power flow along the line.
- May also be constraints on line flows or on corridor flows that depend on particular generators being in-service.

4.1.3.2 Commercial network model

- Simplified transmission model used in market.
- Examples for Kirchhoff's laws:
  - linearization of non-linear equalities to obtain DC power flow,
  - buses aggregated into zones joined by equivalent lines ("commercially significant constraints" in ERCOT zonal market).
- Examples for inequality constraints:
  - limits on real power flow in DC power flow model,
  - limits on flow on commercially significant constraints.
- Discrepancies between commercial network model and physical model dealt with through "out-of-market" actions by independent system operator involving "side payments" to particular market participants.

Transmission network model, continued

4.1.3.3 Economic model

- Further simplified model used in equilibrium analysis,
- Examples:
  - ignore transmission constraints,
  - only consider pricing intervals when transmission constraints are not binding,
  - simplified network model,
  - ignore effect of “out-of-market” actions,
  - assume that market participants ignore the effect of their actions on transmission congestion or on congestion prices.
- Simplifies the profit maximization problem faced by generators.
  - For example, assuming that participants ignore the effect of their actions on congestion removes potential non-concavities from participant profit functions.
  - Therefore, first-order necessary conditions are sufficient for (simplified) profit maximum problem.

- The solid curve shows the actual profit function, π_k, which is non-concave and has two local maximizers, $s_k^\star$ and $s_k^{\star\star}$.
- The dashed curve extrapolates the functional form from negative values, which removes the non-concavity, but also eliminates the maximizer!

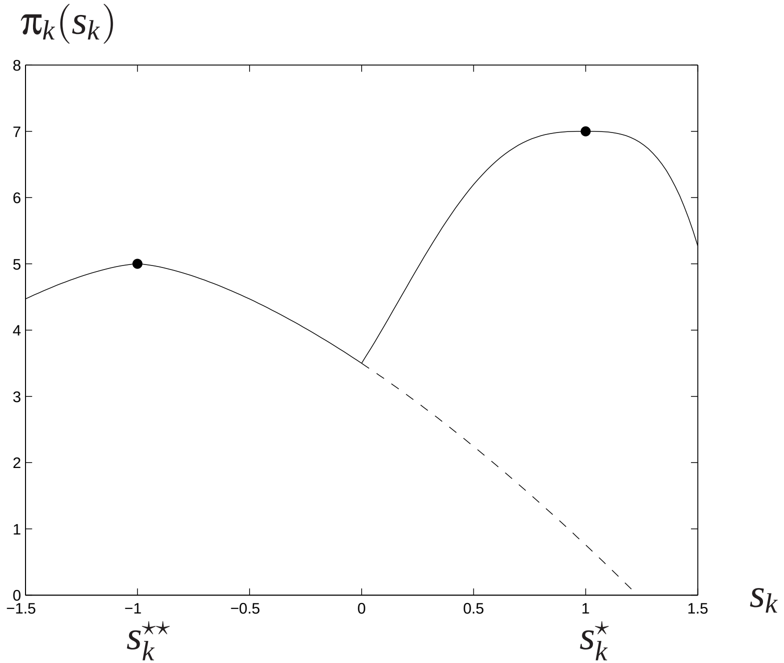

Fig. 4.2. Profit function having multiple maximizers indicated by the bullets ●.

4.1.4 Generator cost function and operating characteristics

4.1.4.1 Physical model

- Thermal generators have energy costs, unit commitment issues, reserves and reactive power capability, ramp and other constraints on operation.
- Energy cost functions for thermal generation are non-linear functions of production.
- Hydro generators have low, roughly constant, marginal costs, but are energy limited.

4.1.4.2 Economic model

- Portfolio models abstract from unit commitment and other issues:
  - ignore discrete variables associated with unit commitment decisions.
- May ignore the joint production of energy and ancillary services by a generator.

4.1.5 *Offer function*

4.1.5.1 Commercial model

- Complex (start-up costs etc) versus simple (energy) offer functions,
- Requirements to hold offers fixed over multiple intervals or fixed despite uncertainty in demand,
- Uncertainty managed through long-term forward contracts, day-ahead markets, real-time markets, and ancillary services.
- Installed capacity markets.

4.1.5.2 Economic model

- Choice of strategic variable may abstract from the commercial model:
  - quantity, as in Cournot model, does not literally represent market rules,
  - supply functions are closer in form to requirements of market rules,
  - number of free parameters in supply function model can have significant implications for the results of the model:
    - for example, too few free parameters in offers usually means that results are an artifact of assumed values of fixed parameters.
- Bilateral contract representation.

4.1.6 Demand

4.1.6.1 Physical model

- Temporal variation and uncertainty,
- Usually small (possibly zero) short-term price elasticity.

4.1.6.2 Commercial model

- Forecast of temporal variation,
- Uncertainty managed through long-term forward contracts, day-ahead markets, real-time markets, and ancillary services.

4.1.6.3 Economic model

- Forecast of temporal variation,
- Estimate of elasticity:
  - (i) May be calibration to observed behavior,
  - (ii) May be representation of “competitive” market participants.

4.1.7 Uncertainty

4.1.7.1 Physical model

- Demand, residual demand, fuel costs and availability, and equipment capacity are stochastic.

4.1.7.2 Commercial model

- Uncertainty in generator capacity and of demand is represented through:
  - day-ahead and real-time markets,
  - reserves and other ancillary services.

4.1.7.3 Economic model

- Many stochastic issues could be incorporated into the models.
- Uncertainty in demand is typically represented, but most other stochastic issues are typically not explicitly represented.
  - Consequently, effect on prices of stochastic issues may be absent.
- Real-time markets may not be explicitly modelled or may be modelled separately, ignoring the joint equilibrium between the markets.

4.1.8 Market operation and price formation

4.1.8.1 Physical model

- Lack of storage and limited elasticity of demand mean that action by ISO is necessary to match supply and demand through utilization of ancillary services.
- For example, real-time market deals with deviations from day-ahead market positions.

4.1.8.2 Commercial model

- Typical commercial model is a uniform clearing price market:
  - "pay-as-bid" is an alternative model.
- The role of ancillary services in matching supply and demand is not explicitly represented in, for example, the day-ahead energy market model,
- Ancillary services become critical under scarcity:
  - price formation under scarcity may not be explicitly specified,
  - may rely on difficult-to-model operator actions and post-market calculations.

Market operation and price formation, continued

4.1.8.3 Economic model

- Models crossing of supply and demand.
- Typically ignores ancillary services:
  - Typically require elastic demand at each bus to obtain well-defined prices when transmission constraints represented.
  - Model results may be extremely dependent on the specification of demand elasticity.
- Typically ignores unit commitment and installed capacity markets:
  - incorporating discrete variables into formulation is computationally difficult.

4.1.9 Nash equilibrium and solution

- A Nash equilibrium is set of participant offers such that no participant can improve its profit by unilaterally deviating from the offer within the market rules:
  - ignores collusion,
  - model of market operation and price formation determines profit.
- In homework problem, equilibrium if everyone achieved the *ex post* maximum profit.

Nash equilibrium and solution, continued

- Suppose strategic variables are s_k for participants $k = 1, \ldots, n$:
  - choice of strategic variable in model is reflection of offer rules and decision process of participant,
  - may only implicity reflect choices as in Cournot model.
- Suppose that profit to participant k is $\pi_k(s_k, s_{-k})$, where $s_{-k} = (s_\ell)_{\ell \neq k}$ is the collection of strategic variables of all the participants besides participant k.
- Then $(s_k^\star)_{k=1,\ldots,n}$ is a **pure strategy Nash equilibrium** if:

$$s_k^\star \in \arg\max_{s_k} \pi_k(s_k, s_{-k}^\star),$$

  where $s_{-k}^\star = (s_\ell^\star)_{\ell \neq k}$.
- Note that $\arg\max_{s_k} \pi_k(s_k, s_{-k})$ is the best response of firm k to the decisions s_{-k} of the other firms, as calculated by Hortaçsu and Puller.
- If we "graph" the best response $\arg\max_{s_k} \pi_k(s_k, s_{-k})$ versus s_{-k} for each k then the equilibrium $(s_k^\star)_{k=1,\ldots,n}$ is the intersection of these best response curves.
- "Single-shot" versus "repeated game."

Equilibrium solution methods

4.1.9.1 Analytical models

- Solve for equilibria analytically.
- Possible for some simple cases:
  - Single pricing interval with certain demand,
  - Cournot model (strategic variables are quantities) with no capacity constraints.
- The collection of first-order necessary conditions for maximizing each participant's profit can be solved:
  - as in homework with Cournot duopoly.
- Conditions for existence for unique equilibrium may be available.

4.1.9.2 Example

- Recall the "symmetric duopoly" with each firm $i = 1, 2$ having marginal cost function:

$$\forall Q_i, c_i'(Q_i) = 20 + 60 Q_i / 2500.$$

- Operating range $[0, \overline{Q}_i]$, where $\overline{Q}_i = 2500$ MW.
- The inverse demand is:

$$\begin{aligned} \forall Q, p^{\mathrm{d}}(Q) &= \max\{50 - (Q - 2800)/2, 0\}, \\ &= \max\{1450 - Q/2, 0\}, \\ &= 1450 - Q/2, \end{aligned}$$

- assuming that $1450 - Q/2 \geq 0$.
- Assume that the strategic variable is quantity.

- For firm $i = 1$, we have that the profit is:

$$\begin{aligned}\pi_1(Q_1, Q_2) &= (1450 - (Q_1 + Q_2)/2)Q_1 - c_1(Q_1), \\ &= -\frac{1}{2}Q_1^2 + \left(1450 - \frac{1}{2}Q_2\right)Q_1 - c_1(Q_1).\end{aligned}$$

- Firm $i = 1$ can choose Q_1, but accepts as fixed the value Q_2 (whatever it might actually be).
- Differentiating π_1 with respect to Q_1 and setting equal to zero to maximize profit, we obtain:

$$\begin{aligned}0 &= \frac{\partial \pi_1}{\partial Q_1}(Q_1, Q_2), \\ &= -Q_1 - \frac{1}{2}Q_2 + 1450 - \left(20 + \frac{60}{2500}Q_1\right).\end{aligned}$$

- That is:

$$1.024Q_1 + 0.5Q_2 = 1430. \tag{4.1}$$

- Similarly, for firm $i = 2$, we have that:

$$\begin{aligned}\pi_2(Q_2, Q_1) &= (1450 - (Q_1 + Q_2)/2)Q_2 - c_2(Q_2), \\ &= -\frac{1}{2}Q_2^2 + \left(1450 - \frac{1}{2}Q_1\right)Q_2 - c_2(Q_2).\end{aligned}$$

- Firm $i = 2$ can choose Q_2, but accepts as fixed the value Q_1 (whatever it might actually be).
- Differentiating π_2 with respect to Q_2 and setting equal to zero to maximize profit, we obtain:

$$\begin{aligned}0 &= \frac{\partial \pi_2}{\partial \partial Q_2}(Q_2, Q_1), \\ &= -Q_2 - \frac{1}{2}Q_1 + 1450 - \left(20 + \frac{60}{2500}Q_2\right).\end{aligned}$$

- That is:

$$0.5Q_1 + 1.024Q_2 = 1430. \tag{4.2}$$

Example, continued

- Solving the simultaneous equations (4.1) and (4.2), we obtain:

$$
\begin{aligned}
Q_1^\star &= 938.3 \text{ MW}, \\
Q_2^\star &= 938.3 \text{ MW}, \\
Q_1^\star + Q_2^\star &= 1876.6 \text{ MW}, \\
p^{\mathrm{d}}(Q_1^\star + Q_2^\star) &= 511.7 \text{ \$/MWh}, \\
c_i'(Q_1^\star) = c_i'(Q_2^\star) &= 42.5 \text{ \$/MWh}.
\end{aligned}
$$

- We have calculated the Nash equilibrium of a Cournot duopoly by simultaneously solving the first-order necessary conditions for maximizing the profit function of each participant:
  - by construction, $Q_1^\star = 938.3$ MW is the profit maximizing quantity for firm 1, given that firm 2 produces $Q_2^\star = 938.3$ MW, and
  - vice versa.
- Note that a different choice of strategic variable might lead to a different result:
  - see in homework.

4.1.9.3 More general derivation of Cournot model

- Consider n firms with quadratic cost functions $c_k : \mathbb{R}_+ \to \mathbb{R}$:

$$\forall Q_k \in \mathbb{R}_+, c_k(Q_k) = \frac{1}{2} e_k Q_k^2 + a_k Q_k,$$

- with $e_k \geq 0$ for convex costs.
- The marginal cost of firm k is c'_k, with:

$$\forall Q_k \in \mathbb{R}_+, c'_k(Q_k) = e_k Q_k + a_k. \tag{4.3}$$

- Ignore capacity constraints.
- Assume demand of the form:

$$\forall P \in \mathbb{R}_+, q^{\mathrm{d}}(P) = N - \gamma P.$$

- Note that previous derivation of Cournot model in Section 2.3.2 used:
  - general convex cost function instead of specific quadratic functional form, and
  - elasticity of demand instead of demand slope.
- However, general features of model are similar.

More general derivation of Cournot model, continued

- Since total supply $\sum_\ell Q_\ell$ must equal demand, inverse demand $p^{\mathrm{d}} : \mathbb{R} \to \mathbb{R}$ is:

$$\forall Q_k, k = 1, \ldots, n, p^{\mathrm{d}}\left(\sum_\ell Q_\ell\right) = \left(N - \sum_\ell Q_\ell\right)/\gamma.$$

- The operating profit for firm k is its revenue minus its operating costs:

$$\pi_k(Q_k, Q_{-k}) = Q_k p^{\mathrm{d}}\left(\sum_\ell Q_\ell\right) - c_k(Q_k),$$

- Necessary and sufficient conditions on Q_k to maximize $\pi_k(Q_k, Q_{-k})$ are linear:

$$\begin{aligned} 0 &= \frac{\partial \pi_k(Q_k^{\mathrm{Cournot}}, Q_{-k})}{\partial Q_k}, \\ &= p^{\mathrm{d}}\left(Q_k^{\mathrm{Cournot}} + \sum_{\ell \neq k} Q_\ell\right) - Q_k^{\mathrm{Cournot}}(e_k + 1/\gamma) - a_k. \end{aligned}$$

More general derivation of Cournot model, continued

- Simultaneously satisfying the conditions for all firms results in n equations.
- Resulting "Cournot price" P^{Cournot} is given by:

$$P^{\text{Cournot}} = \frac{N + \sum_{k=1}^{n} \frac{a_k}{(e_k+1/\gamma)}}{\gamma + \sum_{k=1}^{n} \frac{1}{(e_k+1/\gamma)}}.$$

- Corresponding "Cournot quantities" are:

$$\forall k = 1,\ldots,n, Q_k^{\text{Cournot}} = \frac{1}{(e_k+1/\gamma)}(P^{\text{Cournot}} - a_k).$$

More general derivation of Cournot model, continued

- Price-cost mark-up is:

$$P^{\text{Cournot}} - c_k'(Q_k^{\text{Cournot}}) = \frac{P^{\text{Cournot}} - a_k}{e_k \gamma + 1}.$$

- Consider firms with the same generation technology but different capacities:
  - a_k is the same for all firms, but
  - e_k is smaller for larger firms.
- Generation, market share, and price-cost mark-up is larger for larger firms:
  - as in earlier derivation of Cournot model, firms with larger market share will have higher price-cost mark-up,
  - in the limit for firms with market share approaching zero, the price will equal their marginal costs.

- As a specific numerical example, consider the $n = 5$ firm example specified in Table 4.1.

Firm i	1	2	3	4	5
e_i((\$/MWh)/GW)	2.687	4.615	1.789	1.93	4.615
a_i(\$/MWh)	12	12	8	8	12

Table 4.1. *Five firm cost data from Baldick, Grant, and Kahn.*

More general derivation of Cournot model, continued

- Assume that demand level is specified by $N = 35$ and the demand slope is $\gamma = 0.1$ GW per (\$ per MWh).
- Obtain:

$$
\begin{aligned}
P^{\text{Cournot}} &= 80\$ \text{ per MWh}, \\
Q_1^{\text{Cournot}} &= 5.3911 \text{ MW}, \\
Q_2^{\text{Cournot}} &= 4.6799 \text{ MW}, \\
Q_3^{\text{Cournot}} &= 6.1410 \text{ MW}, \\
Q_4^{\text{Cournot}} &= 6.0684 \text{ MW}, \\
Q_5^{\text{Cournot}} &= 4.6799 \text{ MW}.
\end{aligned}
$$

4.1.9.4 Numerical solution

- The analytical approach may involve first-order necessary conditions that are non-linear or require the solution of differential equations.
- Numerical and differential equation solving methods may then be used to solve for the equilibrium.
- Potential for multiple equilibria is more difficult to investigate in this context.

4.1.9.5 Example

- Following Green, we assume that *demand* $q^{\mathrm{d}} : \mathbb{R}_+ \times [0,1] \to \mathbb{R}$ has a dependence on both price and on time:

$$\forall P \in \mathbb{R}_+, \forall t \in [0,1], q^{\mathrm{d}}(P,t) = N(t) - \gamma P, \tag{4.4}$$

  where:
  - P is the price,
  - t is the (normalized) time,
  - $N : [0,1] \to \mathbb{R}_+$ is the *load-duration* characteristic, and
  - $\gamma \in \mathbb{R}_+$ is minus the slope of the demand curve.
- The load-duration characteristic N represents the distribution of demand over a time horizon, with:
  - the time argument t normalized so that it ranges from 0 to 1, and
  - N non-increasing, so that $t = 0$ corresponds to peak conditions and $t = 1$ corresponds to minimum demand conditions.
- It could also represent the probability distribution of random demand as in Hortaçsu and Puller.

- An affine load-duration characteristic.

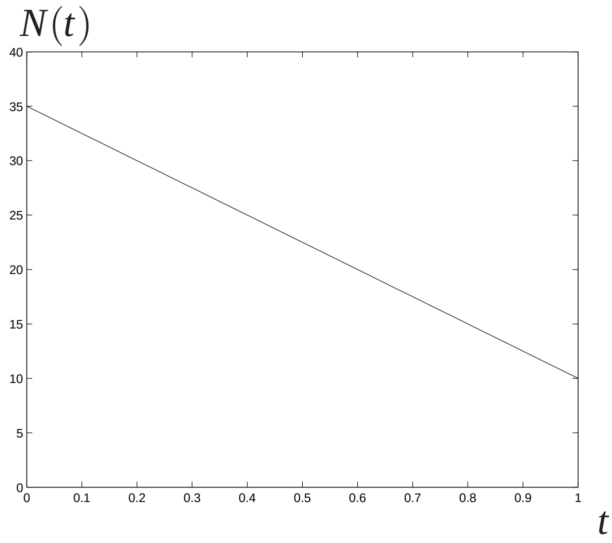

Fig. 4.3. Example load-duration characteristic.

Example, continued

- We assume that firms are labelled $i = 1, \ldots, n$, with $n \geq 2$.
- Assume that the *total variable operating cost function* of the i-th firm is $c_i : \mathbb{R}_+ \rightarrow \mathbb{R}$, with c_i assumed convex.
- Marginal costs are c_i'.
- We assume that market rules require that a single non-decreasing offer be specified for all time in the time horizon specified by the load-duration characteristic:
  - similar to version of homework where offer was used for all three sub-intervals.
- It will turn out that it is easier to analyze the *inverse* of the offer function, called the "supply function."
  - Each firm i specifies a function $s_i : \mathbb{R} \rightarrow \mathbb{R}$.
  - If the supply function is non-decreasing then the corresponding offer function will also be non-decreasing.
  - If the price is P then firm i is prepared to produce $s_i(P)$.

Example, continued

- Suppose that each firm $j \neq i$ specifies its non-decreasing supply function s_j.
- Consider a particular time t.
- Suppose that firm i produced Q_{it} at time t.
- Equating supply and demand at time t we obtain an expression that must be satisfied by the market clearing price P_t at time t:

$$Q_{it} = N(t) - \gamma P_t - \sum_{j \neq i} s_j(P_t).$$

- If firm i commits to meeting the residual demand it faces then we can think of Q_{it} as a function of P_t:
  - note that Q_{it} also depends on the supply functions s_j, $j \neq i$.

Example, continued

- The profit per unit time π_{it} for firm i if the price is P_t is therefore:

$$\begin{aligned}\pi_{it}(P_t) &= Q_{it}P_t - c_i(Q_{it}),\\ &= \left(N(t) - \gamma P_t - \sum_{j\neq i} s_j(P_t)\right)P_t - c_i\left(N(t) - \gamma P_t - \sum_{j\neq i} s_j(P_t)\right).\end{aligned}$$

- Suppose that the supply functions s_j, $j \neq i$ are differentiable.
- Differentiating π_{it} with respect to P_t and setting equal to zero, we obtain:

$$\begin{aligned}0 &= \frac{\partial \pi_{it}}{\partial P_t}(P_t),\\ &= \left(N(t) - \gamma P_t - \sum_{j\neq i} s_j(P_t)\right) + \left(-\gamma - \sum_{j\neq i}\frac{\partial s_j}{\partial P_t}(P_t)\right)P_t\\ &\quad -c_i'\left(N(t) - \gamma P_t - \sum_{j\neq i} s_j(P_t)\right)\left(-\gamma - \sum_{j\neq i}\frac{\partial s_j}{\partial P_t}(P_t)\right).\end{aligned}$$

Example, continued

- Recall the market clearing condition:

$$Q_{it} = N(t) - \gamma P_t - \sum_{j \neq i} s_j(P_t).$$

- Substituting from the market clearing condition, we can re-write the profit maximization condition as:

$$0 = Q_{it} + (P_t - c_i'(Q_{it})) \left(-\gamma - \sum_{j \neq i} \frac{\partial s_j}{\partial P_t}(P_t) \right).$$

- Re-arranging and requiring this condition to hold for each time t, we obtain:

$$\forall t \in [0,1], Q_{it} = (P_t - c_i'(Q_{it})) \left(\gamma + \sum_{j \neq i} s_j'(P_t) \right). \tag{4.5}$$

- Again, larger firms will have a larger price-cost mark-up.
- In the limit for small firms, the price will equal their marginal costs.

- To summarize, and similarly to Hortaçsu and Puller, if the supply functions of every other firm are specified then we can find the *ex post* optimal quantity and price for firm i at time t.
- This defines an implicit relationship between Q_{it} and P_t.
- If the implicit relationship is non-decreasing then we can find a supply function s_i that satisfies it.
- That is, we seek a function s_i that satisfies:

$$\forall P, s_i(P) = (P - c_i'(s_i(P))) \left(\gamma + \sum_{j \neq i} s_j'(P) \right). \tag{4.6}$$

- If the load-duration characteristic consists of discrete values (as in the homework) then (4.6) will only hold at the particular corresponding values of P.
- If the load-duration characteristic is continuous then (4.6) will hold for a continuum of prices.

Example, continued

- If we can find $s_i^\star, i = 1, \ldots, n$ that satisfy (4.6) for every firm i then we have an Nash equilibrium in supply functions:
  - "supply function equilibrium."
- If the load-duration characteristic is continuous, then these conditions specify a set of coupled non-linear differential equations:
  - there are multiple solutions to the non-linear differential equations depending on the "initial conditions,"
  - "least competitive SFE" includes prices that are equal to Cournot prices at peak demand,
  - "most competitive SFE" includes prices that are competitive at peak demand!
- Unfortunately, the differential equations are difficult to solve in general for supply functions that satisfy the non-decreasing constraints:
  - particular cases such as all cost functions identical ("symmetrical SFE") are typically easier to solve.

Example, continued

- Suppose that $n = 3$ with all firms having the same quadratic cost function:

$$\forall Q_i \in \mathbb{R}_+, c_i(Q_i) = \frac{1}{2} e_i Q_i^2 + a_i Q_i,$$

Firm i	1	2	3
e_i(\$/MWh per MWh)	0.5	0.5	0.5
a_i(\$/MWh)	9	9	9

Table 4.2. *Cost and capacity data for three firm example system from Day and Bunn.*

- Demand slope is $\gamma = 0.125$ GW per (\$/MWh)
- Load-duration characteristic is:

$$\forall t \in [0,1], N(t) = 7 + 20(1 - t),$$

- with quantities measured in GW.
- That is, N varies linearly from 27 to 7 GW.

Example, continued

- Solving the differential equations corresponding to the SFE for different "initial conditions" results in various equilibria:
  - because the cost functions are symmetric, a symmetric "initial condition" results in a symmetric equilibrium.
- There is a continuum of equilibria:
  - for each equilibrium, all three supply functions are the same,
  - will illustrate the supply function for one of the firms.

Example, continued

- Figure shows 14 different equilibria.
- The dashed curve shows an equilibrium where the supply functions are affine.

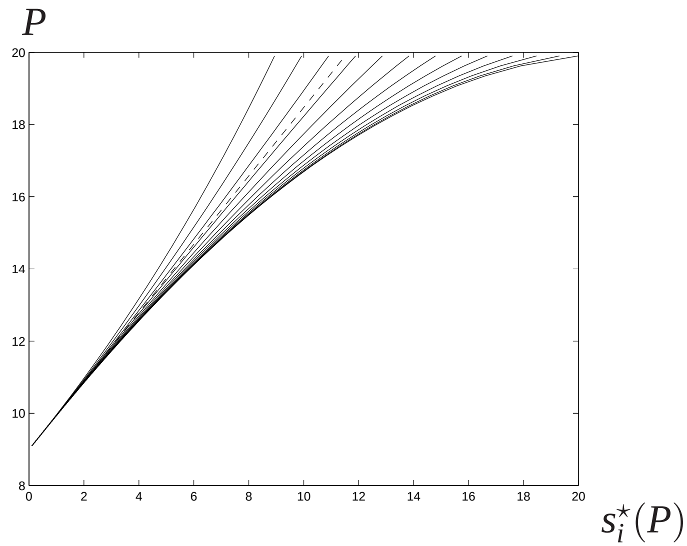

Fig. 4.4. Continuum of equilibria.

Example, continued

- Now let's assume that market rules require that a single affine offer be specified for all time in the time horizon:
  - similar to homework,
  - rules out all but one of the equilibria calculated in the previous example.
- It will again turn out that it is easier to analyze the *inverse* of the offer function, which is also affine:

$$\forall i, \forall P \geq \alpha_i, s_i^{\text{affine}}(P) = \beta_i (P - \alpha_i), \tag{4.7}$$

- where α_i and β_i are coefficients determined by firm i.
- The corresponding offer function is:

$$\alpha_i + q_i/\beta_i,$$

- so that the offer price at zero quantity is α_i.
- The slopes $\beta_i \in \mathbb{R}_{++}, i = 1, \ldots, n$ must be non-negative to ensure that the offer function is well-defined and non-decreasing.

Example, continued

- We will assume that the cost functions are quadratic and of the form:

$$\forall i, \forall Q_i \in \mathbb{R}_+, c_i(Q_i) = \frac{1}{2} e_i Q_i^2 + a_i Q_i,$$

- with $e_i \geq 0$ for each i so that the variable generation costs are convex.
- Marginal costs are c_i', so that:

$$\forall Q_i \in \mathbb{R}_+, c_i'(Q_i) = e_i Q_i + a_i. \tag{4.8}$$

- We ignore capacity constraints.
- Note that a competitive offer would correspond to:

$$\begin{aligned} \alpha_i &= a_i, \\ \beta_i &= 1/e_i. \end{aligned}$$

Example, continued

- Substituting the assumed affine functional form into (4.6) and assuming that the load-duration characteristic is continuous (or, that there are at least two distinct clearing prices) and that price is always at least $\max_i\{\alpha_i\}$, we obtain:

$$\forall i, \forall P, \beta_i(P-\alpha_i) = (P - e_i\beta_i(P-\alpha_i) - a_i)\left(\gamma + \sum_{j\neq i}\beta_j\right).$$

- Equating coefficients of P, we obtain:

$$\forall i, \beta_i = (1-e_i\beta_i)\left(\gamma + \sum_{j\neq i}\beta_j\right). \tag{4.9}$$

- Equating coefficients of the constant terms, we obtain:

$$\forall i, -\alpha_i\beta_i = -(a_i - e_i\beta_i\alpha_i)\left(\gamma + \sum_{j\neq i}\beta_j\right). \tag{4.10}$$

Example, continued

- Substituting from (4.9) into the left-hand side of (4.10) yields:

$$-\alpha_i(1-e_i\beta_i)\left(\gamma+\sum_{j\neq i}\beta_j\right) = -(a_i - e_i\beta_i\alpha_i)\left(\gamma+\sum_{j\neq i}\beta_j\right).$$

- So long as $(\gamma+\sum_{j\neq i}\beta_j) > 0$, we can cancel this factor on both sides to obtain:

$$\forall i, \alpha_i^\star = a_i.$$

- Note that although firm i can choose α_i as it wishes, its profit maximizing choice is consistent with a competitive offer!
- However, $\beta_i^\star$ will generally differ from $1/e_i$.

- Rudkevich shows that here is exactly one non-negative solution to (4.9), which can be found by solving the non-linear equations $g(\beta) = \mathbf{0}$, where:
  - the unknowns are $\beta = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_n \end{bmatrix} \in \mathbb{R}^n$,
  - the function $g : \mathbb{R}^n \to \mathbb{R}^n$ is defined by:

$$\forall \beta, g_i(\beta) = \beta_i - (1 - e_i \beta_i) \left(\gamma + \sum_{j \neq i} \beta_j \right).$$

- See in homework.

Homework exercise: Due Tuesday, April 29 in class

- Consider the "symmetric duopoly" with each firm $i = 1,2$ having marginal cost function:

$$\forall Q_i, c_i'(Q_i) = 20 + 60Q_i/2500.$$

- Operating range $[0, \overline{Q}_i]$, where $\overline{Q}_i = 2500$ MW.
- Note that, in the context of the affine supply function equilibrium formulation, $e_i = 60/2500$ and $a_i = 20$ for each firm.
- The inverse demand in each of three intervals is:

Interval 1 $\forall Q, p^{\mathrm{d}}(Q) = \max\{50 - (Q - 2800)/2, 0\}$,
Interval 2 $\forall Q, p^{\mathrm{d}}(Q) = \max\{75 - (Q - 3500)/2, 0\}$,
Interval 3 $\forall Q, p^{\mathrm{d}}(Q) = \max\{500 - (Q - 4200)/2, 0\}$,

- where Q is in MW and $p^{\mathrm{d}}(Q)$ is in \$/MWh.
- That is, the demand slope is $\gamma = 2$ MW per (\$/MWh).

Homework exercise: Due Tuesday, April 29 in class

- Find the affine supply function equilibrium for this industry.
- That is, assume that the strategic variable is the supply function and that supply functions are restricted to being affine.
- Moreover, assume that a single affine supply function must be specified for all three intervals:
  - this is sufficient for the affine supply function equilibrium analysis to apply.
- Solve (4.9) for this data using the MATLAB function `fsolve` (or any other technique of your choice) with initial guess given by the inverses of the e_i.
- That is, solve $g(\beta) = \mathbf{0}$, where:

$$\forall \beta, g_i(\beta) = \beta_i - (1 - e_i\beta_i)\left(\gamma + \sum_{j \neq i} \beta_j\right).$$

- Calculate the clearing price and quantities in each interval.
- Compare the results to the Cournot results obtained when the strategic variable was assumed to be the quantity.

Homework exercise: Due Tuesday, April 27, by 10pm

- For next week, we will again allow offers to vary for three peak pricing periods with demand:
  - 4150 MW,
  - 4200 MW, and
  - 4250 MW.
- That is, a different offer will be used for each of three pricing periods.
- Suppose that the cost functions for the last homework exercise stayed exactly the same.
- Again assume that the "top" 400 MW of demand in each period will be price responsive, with willingness-to-pay varying linearly from \$500/MWh down to \$100/MWh.
- Update your offers for the peak demand period to try to improve your profits compared to your previous offers:
  - submit offers for all periods, all three offers will be considered.

4.1.9.6 Fictitious play

- For complex models, a natural approach is to successively update the strategic variables starting from some initial guess at the equilibrium value of the strategic variables.
- Each participant may find its profit maximizing response to the other participants' strategic variables and use that to update its own strategic variables:
  - in principle, can incorporate a variety of issues including generation capacity and transmission constraints,
  - in principle, global search could be carried out to deal with non-concave profit function but, in practice, implementations tend to use local optimizers.

Fictitious play, continued

- In principle, converges to "single-shot" pure strategy equilibrium, if it exists:
  - does not represent repeated game, despite update involving repeated updates!
  - "damped" update may be necessary to facilitate convergence,
  - if local optimizer is used then may converge to non-equilibrium.
- Can also sometimes be used to find a mixed strategy equilibrium:
  - strategies are random mixtures of "pure strategies."

4.1.9.7 Example

- Use five firm data again, but include generator capacities as shown in Table 4.3.

Firm i	1	2	3	4	5
e_i((\$/MWh)/GW)	2.687	4.615	1.789	1.93	4.615
a_i(\$/MWh)	12	12	8	8	12
$\overline{Q}_k$(GW)	5.70945	3.35325	10.4482	9.70785	3.3609

Table 4.3. *Five firm cost data from Baldick, Grant, and Kahn.*

- Assume a demand slope of $\gamma = 0.1$ GW per (\$ per MWh) and that $N(t)$ is affine, ranging from 35 to 10 GW as in Figure 4.3.
- Allow piecewise linear supply functions with multiple segments.
- Initially ignore capacity constraints.

- Initial guess is affine supply function equilibrium.

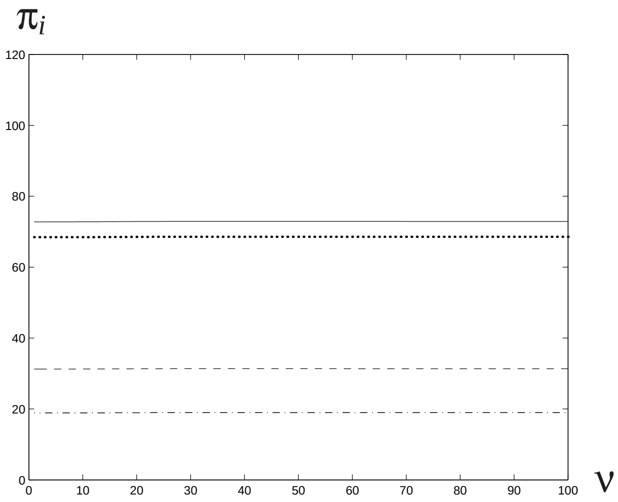

Fig. 4.5. Profits versus iteration for case of no capacity constraints, starting from the affine SFE supply function.

- Supply functions stay the same at each iteration since initial guess is equilibrium!

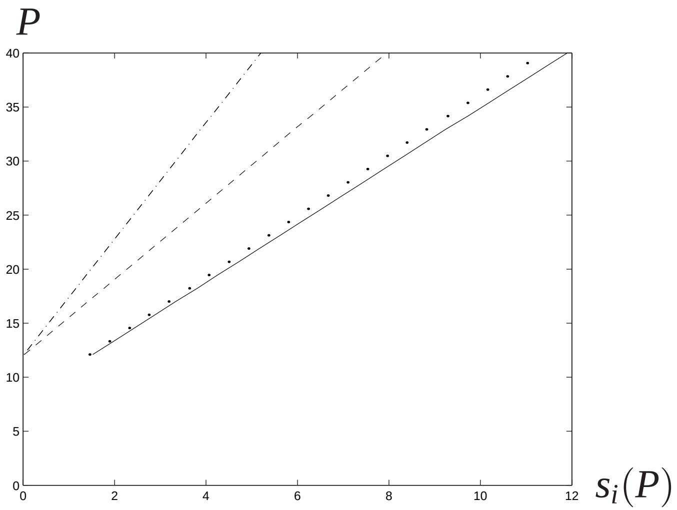

Fig. 4.6. Supply functions at iteration 100 for case of no capacity constraints, starting from the affine SFE supply function.

Example, continued

- Price is affine function of time since load-duration characteristic was assumed to be affine function of time and equilibrium supply functions are affine.

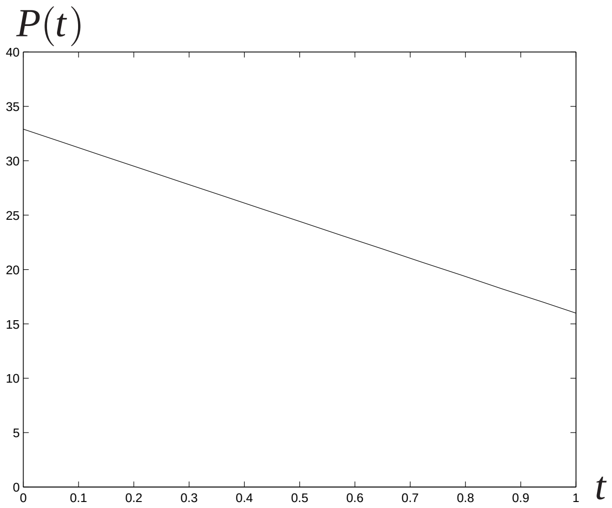

Fig. 4.7. Price-duration curve at iteration 100 for case of no capacity constraints, starting from the affine SFE supply function.

Example, continued

- In this case, initial guess is competitive offers.
- Profits increase from competitive as equilibrium is approached.

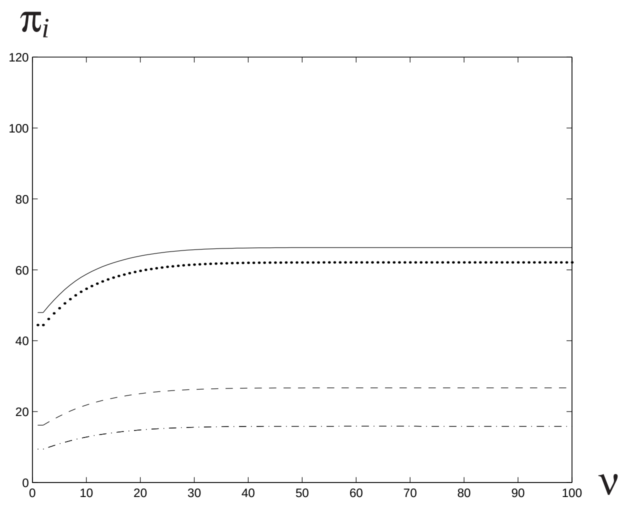

Fig. 4.8. Profits versus iteration for case of no capacity constraints, starting from the competitive supply function.

Example, continued

- Equilibrium is somewhat different to affine SFE.
- Consistent with theoretical conclusion that there are multiple equilibria.
- Only offers for prices less than \$28/MWh are relevant.

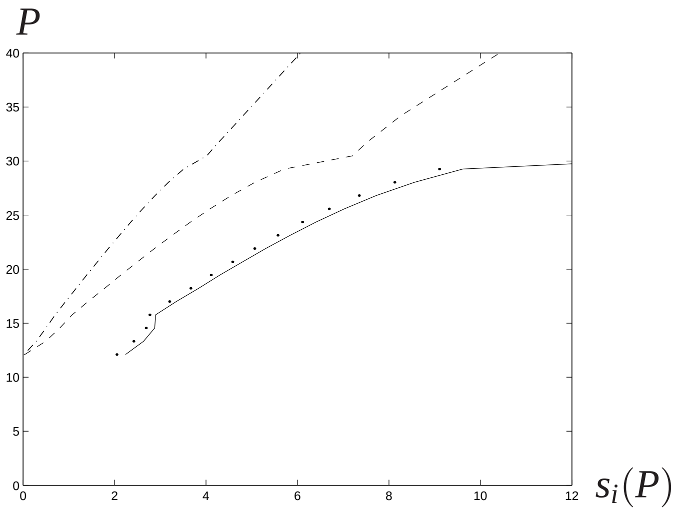

Fig. 4.9. Supply functions at iteration 100 for case of no capacity constraints, starting from the competitive supply function.

Example, continued

- Prices somewhat lower than in affine SFE except for low demand.

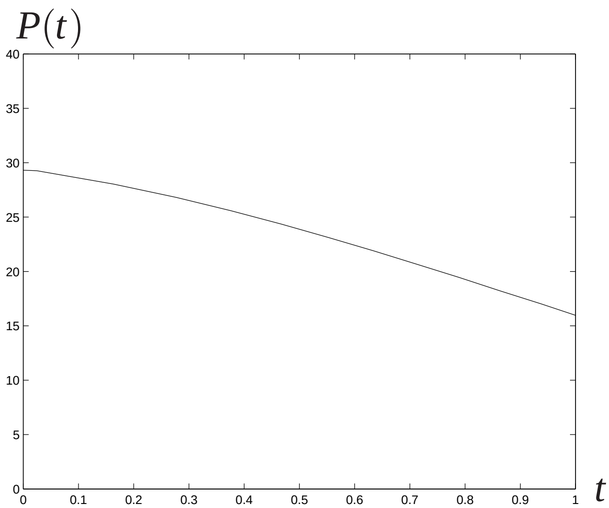

Fig. 4.10. Price-duration curve at iteration 100 for case of no capacity constraints, starting from the competitive supply function.

Example, continued

- Starting from widely different initial guesses result in slightly different equilibria.
- Range of *numerically* calculated equilibria is much less wide than the range of theoretically possible equilibria.

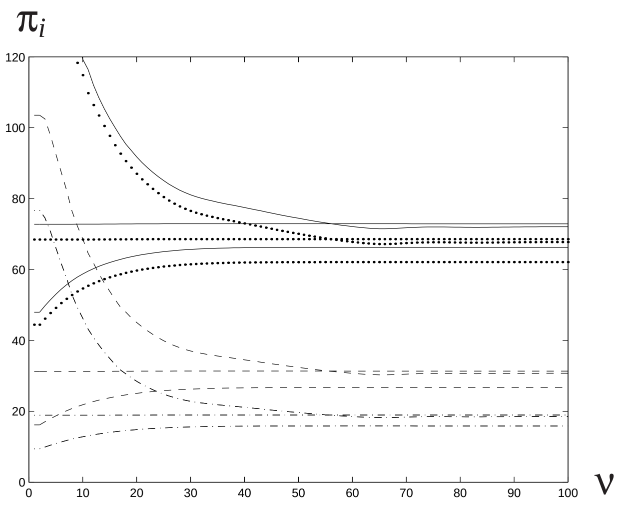

Fig. 4.11. Profits versus iteration for case of no capacity constraints for all starting functions combined.

Example, continued

- Only offers for prices less than approximately \$30/MWh are relevant for comparison.

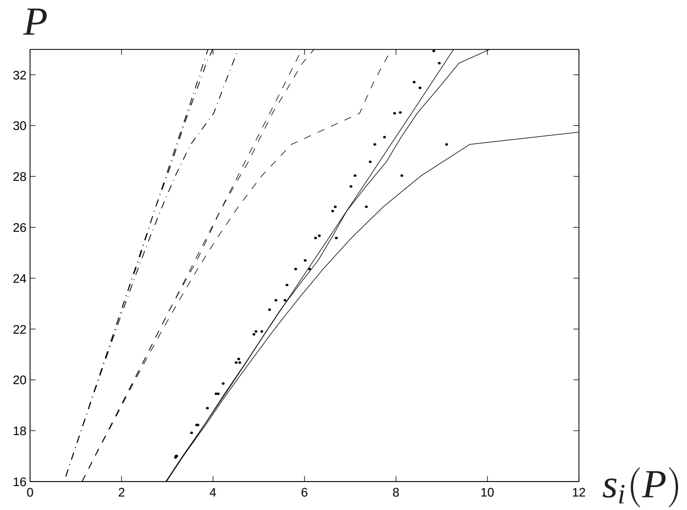

Fig. 4.12. Supply functions at iteration 100 for case of no capacity constraints for all starting functions combined.

- Equilibria only differ noticeably at higher demand levels.

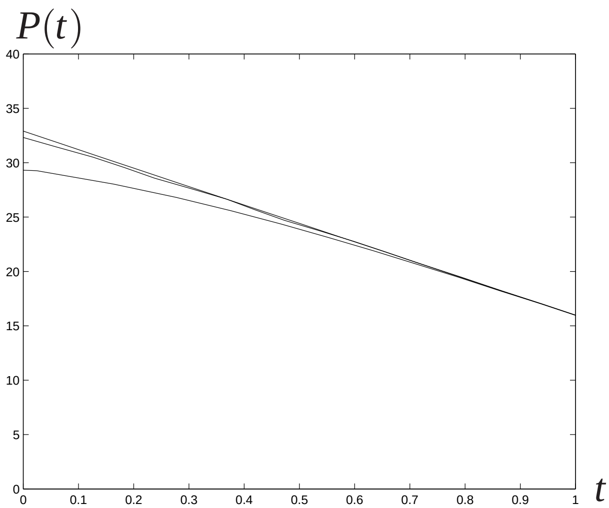

Fig. 4.13. Price-duration curve at iteration 100 for case of no capacity constraints for all starting functions combined.

4.1.9.8 Example with capacity constraints

- Now impose capacity constraints on five firm example.
- Start with competitive offers.

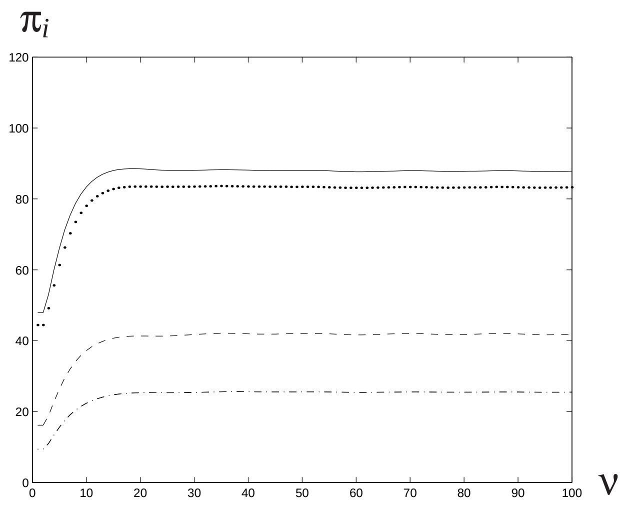

Fig. 4.14. Profits versus iteration with capacity constraints starting from capacitated competitive.

- Supply functions have kinks.

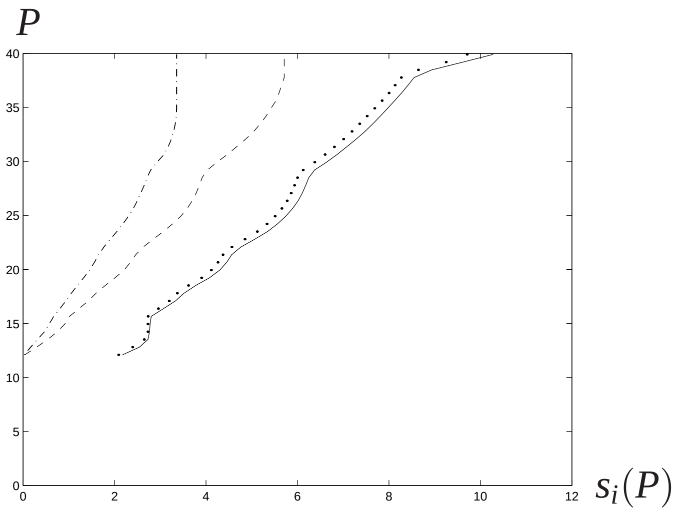

Fig. 4.15. Supply functions at iteration 100 with capacity constraints starting from capacitated competitive.

- Price-duration curve non-affine.

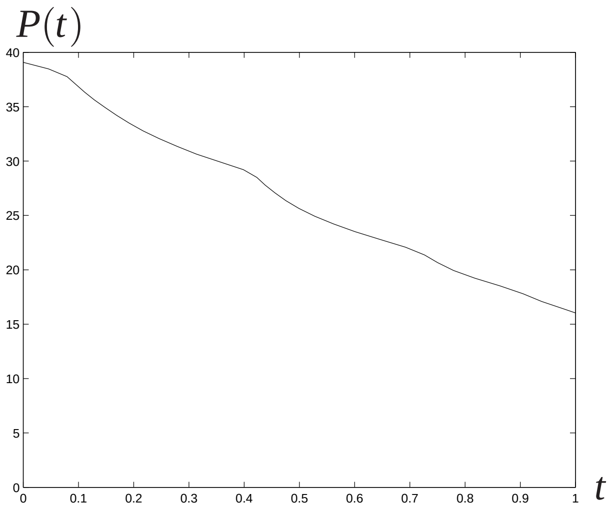

Fig. 4.16. Price-duration curve at iteration 100 with capacity constraints starting from capacitated competitive.

Example with capacity constraints, continued

- Starting from widely different initial guesses again result in slightly different equilibria.
- Range of *numerically* calculated equilibria is very small.

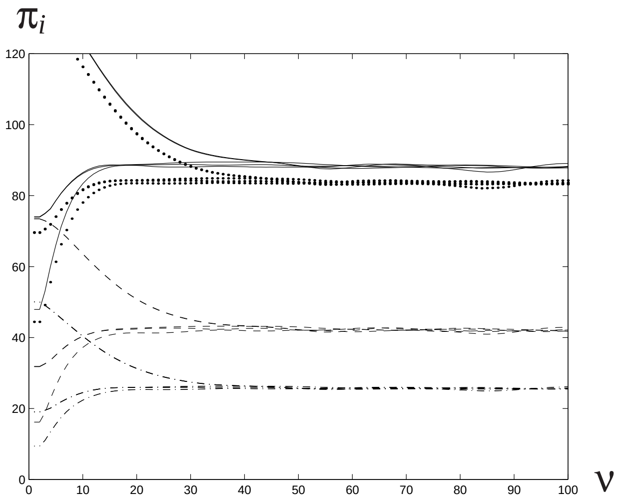

Fig. 4.17. Profits versus iteration with capacity constraints for all starting functions combined.

Example with capacity constraints, continued

- Supply functions at iteration 100 similar despite varying initial guesses.

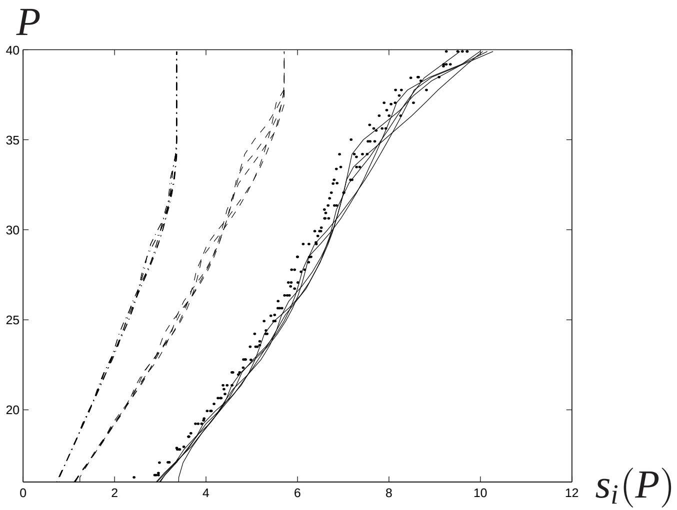

Fig. 4.18. Supply functions at iteration 100 with capacity constraints for all starting functions combined.

Example with capacity constraints, continued

- Price-duration curves at iteration 100 all similar despite varying initial guesses.

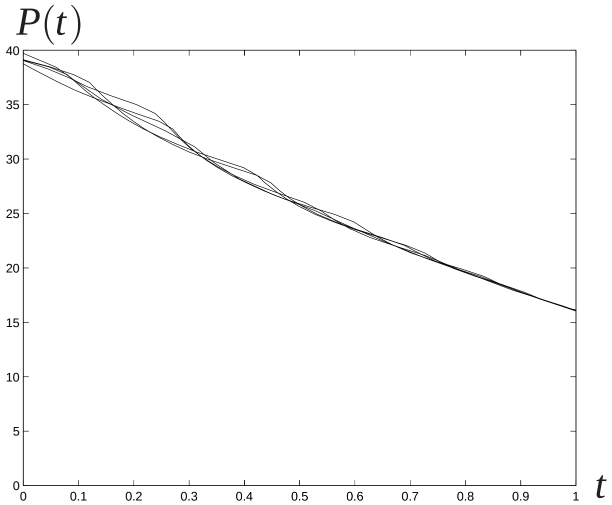

Fig. 4.19. Price-duration curve at iteration 100 with capacity constraints for all starting functions combined.

Fictitious play, continued

- "Agent-based" models fall into this framework, although the "agent" may not be explicitly finding its profit maximizing response.
- "Experimental economics," where human subjects act as market participants are another example of fictitious play:
  - as in group homework where offers are updated each week.

Equilibrium solution methods, continued

4.1.9.9 Mathematical program with equilibrium constraints and equilibrium program with equilibrium constraints

- Model the market clearing mechanism by its optimality conditions.
- Incorporate optimality conditions into the optimization problems faced by each participant:
  - optimization problem is a “mathematical program with equilibrium constraints.”
  - May deliberately simplify the profit maximization problems to avoid non-concave profit functions for participants, particularly in case of generator or transmission capacity constraints.
- Collecting together the problems of every participant and solving for the equilibrium results in an “equilibrium program with equilibrium constraints.”

4.1.9.10 Specialized solution methods

- In some cases, specialized algorithms may be applied to particular types of equilibria.
- For example, Anderson and Hu describe a technique for finding supply function equilibria.

4.1.10 Validity, uses, and limitations of equilibrium models

- Are equilibrium models reasonable?
  - In the ERCOT balancing market, some smaller market participants' behavior is evidently not consistent with a model of profit maximization:
    - as discussed in Hortaçsu and Puller,
    - this may simply be due to discrepancies between the economic and commercial models or due to concerns about regulatory intervention.

Validity, uses, and limitations of equilibrium models, continued

- Are equilibrium models reasonable?
  - Sometimes, there are only "mixed strategy" equilibria:
    - rock, scissors, paper payoffs are shown in table,
    - if either player picks one strategy and continues to pick it then the other player can always win!
    - Nash equilibrium strategy is for each player to randomly pick rock, scissors, or paper.

Payoff (to 1, to 2)		2: Rock	2: Scissors	2: Paper
1	Rock	$(0,0)$	$(1,-1)$	$(-1,1)$
1	Scissors	$(-1,1)$	$(0,0)$	$(1,-1)$
1	Paper	$(1,-1)$	$(-1,1)$	$(0,0)$

Table 4.4. *Payoffs for rock, scissors, paper.*

Validity, uses, and limitations of equilibrium models, continued

- Are equilibrium models reasonable?
  - There is little evidence of randomized offers in actual electricity markets:
    - Simplifications of representation of transmission and generation capacity constraints are typically aimed at ensuring concavity of generator profit function to help assure that pure strategy equilibria exist,
    - Not clear whether this simplification is an appropriate model of participant behavior.
  - There may be multiple equilibria, particularly for supply function equilibria, reducing the predictive value:
    - numerical results and theoretical "stability" analysis suggest that range of observed equilibria is likely to be smaller than theoretically possible range,
    - ongoing research in this area.

Validity, uses, and limitations of equilibrium models, continued

- Are equilibrium models reasonable?
  - There are a large number of modelling assumptions:
    - only a fraction of market rules can be modelled.
  - Choice of parameterization of strategic variables can quantitatively and qualitatively affect equilibrium:
    - as in Cournot and supply function versions of homework problems,
    - requires very careful modelling to avoid the results being an artifact of unrealistic choices of strategic variables.
- Cannot expect to predict outcomes and prices accurately!

4.1.11 Principled analysis of the effect of changes

- Evaluate alternative market rules such as:
  - allowing offers to change from interval to interval versus requiring offers to remain fixed over multiple intervals, and
  - single clearing price versus pay-as-bid prices,
- Evaluate changes in market structure such as mandated divestitures,
- Estimate the effect of transmission constraints.
- Estimate the effect of the level of contracts, such as:
  - physical and financial bilateral energy contracts, and
  - financial transmission rights,
- Evaluate modelling assumptions, such as:
  - the assumed form of cost functions or offer functions,
  - the use of portfolio-based versus unit-specific costs or offers, and
  - the representation of unit commitment.

4.1.11.1 *Strategy to evaluate changes*

- Hold most market rules and features constant.
- Vary one particular issue for a qualitative "sensitivity" analysis.
- Estimate the *change* due to the modeled variation.
- Allows the potential for policy conclusions to be made from studies even in the absence of absolute accuracy:
  - responds to Harvey and Hogan criticism that underlying models were developed for comparing alternatives, not for absolute evaluation.
- Group homework provides examples:
  - allowing offers to change from interval to interval versus fixed offers,
  - inelastic versus elastic demand.
- Case studies:

  (i) Market rules regarding changing of offers.
  (ii) Single clearing price versus pay-as-bid prices.
  (iii) Divestitures.

4.1.11.2 Market rules regarding changing of offers

- Single set of energy offers that must apply across all intervals in the day versus offers that can vary from hour to hour.
- A supply function equilibrium model can represent both cases.
- Many of the detailed features of electricity markets, including transmission constraints, might be ignored.
- Such an analysis was performed by Baldick and Hogan (2002, 2006).
- A rule requiring consistent offers can help to mitigate market power.

4.1.11.3 Single clearing price versus pay-as-bid prices

- Concerns about exercise of market power sometimes prompt suggestions for a "pay-as-bid" market:
  - each accepted offer is paid its offer price instead of the market clearing price,
  - so even if the market clearing price is high due to market power, other offers will only receive their offer price.
- Proposals for pay-as-bid markets usually neglect to realize that offers will change in response to changes in market rules.
- The "revenue equivalence theorem" suggests that equilibrium prices should be the same in both types of markets:
  - in absence of uncertainty, offers in pay-as-bid market will rise to equal what *would* have been the equilibrium clearing price in the single clearing price market!
  - clearing price estimation errors in presence of uncertainty will mean that dispatch is inefficient.
- Not all of the assumptions required for the revenue equivalence theorem actually hold in electricity markets.

Single clearing price versus pay-as-bid prices, continued

- A simplified model of an electricity market can be used to obtain a sensitivity result for the change between single clearing price and pay-as-bid prices.
- In some models of electricity markets, pay-as-bid pricing can result in lower equilibrium prices than in single clearing price markets (Fabria, 2000, and Son, Baldick, and Lee, 2004).
  - Effect is relatively small and unlikely to compensate for downsides of pay-as-bid such as poor dispatch decisions.
  - Although revenue equivalence theorem does not apply rigorously, the result remains approximately true.

4.1.11.4 Divestitures

- Market structure has been changed by mandated divestitures in the England and Wales market in the late 1990s.
- A supply function equilibrium model reproduced the change in prices from before to after the divestitures, given calibration to observed demand pre-divestiture (Baldick, Grant, and Kahn, 2004, and Day and Bunn, 2001).
- Helps to confirm insight that greater number of smaller competitors results in more competitive prices.

4.1.12 Summary

- Discussed equilibrium models, their solution, and uses.
- There has been considerable effort in recent years in developing the theory and application of these models.
- There are strong prospects for improving such models, although their application should be tempered with the understanding that the actual market is likely to include a host of details that remain unmodelled.
- Qualitative sensitivity analysis can be useful, even in the absence of quantitative accuracy.
- Empirical studies, such as the IMM report can elucidate exercise of market power.
- Theoretical studies, such as equilibrium analysis, can inform the empirical studies and help with market design.

Homework exercise: Due Tuesday, May 4, by 10pm

- For next week, we will again allow offers to vary for three peak pricing periods with demand:
  - 4150 MW,
  - 4200 MW, and
  - 4250 MW.
- That is, a different offer will be used for each of three pricing periods.
- Suppose that the cost functions for the last homework exercise stayed exactly the same.
- Again assume that the "top" 400 MW of demand in each period will be price responsive, with willingness-to-pay varying linearly from $500/MWh down to $100/MWh.
- Update your offers for the peak demand period to try to improve your profits compared to your previous offers:
  - submit offers for all periods, all three offers will be considered.

Homework exercise: Due Tuesday, May 4 in class

- Consider the five firm example system with costs shown in the table.
- Solve (4.9) for this data using the MATLAB function `fsolve` (or any other technique of your choice) with initial guess given by the inverses of the e_i.
- That is, solve $g(\beta) = \mathbf{0}$, where:

$$\forall \beta, g_i(\beta) = \beta_i - (1 - e_i \beta_i) \left(\gamma + \sum_{j \neq i} \beta_j \right).$$

- Assume that $\gamma = 0.1$ GW per (\$/MWh).

Firm i	1	2	3	4	5
e_i((\$/MWh)/GW)	2.687	4.615	1.789	1.93	4.615
a_i(\$/MWh)	12	12	8	8	12

Table 4.5. *Five firm cost data from Baldick, Grant, and Kahn.*