Firm-Based Measurements of Market Power in Transmission Constrained Electricity Markets

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Purpose of this presentation

- Introducing the state of art research in market power.
- A taste of the issue of market power.
- All three pre-requisites will appear in this presentation: microeconomics, power system engineering, and optimization theory.

Outline

- Some background knowledge.
- Two new firm-based measurements of market power:
  1. Transmission constrained residual supply index.
  2. Firm-based transmission constrained market power indices.
- Conclusion.

Defining Market Power

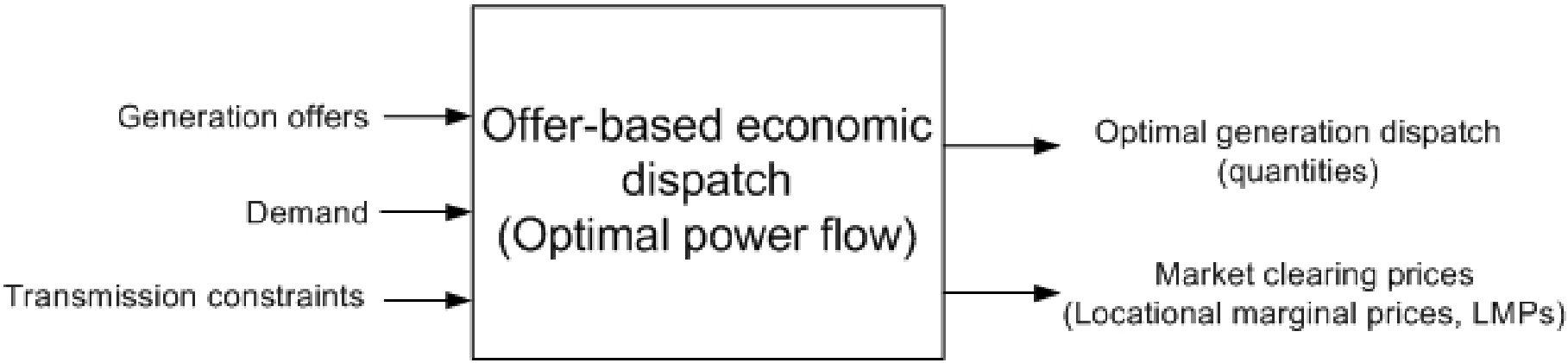
- Market power to a seller is the ability profitably to maintain prices above competitive levels for a significant amount of time.
- Exercising market power by **physical withholding** (curtailing output) or **financial withholding** (bidding higher).
- Market power might cause inefficiency in production, capital allocation, and transfer of wealth from demand to generator.
- Market power is hard to be modeled because it involves strategic behavior by several competitors, especially in the context of transmission constraints.
- In this presentation, two approaches to analyzing market power in the contxt of transmission constraints will be introduced.

Offer-based economic dispatch

- We assume an electricity market that generators make offers to sell electricity to an independent system operator(ISO).
- An offer of generator i is a function $p_i : \mathbb{R} \to \mathbb{R}$ from quantity to price that specifies, for each quantity Q_i produced, the minimum price $p_i(Q_i)$ to produce at that quantity.
- ISO use offer-based economic dispatch algorithm to seek a price that matches demand and supply.
- Behaving competitively means the generation offer equals to the marginal cost.

Offer-based economic dispatch

- The offer-based economic dispatch is actually an mathematical programming program that seeks the minimum production cost to meet the demand.
- Given the offer, demand, and transmission constraints, offer-based economic dispatch finds the optimal dispatch and the prices which provide the right incentives for the generators.
- This process is also called optimal power flow (OPF).



Locational marginal price (LMP)

- Price varies at each bus when there are any congested transmission lines.
- The price at a given bus reflects the cost of adding an extra unit of demand at that bus.
- At any particular bus, all demand pays and all generation is paid the same price.

Transmission Constrained Residual Supply Index

- An index to predict market power under transmission constraints.
- A firm is called **pivotal** when all demand could not be satisfied without the supply from the firm.
- If a firm is pivotal, it has absolute market power.
- The residual supply index (RSI) is a measurement of how pivotal a firm is.
- The transmission constrained residual supply index generalizes the RSI to the case of transmission constraints.

Residual Supply Index (RSI)

- Proposed by California ISO in order to predict market power.
- The RSI for firm i is defined as following:

$$\text{RSI} = \frac{\text{total available supply} - \text{availble supply from firm } i}{\text{demand}}$$

- For example, firm i owns capacity of 500MW, and the total capacity in the system is 1400MW. Assuming the demand is 1000MW, the RSI index for firm i would be $\frac{1400 - 500}{1000} = 0.9$.

Residual Supply Index (RSI)

- The RSI value less than 1 implies that the firm's offer is "pivotal" to satisfy all demand.
- The empirical evidence shows the negative correlation between residual supply index and price-cost markup.
- The RSI ignores the effects of transmission constraints, might be less useful in the context of "locational marginal price"(LMP) markets.

Transmission Constrained Residual Supply Index (TCRSI)

- We developed a new index, the "transmission constrained residual supply index" (TCRSI).
- The TCRSI is a generalization of the RSI into the context of transmission constraints.
- Using a single parameter to scale all the loads conformally, we find the maximum value of this parameter in the absence of any supply from firm i.
- In other words, considering the transmission constraints, we find the maximum demand the rest of the system could serve with the restriction that all loads has to be equally scaled.

Why a single parameter?

- Instead of single parameter to scale all the loads conformally, another possibility to measure maximum residual supply is to treat each load as an free variable and then we maximize their sum.
- The loads usually are distributed throughout the system with some geographical pattern. i.e. demand at urban areas is often much higher than the rural areas.
- The pattern should be kept while we evaluate the maximum residual supply.

Definition of TCRSI

- The TCRSI is the optimal value of the following linear programming (LP) problem:

$$
\begin{aligned}
\underset{\mathbf{q},\, t}{\text{maximize}} \quad & t \\
\text{subject to} \quad & \mathbf{1}^{\dagger}\mathbf{q} - t(\mathbf{1}^{\dagger}\mathbf{d}) = 0 \\
& \mathbf{H_g q} + \mathbf{H_d}(\mathbf{d}t) \leq \mathbf{b} \\
& 0 \leq q_j \leq \bar{q}_j, j \notin R \\
& q_j = 0, j \in R
\end{aligned}
$$

Formulation of TCRSI LP

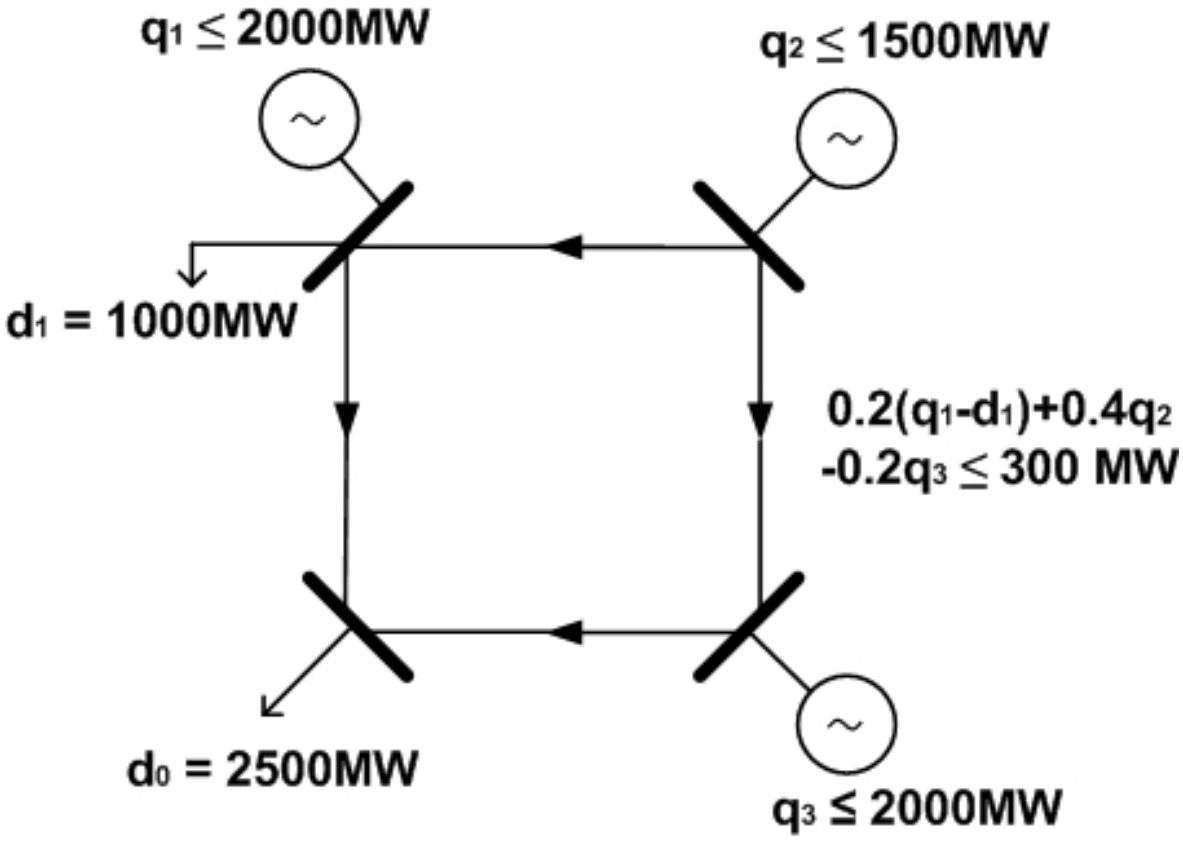
- n generators, ℓ loads, and m transmission constraints.
- $\mathbf{d} \in \mathbb{R}^{\ell}$: load quantities.
- $\mathbf{H_g} \in \mathbb{R}^{m \times n}$,$\mathbf{H_d} \in \mathbb{R}^{m \times \ell}$ the matrices of power transfer distribution factors (PTDFs) for generator and load respectively.
- R: set of indices of generators that firm i owns.
- $\bar{q}_j, j = 1, ..., n$: the maximum available supply of each generator.
- $\mathbf{b}$: the thermal limit of each transmission line.

Implication of TCRSI

- If the TCRSI of firm i is less than 1, it implies that the system could never meet demand without supply from firm i. Now firm i could offer its generation at any arbitrarily high price (or up to price cap) and at least some of its offers are guaranteed to be accepted by the ISO. In this case, firm i has absolute market power.
- If the TCRSI of firm is equal or slightly greater than 1, the system could just meet all the demand. However, the system might have to dispatch some expansive generators. In this scenario, the firm still has some chances to offer strategically: for example, offers its base load generator at price slightly less than the current system's marginal cost and drives up the market clearing prices.
- Generally speaking, the greater the TCRSI, the more generation resources the system operator could dispatch, and less market power owned by firm i.

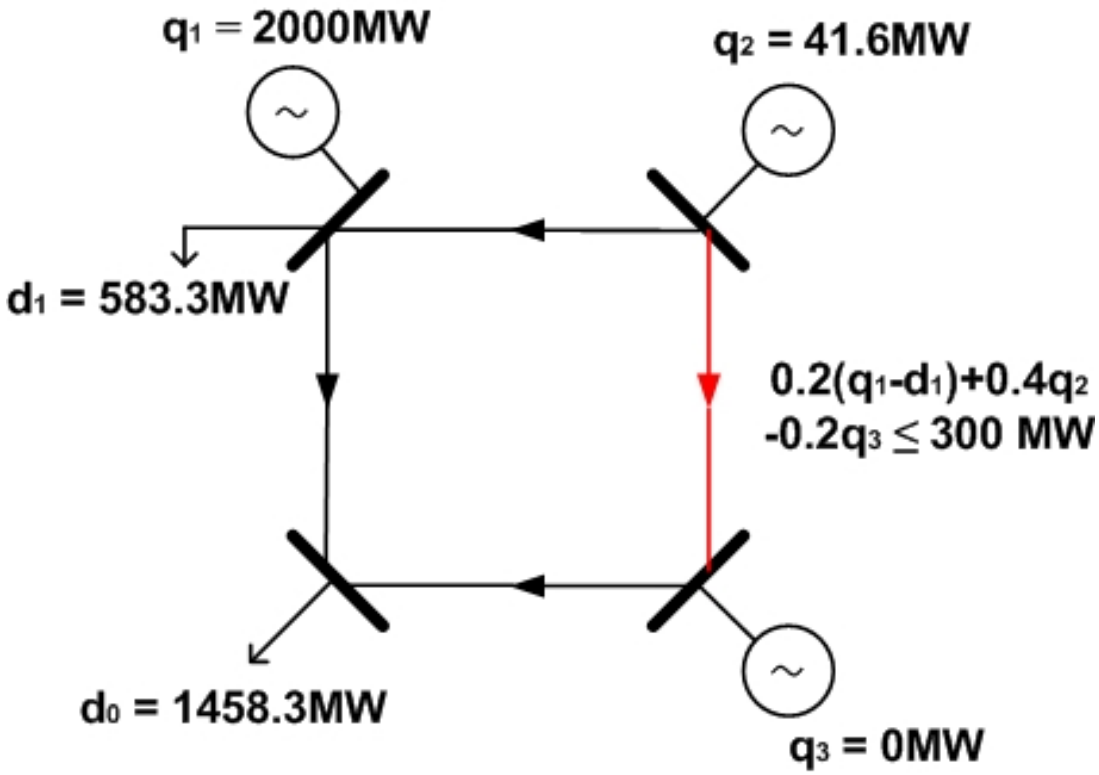
Four Bus System

- A simple four bus example illustrate how the TCRSI works.
- From the RSI analysis, there is no pivotal firm.



Four Bus System

- The RSI for generator 3 is (5500-2000)/3500 = 1.
- However, if we removed generator 3, all the load could not be satisfied since generator 3 is the only generator could produce "counter-flow" on line 2-3.The TCRSI for generator 3 = 0.583.



118 Bus Reliability Test System

- We arbitrarily defined the ownerships of 54 generators into 6 owners and calculate the TCRSIs.
- Interestingly, firm A has least TCRSI, but largest RSI, and smallest capacity share.

Firm	TCRSI	RSI	Capacity Share
A	0.61	3.30	0.09
B	1.00	3.01	0.14
C	0.85	2.83	0.22
D	1.48	3.17	0.13
E	1.23	3.06	0.15
F	1.39	2.76	0.24

ERCOT system

- We arbitrarily defined the ownership data and calculate the TCRSI for ERCOT system. We list the six firms with least the TCRSI value.
- Firm B and D do not have large amount of capacity share but are actually pivotal firm under transmission constraints.

Firm	TCRSI	RSI	Capacity Share
A	0.67	0.97	0.20
B	0.79	1.15	0.06
C	0.81	1.08	0.12
D	0.86	1.15	0.05
E	1.00	1.18	0.03
F	1.00	1.19	0.02

Firm-Based Transmission Constrained Market Power Indices

- Another approach to gauge market power
- Based on economic principles, estimate the price-cost markup and transfer of wealth under profit maximizing hypothesis.
- An application of transmission constrained residual demand Jacobian

Principle-Based Analysis of Firm-based Market Power

- firm i owns generators at buses $k \in R$, $|R| = r$.
- collect the production quantities $q_k, k \in R$, at all these generators into a vector $\mathbf{q} \in \mathbb{R}^r$.
- $p^{\mathrm{d}}_{-k} : \mathbb{R}^r \to \mathbb{R}$ is the resulting market clearing price at bus k given that the firm produces the quantities $\mathbf{q}$.
- collect the inverse residual demands $p^{\mathrm{d}}_{-k}, k \in R$, together into a vector function $\mathbf{p}^{\mathrm{d}} : \mathbb{R}^r \to \mathbb{R}^r$.
- Profit of firm i is as follows:

$$\forall \mathbf{q} \in \mathbb{R}^r, \pi(\mathbf{q}) = \sum_{k \in R} \Big(q_k p^{\mathrm{d}}_{-k}(\mathbf{q}) - c_k(q_k) \Big)$$

Principle-Based Analysis of Firm-based Market Power (Cont.)

- Differentiate π with repect to q_ℓ:

$$0 = \frac{\partial \pi}{\partial q_\ell}(\mathbf{q}) = p^{\mathrm{d}}_{-\ell}(\mathbf{q}) + \sum_{k \in R} q_k \frac{\partial p^{\mathrm{d}}_{-k}}{\partial q_\ell}(\mathbf{q}) - c'_\ell(q_\ell),$$

- Re-arranging, we obtain the price-cost mark-up at bus ℓ

$$p^{\mathrm{d}}_{-\ell}(\mathbf{q}) - c'_\ell(q_\ell) = -\sum_{k \in R} q_k \frac{\partial p^{\mathrm{d}}_{-k}}{\partial q_\ell}(\mathbf{q}).$$

Principle-Based Analysis of Firm-based Market Power (Cont.)

- Two firm-based market power indices are developed:
  1. The quantity-weighted average markup of firm i:

$$\left(-\sum_{\ell=1}^{r} q_\ell \sum_{k=1}^{r} q_k \frac{\partial p_{-k}^{\mathrm{d}}}{\partial q_\ell}(\mathbf{q})\right) / \left(\mathbf{1}^\dagger \mathbf{q}\right) = \left(-\mathbf{q}^\dagger \frac{\partial \mathbf{p}^{\mathrm{d}}}{\partial \mathbf{q}}(\mathbf{q})\mathbf{q}\right) / \left(\mathbf{1}^\dagger \mathbf{q}\right).$$

  2. The estimated transfer of wealth to firm i:

$$-\sum_{\ell=1}^{r} q_\ell \sum_{k=1}^{r} q_k \frac{\partial p_{-k}^{\mathrm{d}}}{\partial q_\ell}(\mathbf{q}) = -\mathbf{q}^\dagger \frac{\partial \mathbf{p}^{\mathrm{d}}}{\partial \mathbf{q}}(\mathbf{q})\mathbf{q}.$$

Inverse Residual Demand Jacobian

- The key to calculate the indices is how to derive inverse residual demand Jacobian.
- The prices p are defined through an optimal power flow (OPF) programs. (p is defined as part of the lagrange multipliers that together with primal variables satisfy the optimality condition.)
- Imagining an OPF program that is parameterized by the firm's production q. That is, p is a function of q.
- The derivatives could be derived by asking: if we perturbed q a little bit, what is the resulting p?

Inverse Residual Demand Jacobian

- The naive way to evaluate inverse residual demand Jacobian is to use finite difference method: change one entry of q slightly at a time, resolve the OPF and get the new prices p'.
- Assuming the set of binding constraints is fixed, the optimality condition could be expressed as a set of non-linear equations. The sensitivity analysis technique of non-linear simultaneous equations could be utilized.

Transmission Constrained Inverse Residual Demand Jacobian (TCRDJ)

To calculate a column of $\frac{\partial \mathbf{p}^{\mathrm{d}}}{\partial \mathbf{q}}$, say, $\frac{\partial \mathbf{p}^{\mathrm{d}}}{\partial q_\ell}$, where ℓ is price reference bus:

1. First calculate the derivative of price at reference bus w.r.t q_ℓ

$$\frac{\partial p^d_{-\ell}}{\partial q_\ell} = \left[\mathbf{1}_{n-r}^T \mathbf{\Lambda}\left(\mathbf{H}^T(\mathbf{H}\mathbf{\Lambda}\mathbf{H}^T)^{-1}\mathbf{H}\mathbf{\Lambda}\mathbf{1}_{n-r} - \mathbf{1}_{n-r}\right)\right]^{-1}.$$

2. Then we can derive the derivatives of shadow price of transmission constraints:

$$\frac{\partial \boldsymbol{\mu}}{\partial q_\ell} = -(\mathbf{H}\mathbf{\Lambda}\mathbf{H}^T)^{-1}\mathbf{H}\mathbf{\Lambda}\mathbf{1}_{n-r}\frac{\partial p^d_{-\ell}}{\partial q_\ell}.$$

3. By the definition of locational marginal pricing:

$$\frac{\partial \mathbf{p^d}}{\partial q_\ell} = \bar{\mathbf{1}}_r \frac{\partial p^d_{-\ell}}{\partial q_\ell} + \mathbf{H_R}^T \frac{\partial \boldsymbol{\mu}}{\partial q_\ell}.$$

Transmission Constrained Inverse Residual Demand Jacobian (TCRDJ)

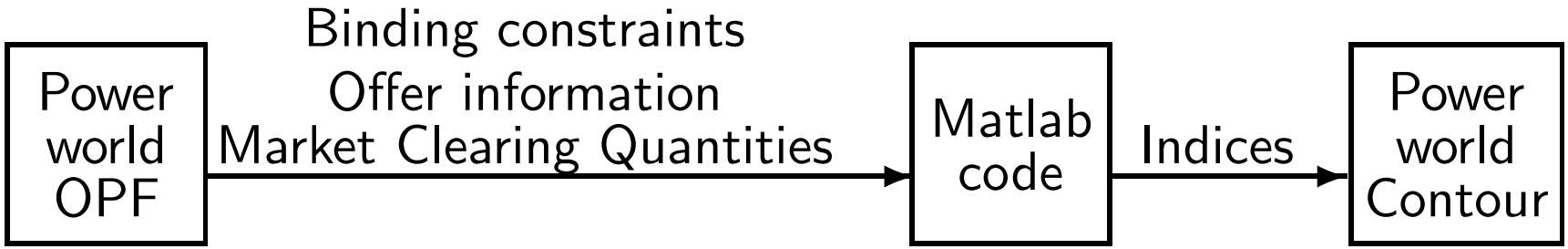
- R is the set of indices of generators that firm i owns.
- $\mathbf{H} \in \mathbb{R}^{b \times (n-r)}$ is the matrix of shift factors for injection of generators $g \notin R$ for the binding transmission constraints.
- $\mathbf{H_R} \in \mathbb{R}^{b \times r}$ is the matrix of shift factors of generators $g \in R$.
- $\mathbf{\Lambda} \in \mathbb{R}^{(n-r) \times (n-r)}$ is a diagonal matrix whose diagonal entries are the inverse of the derivatives of the offers of generators $g \notin R$, evaluated at the market clearing condition.

Transmission Constrained Inverse Residual Demand Jacobian

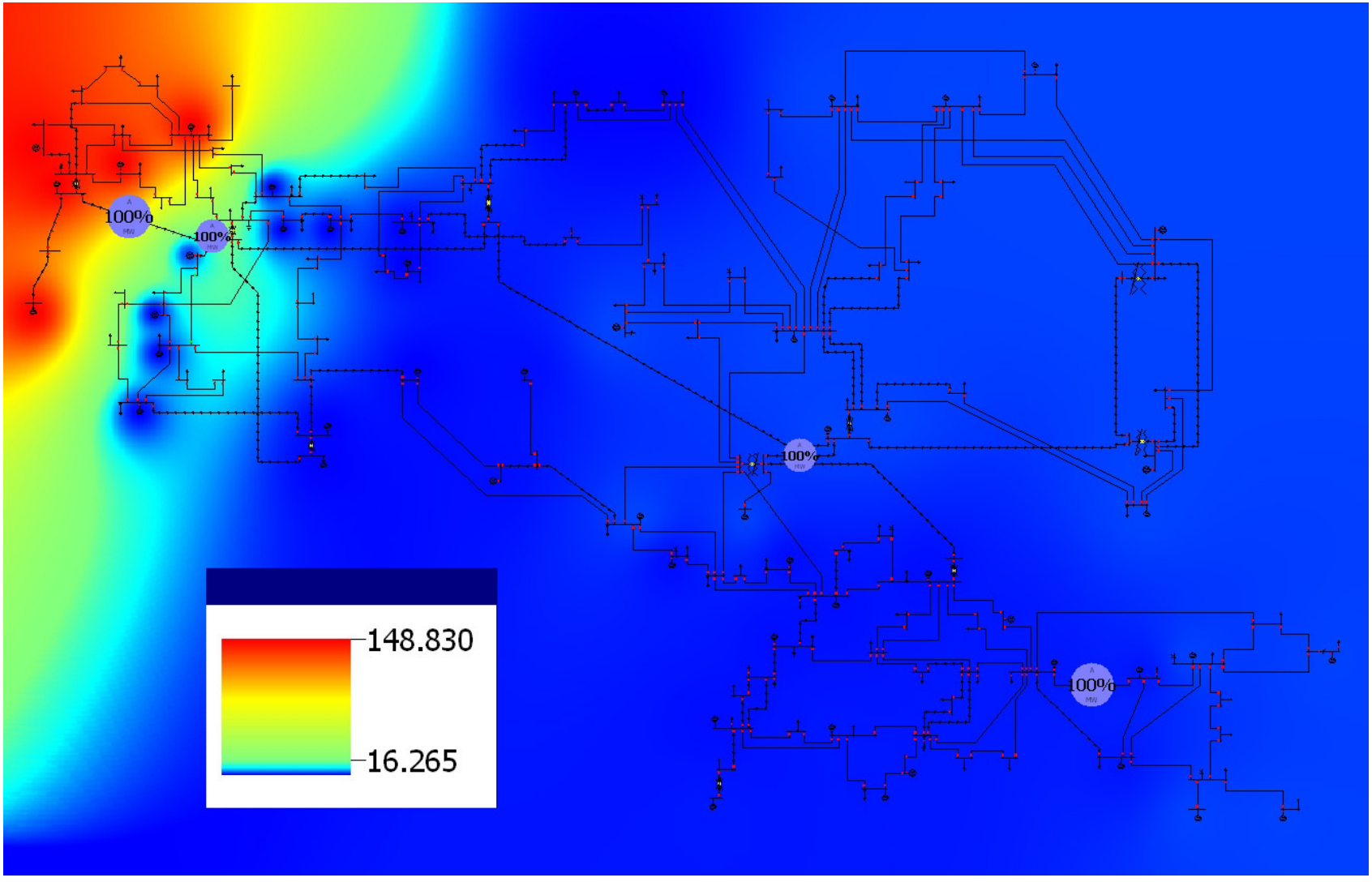
- Firm-based transmission constrained market power indices calculation requires following information:
  1. Shift factors for binding transmission constraints.
  2. Offer information.
  3. Market clearing quantities.
- It is post-OPF analysis, which relies on the result of OPF.

Implementation Framework

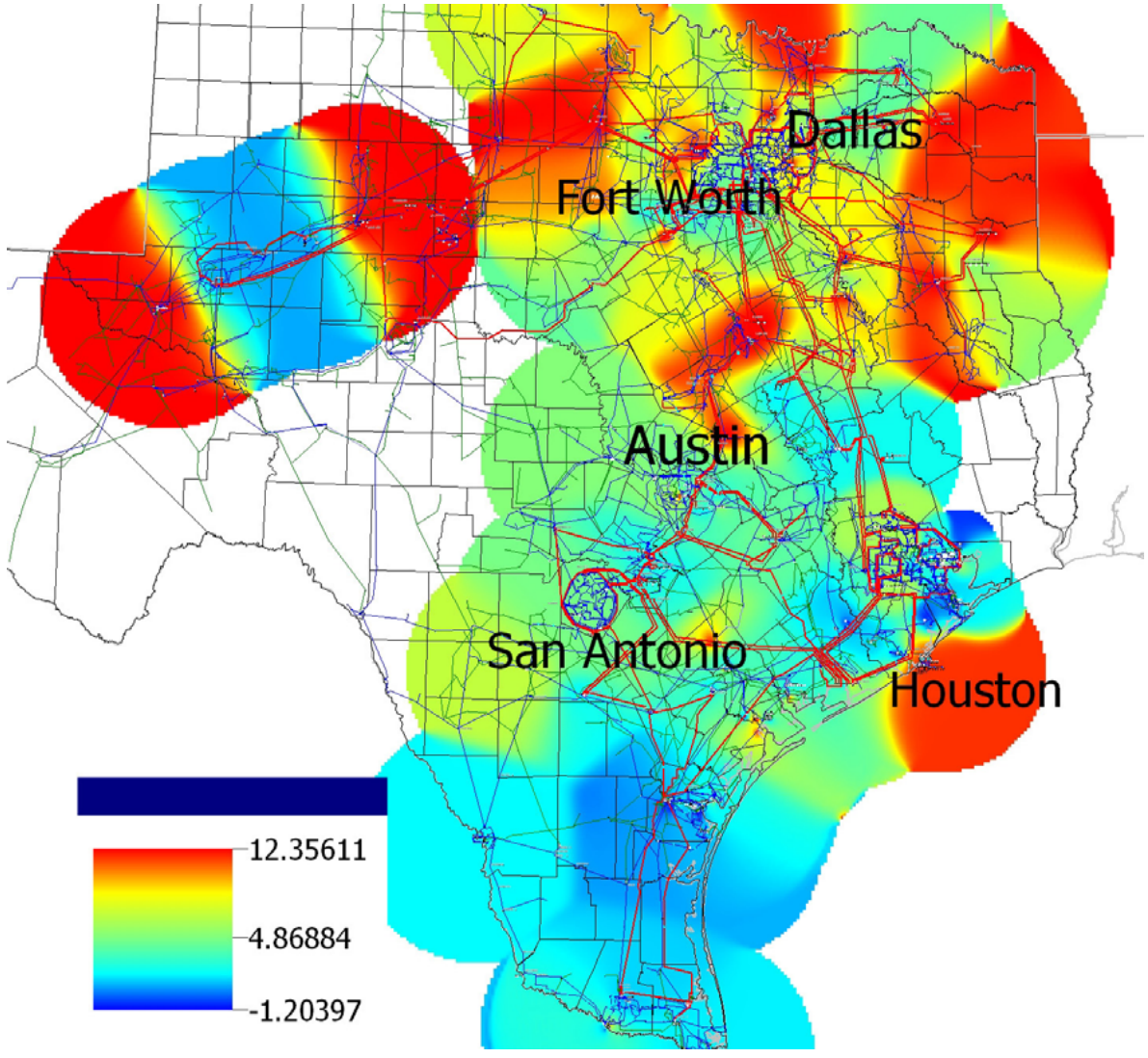
- We use Powerworld as the OPF solver to produce the market clearing results.
- Based on the market clearing results from Powerworld, we calculate the indices using MATLAB.
- Powerworld is utilized again to visualize the indices.



Quantity-Weighted Average Mark-Up of Reliability Test System



Quantity-Weighted Average Mark-Up of ERCOT System (on a log scale)



Comparison of TCRSI and Indices

- So far, we discussed two firm-based market power analysis approaches: the TCRSI and market power indices.
- These two approaches are different from many perspectives, but they also have some common characteristics.

TCRSI vs. Market Power Indices

Comparison:

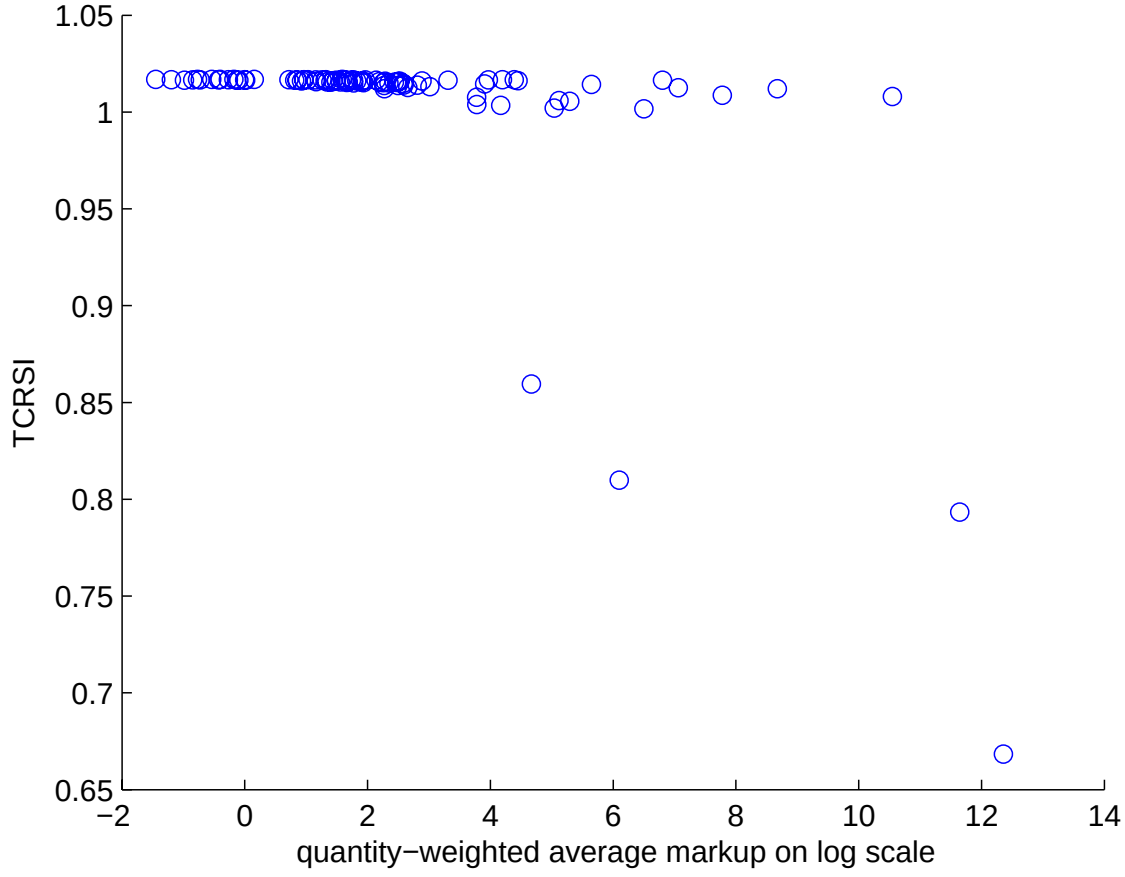
TCRSI	Market Power Indices
pre-OPF	**post**-OPF
No offers information needed	Offers information needed
Predicting market power	Evidence of **exercising** market power
"Large-signal" analysis	**"Small-signal"** analysis

TCRSI vs. Market Power Indices

- Despite the differences from many perspective, these two approaches have a common charactericstic: they both focus on what a firm would do, given the behavior of others.
- In other words, when we analyze a given firm using these two approaches, we leave all the generation from the firm out of the stories.

Scatter plot of TCRSI and indices of ERCOT

- From the scatter plot, we can observe that the TCRSI and market power indices are "complementary" in some case.
- As a result, we suggest that both of these approaches can be integrated into market power analysis flow.



Limitations and Conclusion

- Firm-based market power indices are based on the hypothesis of profit maximization. In other words, it might mistakenly flag the firms that are not maximizing their profit.
- The TCRSI only recognized the pivotal firms. However, sometimes a firm could have market power even without being pivotal.
- Two different approaches to analyzing market power are proposed.
- Market power is usually hard to be analyzed, especially in the context of transmission constraints. Thus we might need more than one approach to get a better picture.