A Model for a Zonal Operating Reserve Demand Curve

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Outline

- What is operating reserves?
- Why do we need elastic reserves demand curve?
- An example of the energy/reserves co-optimized market.
- Why do we need zonal reserves procurement?
- Value of unserved energy without transmission constraints.
- Value of unserved energy considering inter-zone congestion.
- Conclusion.

Introduction

- This material is based on a work by William Hogan, "A Model for a Zonal Operating Reserve Demand Curve", presented at MIT in October 2009.
- There are two concepts we will focus: "Zonal" and "Elastic demand curve".
- Briefly introducing the operating reserves in electricity markets.

Operating reserves

- Operating reserves is generation capacity available to system operator to continuously meet the demand due to demand fluctuation or outages of resources. It is the main part of the ancillary services.
- Operating reserves are classified by how quick they could response to the system operator: regulation(few seconds), spinning reserve(10 minutes), non-spinning reserve(10,30 minutes).In the following materials, we treat them all as operating reserves.
- System operator procures reserves through the markets (semi-annual, day-ahead or real-time).

Why elastic demand curve?

- Currently, the system operator procures an fixed amount of reserve from the market. The demand for reserves is perfectly inelastic.
- Ancillary service markets is coupling with the energy markets, higher demand of ancillary service might also drives up the energy prices significantly.
- An elastic demand curve for reserves that precisely represents the benefit of additional reserves could deploy the reserve resource more efficiently and also mitigates the market power.
- Constructing the demand curve from first principles is necessary to truthfully represent the marginal benefit of reserves.

Energy-Reserve co-optimized market

- Energy and reserves are used to be procured in separate markets, traded as different commodities.
- However, the energy and reserves are actually tightly coupled. Generation resource are utilized as either energy or reserves.
- Nowadays, most ISOs adopted the co-optimized methodology to procure energy and reserves at the same market clearing process which considers the interaction of energy and reserves.

Example of co-optimized energy and reserve

- $\bar{P}_i$ is the capacity of generator i, $\bar{R}_i$ is the capacity of spinning reserve. Note that the dispatched energy plus procured reserve needs to be smaller than capacity of the generator.
- The system specification:

$$\bar{P}_1 = 300(\text{MW}), \bar{R}_1 = 50(\text{MW}), c_1' = 20(\$/\text{MWh})$$
$$\bar{P}_2 = 150(\text{MW}), \bar{R}_2 = 70(\text{MW}), c_2' = 50(\$/\text{MWh})$$
$$\bar{P}_3 = 50(\text{MW}), \bar{R}_3 = 30(\text{MW}), c_3' = 80(\$/\text{MWh})$$

Assume marginal cost of reserves are all 0.

Example of co-optimized energy and reserve

- Considering the case that demand is 350MW and reserve requirement is 90MW.
- The optimal dispatch:

$$P = \begin{bmatrix} \mathbf{300} \\ 50 \\ 0 \end{bmatrix} \text{(MW)}, R = \begin{bmatrix} 0 \\ 60 \\ \mathbf{30} \end{bmatrix} \text{(MW)}$$

- Bold font in P implies that the capacity constraints is binding ($P + R = \bar{P}$), while bold fonts in R implies the reserve capacity constraint is binding ($R = \bar{R}$).
- Define the clearing price as the additional cost to procure a unit more of energy/reserve.

Example of co-optimized energy and reserve

- The optimal dispatch:

$$P = \begin{bmatrix} \mathbf{300} \\ 50 \\ 0 \end{bmatrix} \text{(MW)}, R = \begin{bmatrix} 0 \\ 60 \\ \mathbf{30} \end{bmatrix} \text{(MW)}$$

- Generator 2 would be dispatched more for the additional unit of demand. Thus the clearing price is 50 \$/MWh.
- Generator 2 would provide for the additional unit of reserve. The clearing price is 0 \$/MWh.

Example of co-optimized energy and reserve

- Considering the case that demand is still 350MW but reserve requirement is 140MW.
- The optimal dispatch:

$$P = \begin{bmatrix} \mathbf{260} \\ \mathbf{80} \\ 10 \end{bmatrix} \text{(MW)}, R = \begin{bmatrix} 40 \\ \mathbf{70} \\ \mathbf{30} \end{bmatrix} \text{(MW)}$$

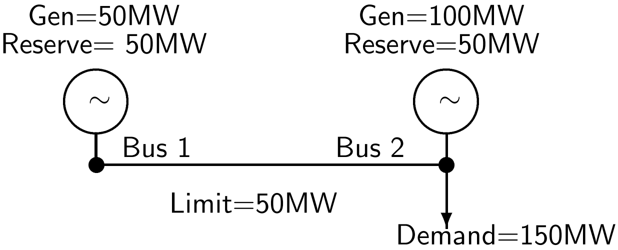
- The clearing price for energy is 80$/MWh.
- Only GEN 1 could provide the additional MW of reserve. However, we need to decrease a MW of the energy output of GEN 1 and use GEN 3 to compensate that MW. The cost to provide the additional MW of reserve is 80-20 = 60$/MWh, which is clearing price for reserve by definition.

Example of co-optimized energy and reserve

- Even though the offers of reserve are 0, the clearing price of reserve is non-zero.
- The reserve price reflects the opportunity cost of GEN 1 to provide reserve.
- Demand of energy is fixed, but the increase of reserve requirement drives up both prices of energy and reserve.
- Therefore, an appropriate level of reserve requirements would be desired.

Why zonal reserve?

- Zonal reserve requirements are aimed to allocate reserve more precisely, considering the possible transmission congestions that block the delivery of reserves.
- Considering a two bus example, where the total reserve equals to the output of largest single generator. Assume that reserves are provided by different generators.
- When forced outage occurs at generator at bus 2, the transmission congestion limits the shipment of reserves from bus 1. Some demand has to be involuntarily curtailed.



Probability of lost load

- Ignore the network features for the first illustration. The amount of unserved load is defined as: $\text{Max}(0, \Delta\text{Load} + \text{Outage} - \text{Operating Reserves})$.
- This produces the familiar loss of load probability (LOLP) calculation:

$$\text{LOLP} = Pr(\Delta\text{Load} + \text{Outage} \geq r) = \bar{F}_{LOL}(r)$$

- A common characterization of a reliability constraint is that there is a limit on the LOLP. This imposes a constraint on the required reserve r: $\bar{F}_{LOL}(r) \leq LOLP_{max}$.

Expected value of lost load

- An alternative approach is to consider the expected unserved energy (EUE) and the value of lost load (VOLL).
- Suppose the VOLL per MWh is v, and the probability density function of change of net demand during an hour (demand+outage) is f, then we can obtain the EUE and its total value (VEUE) as:

$$
\begin{aligned}
EUE(r) &= \int_r^{\infty} f(x)(x-r)dx \\
&= \int_r^{\infty} \bar{F}_{LOL}(x)dx \\
VEUE(r) &= vEUE(r)
\end{aligned}
$$

Model of locational operating demand curve

- A difficulty with defining a locational operating reserve demand curve is the complexity of the interactions among locations plus interactions with the transmission grid.
- Given the value of expected unserved energy as a function of reserve at each zone and the transmission capacity reservation between zone, we could formulate the economic dispatch problem as follows:

$$
\begin{aligned}
\underset{y^0, d^0, g^0, r, \bar{r}}{\text{maximize}} \quad & B(d^0) - C_g(g^0) - C_r(r) - ZVEUE(r, \bar{r}) \\
\text{subject to} \quad & y^0 = d^0 - g^0 \quad \text{(net import at each bus)} \\
& 1^\dagger y^0 = 0 \quad \text{(power balance constraint)} \\
& H^0 y^0 \leq b^0 \quad \text{(transmission constraints)} \\
& g^0 + r \leq Cap^0 \quad \text{(generator capacity constraints)}
\end{aligned}
$$

Model of locational operating demand curve

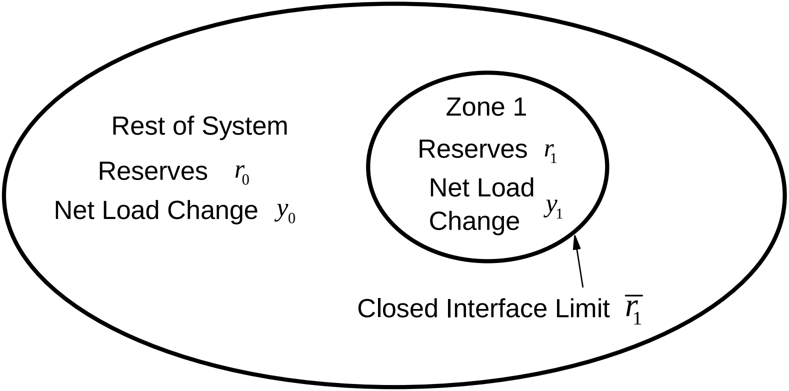
- With sufficient regularity assumptions, we linearize the ZVEUE at some point $(\hat{r}, \hat{\bar{r}})$. So the problem now comes to how to evaluate the gradient of ZVEUE function efficiently.

$$
\begin{aligned}
\underset{y^0, d^0, g^0, r, \bar{r}}{\text{maximize}} \quad & B(d^0) - C_g(g^0) - C_r(r) - \nabla ZVEUE(\hat{r}, \hat{\bar{r}})^t (r, \bar{r}) \\
\text{subject to} \quad & y^0 = d^0 - g^0 \\
& 1^\dagger y^0 = 0 \\
& H^0 y^0 \leq b^0 \\
& g^0 + r \leq diag(\hat{u}) Cap^0 \\
& A^0 y^0 + \bar{r} \leq \bar{r}_{int}
\end{aligned}
$$

Locational operating reserve demand curve

- The characteristics of Hogan's locational reserve demand curve model:
  1. It uses the simple model of loss of load from random changes in demand as a starting point.
  2. It considers the tradeoffs between normal energy dispatch and reservation of interface capacity (the effects of $\bar{r}$).
  3. Under some conditions, reserves in one location can support outages in another location.

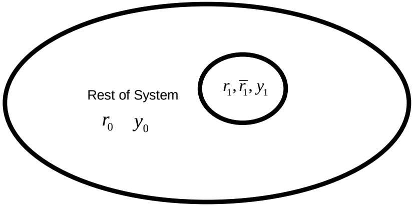
Zonal Interface Limit on Emergency Transfers

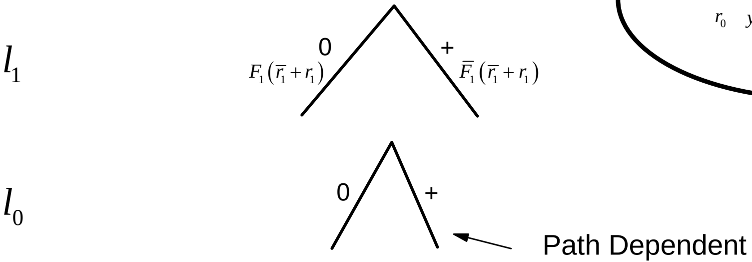


$$y_0 \sim f_0, y_1 \sim f_1, F_o\left(y_0\right) = \int_{-\infty}^{y_0} f_0\left(x_0\right) dx_0, F_1\left(y_1\right) = \int_{-\infty}^{y_1} f_1\left(x_1\right) dx_1$$

Loss of Load Probability Structure

$$ZVEUE\left(r_0,\overline{r}_1,r_1\right)=E_y\left[\underset{l_i\geq 0}{Min}\quad\left\{v_0 l_0+v_1 l_1 \middle| y_0+y_1-l_0-l_1\leq r_0+r_1,\, y_1-l_1\leq \overline{r}_1+r_1\right\}\right]$$



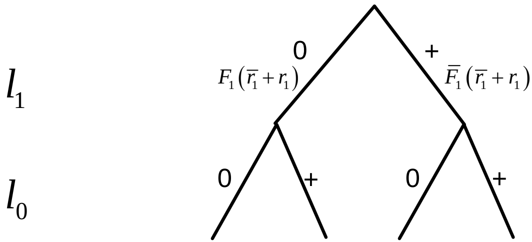


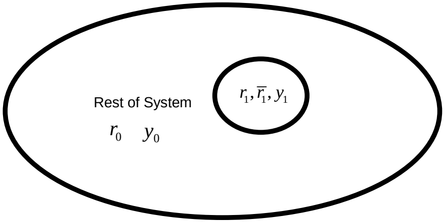
$$y_i \sim f_i,\, F_i\left(y_i\right)=\int_{-\infty}^{y_i} f_i\left(x_i\right)dx_i \qquad VOLL_0=v_0\leq VOLL_1=v_1.$$

Loss of Load Probabilities

$$ZVEUE\left(r_0,\overline{r}_1,r_1\right)=E_y\left[\underset{l_i\geq 0}{Min}\quad\left\{v_0 l_0+v_1 l_1 \,\middle|\, y_0+y_1-l_0-l_1\leq r_0+r_1,\ y_1-l_1\leq\overline{r}_1+r_1\right\}\right]$$

Reserve Incremental Values

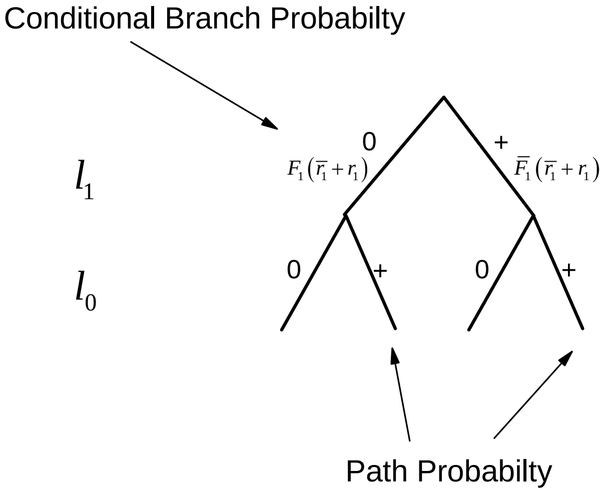


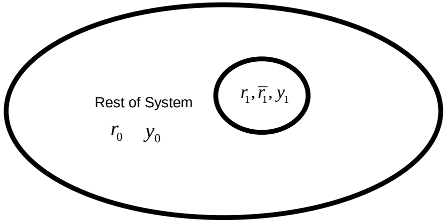


r_1	0	v_0	v_1	v_1
$\overline{r}_1$	0	0	v_1	v_1-v_0
r_0	0	v_0	0	v_0

Loss of Load Probabilities

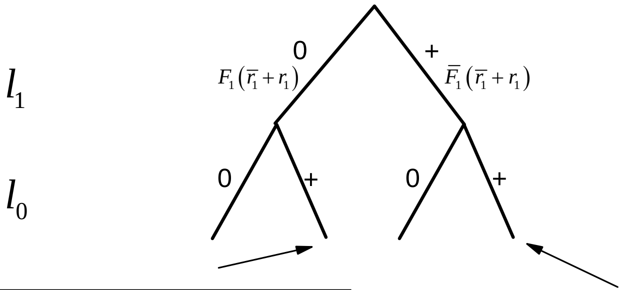
$$ZVEUE\left(r_0,\overline{r}_1,r_1\right)=E_y\left[\underset{l_i\geq 0}{Min}\quad\left\{v_0l_0+v_1l_1\,\middle|\,y_0+y_1-l_0-l_1\leq r_0+r_1,\;y_1-l_1\leq\overline{r}_1+r_1\right\}\right]$$

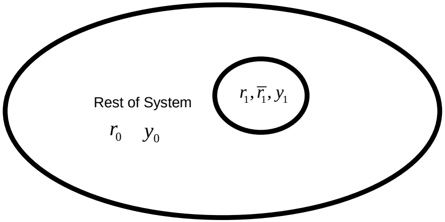




Demand Curve Elements

Path Probabilty



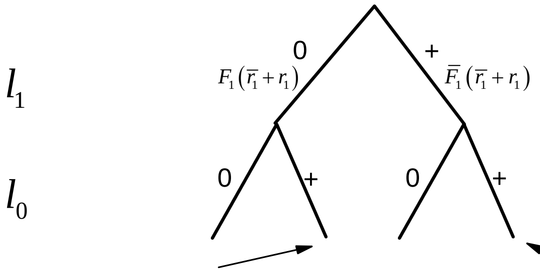


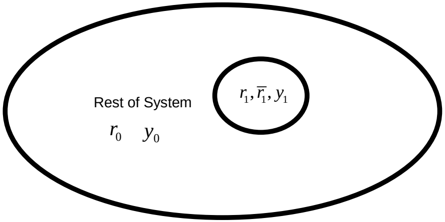
$$\int_{-\infty}^{\bar{r}_1+r_1} \int_{r_0+r_1-x_1}^{\infty} \prod_{i=0}^{1} f_i(x_i)\, dx_i = \int_{-\infty}^{\bar{r}_1+r_1} \bar{F}_0(r_0 + r_1 - x_1)\, f_1(x_1)\, dx_1$$

$$\int_{\bar{r}_1+r_1}^{\infty} \int_{r_0-\bar{r}_1}^{\infty} \prod_{i=0}^{1} f_i(x_i)\, dx_i = \bar{F}_1(\bar{r}_1 + r_1)\, \bar{F}_0(r_0 - \bar{r}_1)$$

Demand Curve Elements: Rest of System

$$p_{r_0} = v_o \left[\int_{-\infty}^{\bar{r}_1 + r_1} \bar{F}_0\left(r_0 + r_1 - x_1\right) f_1\left(x_1\right) dx_1 + \bar{F}_1\left(\bar{r}_1 + r_1\right) \bar{F}_0\left(r_0 - \bar{r}_1\right) \right]$$



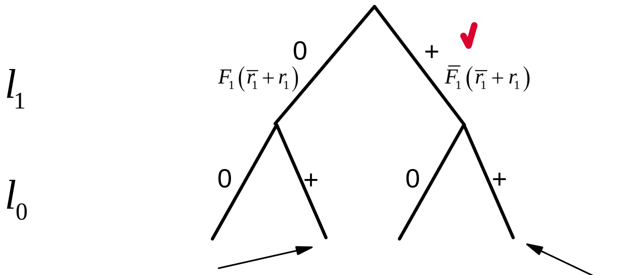


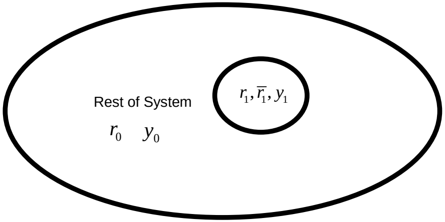
$$\int_{-\infty}^{\bar{r}_1 + r_1} \int_{r_0 + r_1 - x_1}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i = \int_{-\infty}^{\bar{r}_1 + r_1} \bar{F}_0\left(r_0 + r_1 - x_1\right) f_1\left(x_1\right) dx_1$$

$$\int_{\bar{r}_1 + r_1}^{\infty} \int_{r_0 - \bar{r}_1}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i = \bar{F}_1\left(\bar{r}_1 + r_1\right) \bar{F}_0\left(r_0 - \bar{r}_1\right)$$

Demand Curve Elements: Zone 1

$$p_{r_1} = v_1 \bar{F}_1(\bar{r}_1 + r_1) + v_o \left[\int_{-\infty}^{\bar{r}_1 + r_1} \bar{F}_0(r_0 + r_1 - x_1)\, f_1(x_1)\, dx_1 \right]$$





$$\int_{-\infty}^{\bar{r}_1 + r_1} \int_{r_0 + r_1 - x_1}^{\infty} \prod_{i=0}^{1} f_i(x_i)\, dx_i$$

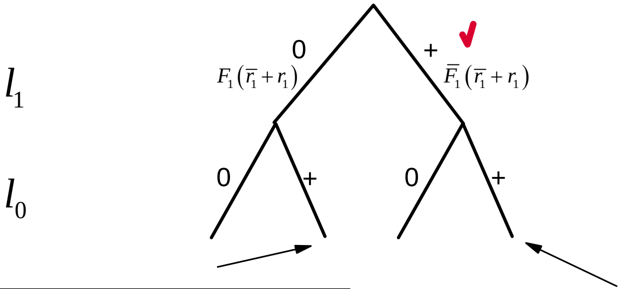
$$= \int_{-\infty}^{\bar{r}_1 + r_1} \bar{F}_0(r_0 + r_1 - x_1)\, f_1(x_1)\, dx_1$$

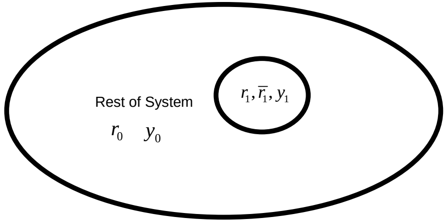
$$\int_{\bar{r}_1 + r_1}^{\infty} \int_{r_0 - \bar{r}_1}^{\infty} \prod_{i=0}^{1} f_i(x_i)\, dx_i$$

$$= \bar{F}_1(\bar{r}_1 + r_1)\, \bar{F}_0(r_0 - \bar{r}_1)$$

Demand Curve Elements: Interface

$$p_{\bar{r}_1} = v_1 \bar{F}_1(\bar{r}_1 + r_1) - v_0 \left[\bar{F}_1(\bar{r}_1 + r_1) \bar{F}_0(r_0 - \bar{r}_1) \right]$$





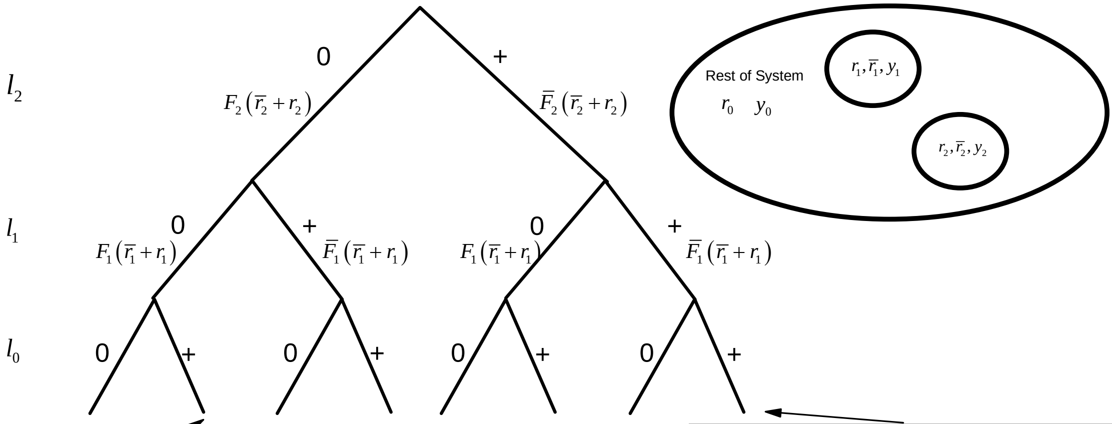
$$\int_{-\infty}^{\bar{r}_1 + r_1} \int_{r_0 + r_1 - x_1}^{\infty} \prod_{i=0}^{1} f_i(x_i)\, dx_i$$

$$= \int_{-\infty}^{\bar{r}_1 + r_1} \bar{F}_0(r_0 + r_1 - x_1)\, f_1(x_1)\, dx_1$$

$$\int_{\bar{r}_1 + r_1}^{\infty} \int_{r_0 - \bar{r}_1}^{\infty} \prod_{i=0}^{1} f_i(x_i)\, dx_i$$

$$= \bar{F}_1(\bar{r}_1 + r_1) \bar{F}_0(r_0 - \bar{r}_1)$$

Demand Curve Elements



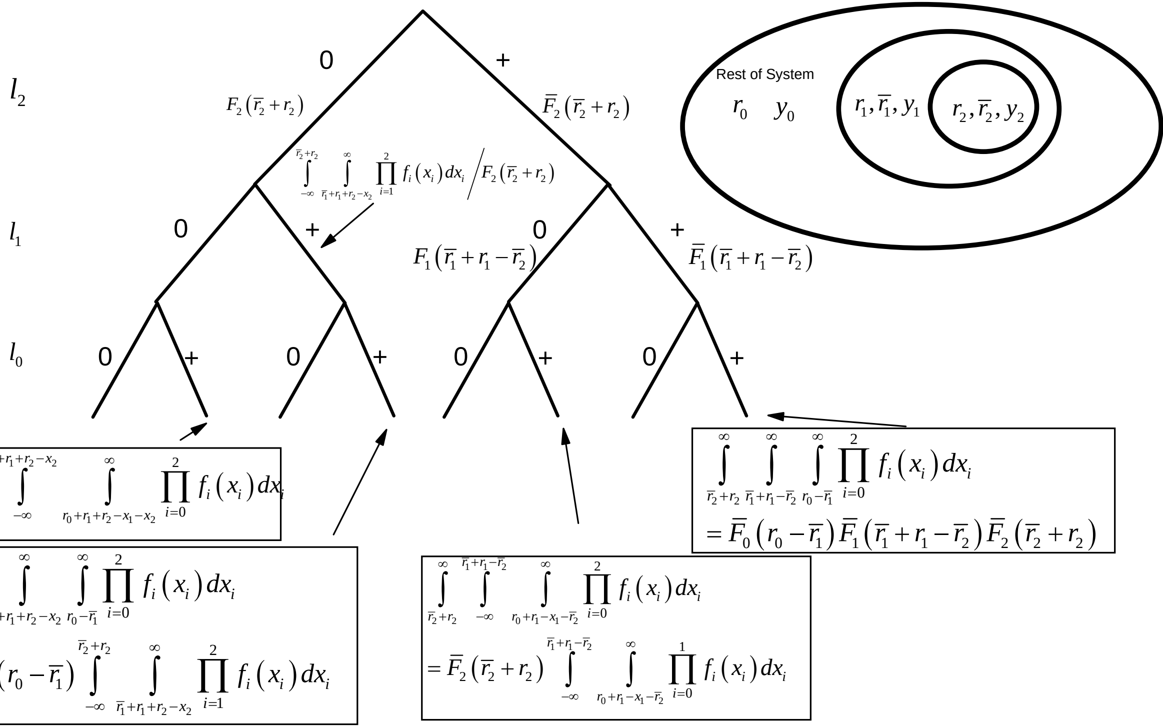
$$\int_{-\infty}^{\bar r_2 + r_2} \int_{-\infty}^{\bar r_1 + r_1} \int_{r_0 + r_1 + r_2 - x_1 - x_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\, dx_i$$

$$\int_{-\infty}^{\bar r_2 + r_2} \int_{\bar r_1 + r_1}^{\infty} \int_{r_0 + r_2 - \bar r_1 - x_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\, dx_i$$
$$= \bar F_1(\bar r_1 + r_1) \int_{-\infty}^{\bar r_2 + r_2} \int_{r_0 + r_2 - \bar r_1 - x_2}^{\infty} \prod_{i=0,2} f_i(x_i)\, dx_i$$

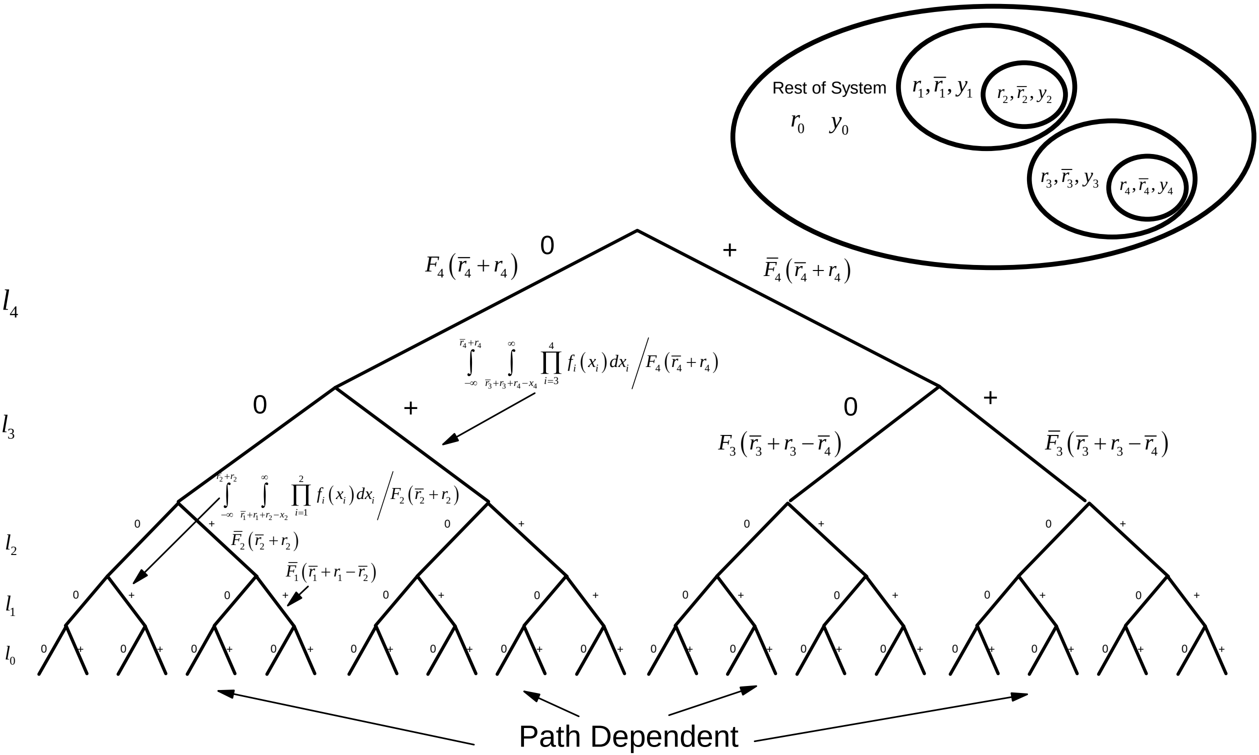
$$\int_{\bar r_2 + r_2}^{\infty} \int_{-\infty}^{\bar r_1 + r_1} \int_{r_0 + r_1 - x_1 - \bar r_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\, dx_i$$
$$= \bar F_2(\bar r_2 + r_2) \int_{-\infty}^{\bar r_1 + r_1} \int_{r_0 + r_1 - x_1 - \bar r_2}^{\infty} \prod_{i=0}^{1} f_i(x_i)\, dx_i$$

$$\int_{\bar r_2 + r_2}^{\infty} \int_{\bar r_1 + r_1}^{\infty} \int_{r_0 - \bar r_1 - \bar r_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\, dx_i$$
$$= \bar F_0(r_0 - \bar r_1 - \bar r_2)\, \bar F_1(\bar r_1 + r_1)\, \bar F_2(\bar r_2 + r_2)$$

Nested Demand Curve Elements



Mixed Demand Curve Elements



Conclusion of Hogan's work

- The probability trees provide a workable means for beginning with the locational probability distributions of load and outages and calculating the resulting demand curves.
- The implied demand curve illustrate critical properties:
  1. **Interaction**: The demand curves are interdependent.
  2. **Interface Demand**: In addition to the demand for operating reserves, there is an implied demand curve for the interface transfer limit.

Conclusion

- One limitation of Hogan's approach is that it can not extend to the meshed zone model.
- ISO might work towards the elastic reserve demand in the future to improve the market efficiency.
- To precisely estimate the benefit of reserve is non-trivial and is an active research area.