Border flow rights and Contracts for differences of differences Technical reference

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Abstract

In this paper a property rights model for electric transmission is proposed and its properties analyzed. The proposed rights, called "border flow rights," support financial hedging of transmission risk and merchant transmission expansion through associated financial rights, called "contracts for differences of differences." These financial rights allow for forward trading of both energy and transmission by a unified exchange, avoiding the bifurcation in current markets between decentralized long-term energy trading and centralized long-term transmission trading. Such long-term trading can help to support the financing of both generation and transmission assets. We consider incentive properties of such a right in the absence of market power.

Keywords

Electricity market, Property rights, Transmission investment, Financial transmission rights, Energy and transmission trading.

I. Introduction

This paper builds on recent work by Gribik, Shirmohammadi, *et al.* [1], [2] that describes long-term property rights for transmission expansion, on provisions in the Australian electricity code [3], and on discussions of transmission in [4], [5], [6]. We propose a property rights model for existing and new transmission investment, called "border flow rights." By a property rights model for transmission, we mean a definition of an underlying revenue stream that accrues to the owner of a transmission line. In particular, under a basic implementation of the border flow rights model, the owner of a transmission line or lines is paid the locational marginal price for energy that it delivers to the rest of the system and pays the locational marginal price for energy that it receives from the rest of the system. The border flow rights model therefore values transmission by its contingency-constrained transport of lower value energy to locations having higher value energy. Border flow rights provide an approximation to efficient marginal incentives for transmission expansion funded by coalitions of beneficiaries.

The basic formulation can also be expanded to value transport of real and reactive power and transport of real and reactive reserves. (See [7] for a development that includes both real and reactive power.)

Border flow rights accommodate trading of financial transmission congestion hedging instruments called "contracts for differences of differences" (CFDDs). Trading based on CFDDs is reminiscent of "contract path" transactions [8] but, unlike the contract path mechanism, incorporates Kirchhoff's laws and represents contingency constraints. CFDDs can be arranged between parties analogously to "contracts for differences" (CFDs) that are used to hedge locational marginal price variation at a given location [6, section 3][9, section 3-2.1]

CFDDs define an alternative to "financial transmission rights" (FTRs) [8][9, chapter 5-9] that, in principle, can be traded without a central exchange. Trading of CFDDs is unlike the auctioning of FTRs in current markets that *requires* the Independent System Operator (ISO) to be intimately involved in the forward trading of transmission in order to guarantee "revenue adequacy" for the ISO [8, appendix][9, page 439]. In contrast, the ISO need not be involved in trading of CFDDs because revenue *neutrality* for the ISO is guaranteed by the border flow rights model. This feature avoids the need for ISOs to be involved in FTR trading,

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contrasting fundamentally with current formulations of transmission property rights including that in Gribik, Shirmohammadi, *et al.* [1], [2]. In current markets, "basis spreads" [10] provide a similar hedging instrument in current markets, but are evidently not built on an underlying revenue stream.

Although contracts for differences of differences can, in principle, be traded without an exchange, such an exchange is likely to help in matching providers of transmission services with users of transmission services, having a role that is somewhat more extensive than the role of a clearing-house for energy contracts. The exchange could facilitate trade of *both* forward transmission and forward energy contracts simultaneously over various timescales. This would avoid the current separation of forward trading of transmission, involving ISOs, from forward trading of energy, which is generally not performed by ISOs [11, section 3].

In the context of energy, CFDs allow the co-existence of short-term offer-based economic dispatch by the ISO with longer term financial contracts to establish forward financial positions, without either constraining the other. In contrast, currently implemented FTR mechanisms require the ISO to be intimately involved in the allocation and reconfiguration of forward transmission rights because congestion rental accruing to the ISO provides the revenue stream to fund the FTRs [8, appendix]. Contracts for differences of differences, as proposed in this paper, would allow the ISO to instead focus solely on short-term dispatch, while forward financial positions could be established without ISO involvement and without ISO financial exposure. This would allow Independent System *Operators* to focus on operational issues.

The organization of the paper is as follows. Section II contains a brief literature survey and contextualizes the goals of the paper. In Section III, we discuss the hedging of transmission and energy prices. In Section IV, we discuss the remuneration of transmission investment, with Section V presenting an example. Section VI discusses the proposed financial hedging mechanism, the relationship to the contract path paradigm, and option rights. Alternatives for trading of the rights are sketched in Section VII using the example from Section V to illustrated the alternatives. Merchant transmission is discussed in Section VIII, using the example from Section V to illustrate. Section IX concludes. An Appendix establishes some theoretical results that simplify and generalize the analysis in Gribik, Shirmohammadi, *et al.* [2] and applies the results to the example system of Section V.

II. Literature survey and goals

Financial transmission rights have been defined and discussed in several papers. For example, see: Hogan [8], [12], [13]; Chao and Peck [14], [15]; Oren [16]; and Bushnell and Stoft [17], [18], [19]. Point-to-point financial transmission rights, as typically defined as being issued by the ISO, must be reconfigured centrally in order that the collection of rights satisfies the "simultaneous feasibility test" (SFT) [8, appendix][9, page 439]. The first goal of the present paper is to remove the need for the ISO to be the issuer of rights, allowing for reconfiguration of transmission rights by an entity other than the ISO.[1] This goal is achieved by defining transmission property rights in terms of an underlying revenue stream that depends only on the prices and flows resulting from offer-based security-constrained economic dispatch.

A second goal is to remove the risk to the ISO of revenue shortfall under transmission outage conditions. The risk is devolved to the owners of the transmission assets.

The third goal of the present paper is to define a property right and associated financial right that supports merchant transmission expansion. Hogan discusses merchant based transmission expansion in [21], while Joskow and Tirole present various problems with merchant expansion in [22].

We make no attempt to solve all the problems associated with merchant transmission that are described in [22]. Experience with merchant transmission in Australia is described in [23], [24], while experience with merchant transmission in Argentina is described in [25], [26]. Joskow describes transmission policy in the United States in [27].

Moreover, the main focus in the paper is on energy rather than on reserves or reactive power. We do not

[1] Although the ISO need not participate in the reconfiguration of the transmission rights, the ISO is still assumed to perform centralized offer-based security-constrained economic dispatch. That is, the dispatch function is not decentralized. Approaches to decentralized dispatch in the presence of congestion are discussed in [20].

directly consider the value of transmission in enhancing reliability by the sharing of reserves, except as it affects the contingency-constrained capacity of the network to transport energy. We do not directly consider reactive power. The formulation can be expanded to include explicit representation of reserves and reactive power and payment to transmission for "transporting" reserves and reactive power. In a related context, both real and reactive power is explicitly considered in [7].

Finally, we observe that we omit discussion of market power in both the energy and transmission markets, but appreciate that it can be a problematic issue, with important inter-relationships between energy and transmission prices. Furthermore, our main result on incentives for transmission construction explicitly ignores market power. For discussions of market power see, for example, [9, part 4][28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39]. Moreover, we do not consider the value of transmission in mitigating locational market power. In particular, see [33] for cases where a line with small capacity and zero flow has an important role in mitigating market power in two inter-connected markets. Our proposal would not directly remunerate such a line.

III. Hedging of transmission and energy prices

From a normative perspective, pricing of transmission services to customers should incent the efficient use of scarce transmission capacity. It is well known that locational marginal prices (LMPs) provide efficient incentives for generators and consumers and for transmission use [40]. In contrast, "contract path" pricing of transmission services usually provides an inefficient signal to users of transmission services [8].

Since LMPs and LMP differences are volatile, however, market participants typically desire financial instruments to hedge against the variation in LMPs and LMP differences. In the absence of transmission constraints and ignoring losses, CFDs can be used to hedge LMP volatility. However, when there are binding transmission constraints, CFDs alone cannot hedge a transaction from a generator to consumer that are not co-located, since the LMP at the generator and consumer will differ and the difference will vary over time. As Bushnell and Stoft point out, exposure to volatility in LMP differences cannot be costlessly hedged by energy trading alone [6, section 2.2].

Financial transmission rights (FTRs) [13] are in use in several electricity markets to hedge the volatility of LMP differences when there are transmission constraints. Such rights are either allocated to pre-existing transmission rights holders, or sold through a centralized auction or sequence of auctions, or both.

In all existing market implementations, the auctions for FTRs are conducted by the ISO, which sells the rights to bidders on the basis of their willingness-to-pay for the transmission rights. A purchaser of a "point-to-point" obligation FTR receives, over the contract duration of the FTR, the right to a revenue stream from, say, the day-ahead market, equal to the FTR contract quantity multiplied by the difference between the LMP at the point of withdrawal minus the LMP at the point of injection [13]. This revenue stream is paid from the congestion rental accruing to the ISO from offer-based security-constrained economic dispatch (OBSCED). Financial instruments other than point-to-point obligation rights present some difficulties because of the implications for the SFT but are being implemented in some markets [41, page 49].[2]

To ensure that the congestion rental is *adequate* to cover the payments to FTR holders, the allocated and sold rights must satisfy the SFT. That is, the allocated rights together with the sold rights must satisfy the transmission constraints, including contingency constraints, in order to ensure "revenue adequacy." (See [8, appendix],[44], [45] for formal statement and proof of this result.) The involvement of the ISO in the auction and the use of the SFT is predicated on the assumptions that:

1. the FTRs are paid out of the congestion rental accruing to the ISO in the OBSCED, and
2. the ISO should remain on net, at least approximately, revenue neutral, with congestion rental from the OBSCED on average at least covering its obligation to FTR holders.

In later sections, we will change the first assumption and in so doing create a transmission rights mechanism that is *exactly* revenue neutral for the ISO.

[2] Alternative financial rights include "flowgate" rights [14], [42], point-to-point option rights, and "contingent" rights [43][41, page 53–54]. Hogan describes some difficulties with flowgate formulations in [13]. Options also pose difficulties [9, page 440].

Transmission rights are often sold for durations that are much longer than a day, during which time some of the transmission lines represented in the SFT "test system" may actually be out of service. When lines are out of service, the revenue adequacy of the issued FTRs will not necessarily hold true. The ISO has several alternatives under these circumstances, such as:

1. it can assume some risk of revenue shortfall (presumably charging it as an "uplift" or averaging the shortfall from pricing intervals when there is an outage against other periods of positive net revenue),
2. the ISO can implement a derating policy or scale down the FTR payments,
3. the ISO can deliberately sell less rights than are implied by the SFT, or
4. the ISO can charge shortfalls to transmission owners.

The first alternative reduces the value of performing the simultaneous feasibility test, which is fundamentally to prevent revenue shortfall. The second alternative blunts the ability of the FTR to hedge transmission charges since transmission customers presumably want to hedge LMP differences whether or not there is a transmission outage, while the third means that some transmission capability is not being offered to the market. That is, none of the first three alternatives are entirely satisfactory in the context of hedging LMP differences. The fourth alternative is used, for example, in New York for maintenance outages [46, pages 46–47][41, pages 52–53].

Furthermore, if a purchaser of point-to-point FTRs finds that its needs change, it will generally not be able to sell the right completely "over the counter" to another party, unless the other party requires an FTR between electrically very similar points.[3] Consequently, relatively frequent auctions are required to enable reconfiguration of the transmission rights as transmission needs change.

To summarize, to ensure revenue adequacy the ISO must only sell transmission rights that collectively satisfy the simultaneous feasibility test, given an assumed test system. During periods when the actual capability falls short of that in the test system, the ISO must have some policy for either covering the revenue shortfalls or derating the system or must otherwise undersell the capability to minimize the likelihood of failing to be revenue adequate. Moreover, the ISO must repeat the auctions on a regular basis to enable reconfiguration of the rights.

These requirements for FTR auctions contrast greatly with financial hedging of energy bought and sold at a single bus. A CFD can be arranged between generators and consumers (or a load serving entity purchasing on behalf of consumers) or through an exchange that is not associated with the ISO. CFDs hedge the volatility of LMPs at a single location and enable financial bilateral contracts in the context of OBSCED. Generators and consumers have opposite tastes for exposure to LMP fluctuations: a high price is "good" for a generator and "bad" for a consumer and vice versa. Consequently, each can costlessly hedge price risk for a specific contractual quantity by signing a CFD.

CFDs can be arranged without any intervention by the ISO and without the CFD posing any revenue risk to the ISO in the event of generation outages.[4] CFDs can be arranged for short durations to support short-term opportunities or for long durations to support the financing of new investment. Moreover, because of their financial character, variations on the basic "obligation" CFD, such as options and collars, can be flexibly defined by contracting parties [47].

In practice, the convenient matching of generators and consumers is likely to benefit from a public exchange. For example, [45] describes a model of a sequence of auctions that can be used to arrange such financial contracts. However, there is no material need for the ISO to be involved in trading of CFDs. The rest of this paper is aimed at the development of a property right definition for transmission and an associated financial mechanism that does allow for decentralized trading of transmission services and which is analogous to CFDs but applied to transmission. The property right that we propose makes transmission owners responsible for the financial implications of outages, as is the case for maintenance outages in New York [46,

[3] In some formulations, "hubs" are defined to allow a point-to-point FTR to be specified as the sum of a "point-to-hub," a "hub-to-hub," and a "hub-to-point" right. In this case, the "hub-to-hub" right is more likely to be easily tradable; however, the "point-to-hub" and "hub-to-point" rights remain specific to the purchaser, requiring a central auction for reconfiguration.

[4] Of course, there is risk to the ISO of parties to a CFD failing to pay the ISO for their energy based on LMPs. However, this risk is not due to the CFD itself and would presumably exist even in the absence of CFDs.

pages 46–47][41, pages 52–53].

IV. Remunerating transmission investment

From a normative perspective, the remuneration of transmission investment should incent efficient transmission investment decisions. This implies that the property right conferred upon an investor in new transmission should produce a revenue stream that incents the efficient level of transmission investment.

In sections IV-A–IV-C, we discuss remuneration schemes that involve financial transmission rights, flowgate rights, and remuneration based on the sensitivity of optimal welfare.

A. Financial transmission rights

FTRs have been proposed as a property right that, under some assumptions, provides efficient incentives for new transmission construction. Under the FTR mechanism, the builder of a line *nominates* an FTR that, together with the other allocated and sold rights, is feasible for the newly expanded system [21], [48].[5] (Such a rights holder might then offer its capacity into the ISO reconfiguration auction to sell to transmission users, possibly involving reconfiguration of this and other rights into different point-to-point rights.)

Bushnell and Stoft [17], [18], [19] show that if an investor is rewarded with such a right then, under the restrictive assumption that the implied operating point for the SFT is the same as the actual operating point for the OBSCED, the investor is incented to make only beneficial modifications to the network because the payment to the investor matches the benefits to the system. "Detrimental" modifications to the network by the investor result in losses to the investor. Hogan also discussed incentives for investment in [21].

In fact, it is unlikely that the implied operating point for the SFT will be the same as the actual operating point of the OBSCED because of diurnal and seasonal variation in demand and the resulting variation in patterns of dispatch over the contract duration of the FTR. If patterns of actual dispatch turn out to be different to that implied in the SFT then the value of the FTR might be relatively small, even if the transmission project itself conferred significant benefits to the system. The ISO will accumulate some of the congestion rental under these conditions.

Because the ISO may accumulate some congestion rental when dispatch differs from the implied SFT operating point, it may not be possible for the builder of the line to nominate an FTR that will provide payment to the builder that reflects the value of the line to the system under all dispatch conditions. The implication is that the builder will not, in practice, be incented to make the most beneficial modifications to the network. Moreover, owners of other, pre-existing, transmission lines have no incentive to maintain the capacity of lines that support the nominated FTR of the investor since their line capacity does not benefit them.

For an extreme example of the implications of the requirement of FTR nomination, suppose that a new line was proposed to join two regions, A and B. Suppose that prices were anticipated to be lower in A than in B most of the time. Therefore, the builder would presumably nominate an FTR from A to B.

However, suppose that the LMP was higher at A than at B for a significant fraction of time. This could be because the flow turned out to be from B to A during these times. The nominated FTR from A to B would have *negative* value during these times, but the transmission project itself might produce considerable value by allowing transmission from B to A. We will see an example of this in Section V.[6]

[5] A complication with this scheme is that there may be some unallocated and unsold capacity still "on the table" before the new transmission is built. In this case, we may prefer to only allow the investor in new transmission to obtain rights to capacity that it actually "contributed." This significantly complicates the nomination process, as discussed in [48], [49], [50], [51].

[6] This situation is not merely theoretical. For example, in the case of the interconnection between the electricity systems of the states of New South Wales and Queensland in Australia, it was initially anticipated that power would be exported from New South Wales to Queensland. However, Littlechild reports that power has often flowed from Queensland to New South Wales [24, page 3 and footnote 7]. Similarly, flows on the line from Almafuerte to El Bracho in Argentina have been in the opposite direction to the initial expectation [26, page 7]. As a third example, the direction of flow on the notoriously limiting Path 15 in California changes with season. Furthermore, in the presence of merchant *generation*, there are likely to be unanticipated patterns of power flow on lines. For example, in West Texas, the transmission system was historically constrained from East to West, with imports into certain areas in West Texas import-constrained. However, with the advent of considerable new wind generation capacity, the constraints are now typically from West to East and are export constraints.

A partial solution to this problem is to perform a sequence of auctions with relatively short contract periods and to allow re-nomination of FTRs for each contract period [48, page 17][41, page 56]. For example, there could be an auction for each season. However, seasonal nominations do not completely solve the problem unless the season correlates strongly with the direction of flow.

To summarize, the value of the FTR depends critically on the nomination of the FTR by the builder and does not evidently allow the builder to capture the value of transmission flows that were not anticipated in the FTR nomination nor to capture the value of flows that are very different to that modeled in the SFT.[7] FTRs do not reflect the "underlying" value of the line to transmit power. They reflect only the underlying value under dispatch conditions that are similar to the implied operating point for the SFT test system.

The right conferred by an FTR should be compared to the property right conferred on a generator that interconnects with the system. In an OBSCED market, such a generator is deemed to have the right to a revenue stream equal, at each time, to its energy generation times the LMP at its bus. The *financial* counterpart of this revenue stream is a financial energy contract at the bus, such as a CFD; however, it is important to note that the *underlying* revenue stream paid to the generator does not depend on any financial nomination such as a CFD: the underlying revenue stream of LMP times energy is conferred upon the generator for its production of energy whether or not it is party to a CFD. The revenue stream pays a generator for the value of the energy it produces at each time.

In contrast, consider the revenue stream accruing to an FTR that is nominated by the builder of the line. The revenue stream depends explicitly on the FTR nomination as well as on the intrinsic line characteristics and the LMPs. In the case of FTRs there is no underlying revenue stream for transmission services that is analogous to a generator being paid LMP times energy. Unlike the case of a generator, one cannot directly observe the value of a transmission expansion (or of existing transmission) in terms of a revenue stream to an asset that is *independent* of a financial nomination.

To summarize, awarding FTRs to incremental transmission investment creates an asymmetry between the right conferred on a transmission investor and the right conferred on a generation investor. Unlike the case for a generator, in the case of transmission there is no underlying revenue stream that is independent of the financial rights nomination. The situation is illustrated in Table I, which compares current implementations of energy and transmission markets. As shown in Table I by the question marks, there is no underlying revenue stream for transmission assets defined in current implementations of energy and AC transmission markets.[8] In Table I, as well as obligation CFDs, various other financial energy products are possible [47].

Even putting aside the problem of nomination of the FTR, there is another issue that is both the principal advantage of and also another problem of the FTR mechanism. Nomination of FTRs assigns all the benefits of incremental transmission to the FTR nominator. That is, the previously "unused" capacity of other lines in the system is effectively appropriated by the FTR nominator. This has the advantage that the nominator is motivated to propose upgrades that maximize overall benefits to the system (ignoring the issue of the deviation of the implied operating point for the SFT from the actual operating point of the system). However, the owners of other lines have no incentive to provide or maintain their capacity in the face of such a mechanism because they do not benefit from it.[9] Moreover, as we will see explicitly in Section V-A.2, the amount of FTR that can be nominated depends on the order of construction of transmission and the nomination of FTRs by other transmission builders.

[7] "Bi-directional" option FTRs can, in principle, solve this problem; however, it is an open question as to how to appropriately represent options in the SFT in a way that is computationally tractable and does not severely limit the amount of FTRs that can be sold, particularly when an AC transmission model is used [12][41, page 53]. We will see in Section VI-C that option instruments can be constructed easily with the financial rights we propose in this paper.

[8] There already exists such an underlying revenue stream for DC transmission, namely the difference between the value of the energy bought and the energy sold. The border flow right proposed in this paper is consistent with this revenue stream.

[9] Although the nominator of the FTR is incented to not propose detrimental upgrades, there is no such incentive for transmission owners that are not party to the FTR. If all transmission had been built under the FTR mechanism this would not be problematic; however, actual transmission systems typically have significant pre-existing capacity that is not built under an FTR system and which must be allocated in some way.

TABLE I

COMPARISON OF CURRENT IMPLEMENTATIONS OF ENERGY AND AC TRANSMISSION MARKETS.

Asset:	Product:	Underlying revenue stream:	Financial instrument:
Generation	Energy	Energy $\times$ LMP	Contract for differences
Transmission	Transport	???	Financial transmission right

B. Flowgate rights

An alternative rights mechanism involves conferring flowgate rights for the contributed capacity of a new line. This right can be defined in terms of an underlying revenue stream. In particular, the property right for a flowgate could be defined to be the right to receive the product of the capacity of the flowgate multiplied by the shadow price on this capacity constraint in the OBSCED. Since lines have capacity limits for flow in each direction, there are effectively two flowgates corresponding to each line and we will assume that the builder of a line receives flowgate rights for both directions.

The revenue stream for a flowgate as just defined is independent of any financial nomination and so is more satisfactory from this perspective. It also confers payments for transmission of power in either direction. Moreover, it is also possible to sell portions of the capacity of a flowgate "over the counter" for different durations without complications.

However, by design, this flowgate right definition will assign the benefits of new capacity to the congested line. As we will see explicitly in Section V-A.3, this will provide no incentive for other lines in the system to maintain their capacity.

C. Sensitivity based rights

To remedy the deficiency of flowgate rights, Gribik, Shirmohammadi, *et al.* [2] propose a *financial* right that depends on both the capacity and the electrical *admittance* of the line [52, chapters 3 and 4]. Gribik, Shirmohammadi, *et al.* suggest a financial transmission right that involves payment based on the sensitivity of optimal welfare to both capacity and admittance.[10]

The border flow right we define has a similar character to that proposed in Gribik, Shirmohammadi, *et al.* [1], [2] in that the underlying revenue stream for the border flow right is based on a similar sensitivity calculation. However, in Section VI we will use this underlying revenue stream to define a financial right. Because of its relationship to the sensitivity of optimal welfare, the border flow right provides incentives to build transmission that are analogous to the guidance provided by sensitivity based transmission planning as described by Dechamps and Jamoulle [53] and Pereira and Pinto [54] and developed in the AC case by Cruz *et al.* [55].[11] In particular, since the underlying revenue stream for the border flow right is based on the sensitivity of optimal welfare, we will see that the border flow right incents efficient *marginal* transmission

[10]Gribik, Shirmohammadi, *et al.* demonstrate the case for both real power and reactive power prices, but we will restrict our discussion in the body of the paper to real power. The principle in Gribik, Shirmohammadi, *et al.* can be generalized to any vector of quantities that are conserved nodally and to any parameters that enter into the power flow equations and the constraints *linearly*, when both the base-case and contingency-case constraints are represented explicitly. For example, one could imagine a payment scheme based on separate prices for real power, reactive power, real power reserves, and reactive power reserves using, for a transmission line, the sensitivities to susceptance, conductance, and line flow limits. Payment to a phase-shifting transformer would be based on the sensitivity to susceptance, conductance, and tap-changer limits. Each of the quantities of real power, reactive power, real power reserves, and reactive power reserves is conserved nodally and each of the parameters of susceptance, conductance, line flow limits, and tap-changer limits enter into the power flow equations and constraints linearly. Payment in a market including real power, reactive power, real power reserves, and reactive power reserves would value transmission for its contingency-constrained transport of these commodities from lower to higher value locations. The development in the Appendix accommodates these generalizations and the case for real and reactive power has been described in [7]. If the parameters appear *non-linearly* in the power flow equations or constraints then a similar sensitivity based scheme will have the same marginal incentive properties; however, such a scheme will no longer be revenue neutral for the ISO.

[11]I am indebted to Professor Rubén Darío Cruz of Universidad Industrial de Santander, Colombia, for bringing this interpretation to my attention.

expansion by coalitions of beneficiaries when all expansion is "price-taking" in the sense of not affecting prices. (See Theorem 5 and Corollary 6 in the Appendix.)[12]

Moreover, by conferring payment on parallel lines and not just on the lines with binding capacity constraints, the payment mechanism more directly encourages competitive suppliers of transmission. For example, consider a line with flow that is less than its capacity, but for which there is a positive difference in LMPs between its ends. Under the right proposed by Gribik, Shirmohammadi, *et al.*, such a line receives payment and can also increase its revenue stream by increasing its admittance and so attracting more power flow on it. By doing so, such a line will be allowing more power to be transferred from nodes with low prices to nodes with high prices and will therefore be increasing welfare.[13]

C.1 No contingency constraints

If we first ignore contingency constraints then the underlying revenue stream we propose for an owner of a transmission line joining nodes k and ℓ is given by:

$$p_k P_{\ell k} + p_\ell P_{k\ell}, \tag{1}$$

where:
- the LMPs at buses k and ℓ are p_k and p_ℓ, respectively,
- the power flow from the line into bus k is $P_{\ell k}$, and
- the power flow from the line into bus ℓ is $P_{k\ell}$.

This payment is equal to the congestion rental on a line in a system consisting of a single radial line between buses k and ℓ, where congestion rental is defined to be the difference between the demand payments and the payments to generators. However, as in [5, equation (2)], we are proposing this payment for all lines, even in non-radial systems. As observed in Gribik, Shirmohammadi, *et al.* [2], (1) is a redistribution of the congestion rental to individual lines [37, section 3.5].

The equivalence between payment based on sensitivity of optimal welfare and payment based on (1) is proved in Theorem 2 in the Appendix, simplifying and generalizing the development in Gribik, Shirmohammadi, *et al.* [2]. The LMPs and the flows on the lines are determined by the ISO as the result of an OBSCED. The payment is made in each pricing interval based on the LMPs and the flows for that pricing interval. (For clarity, we have suppressed the explicit dependence of LMPs and flows on time.) Since $P_{k\ell}$ and $P_{\ell k}$ are generally of opposite sign and since positive flow from the line into the bus will usually be at the higher price bus, the revenue stream is usually positive. Cases where the revenue stream is negative are discussed in Section VI-C.[14]

The revenue stream defined in (1) is analogous to the revenue stream paid to a generator for generation and paid by a consumer for its consumption. In particular, a generator is paid at the LMP for its generation, a consumer pays at the LMP for its consumption and, according to (1), a transmission line is paid at the LMP for energy that it delivers to the system and pays at the LMP for energy it receives from the system. Moreover,

[12] As with all marginal schemes, the presence of scale economies and lumpiness may lead to insufficient capacity being built. In addition to the problems due to scale economies and lumpiness, transmission expansion is also complicated by the "network externalities" due to Kirchhoff's laws. The right proposed by Gribik, Shirmohammadi, *et al.* and the border flow right defined here deal correctly with the network externalities, assuming that the OBSCED represents the network faithfully; however, it does not solve the issues raised by scale economies and lumpiness. For an empirical study of scale economies in transmission construction, see [56].

[13] It is interesting to note that admittance is "always on the margin" in the sense that there will be a non-zero price for admittance if there is at least one transmission constraint binding in the system, whereas capacities of particular lines are not always limiting. I am indebted to Richard O'Neill of the Federal Energy Regulatory Commission for bringing this interpretation to my attention.

[14] Although (1) and variants of it are well-accepted as appropriate ways to charge transmission customers for *use* of transmission services and to remunerate owners of DC transmission [3][24, footnote 64], there have been few formal analyses of a scheme such as (1) for remunerating AC transmission *owners.* In their seminal work, Caramanis, Bohn, and Schweppe specify remuneration consistent with (1), but do not analyze the incentives in detail [4, section 12 and (6)]. Oren *et al.* criticize such a scheme from a market power perspective [57, section IV.A]. Pérez-Arriaga *et al.* describe such a scheme in [5] but criticize it from the perspective of cost recovery in the presence of economies of scale and lumpiness. Bushnell and Stoft allude to the notion of a CFDD in their discussion of "the insurance component" of an FTR, but do not pursue the implications [6, footnote 7]. Gribik, Shirmohammadi, *et al.* explore the scheme in detail [2] in the context of defining a *financial* right. Unlike Gribik, Shirmohammadi, *et al.* we are defining an underlying revenue stream for transmission, not a financial right. We will, however, introduce a financial right associated with this underlying revenue stream.

the ISO is *exactly* revenue neutral under this payment scheme under all dispatch conditions because power is conserved nodally. That is, all the power entering a bus also leaves that bus. Consequently, since all energy is bought and sold at the LMP, the net revenue to the ISO is exactly zero by definition. (See Theorems 1 and 3 in the Appendix for precise statements and proof.)

The proposed revenue stream to transmission owners is in contrast with the situation where only generation is paid at the LMP and only consumers pay at the LMP. In the latter case, the congestion rental accrues to the ISO. As discussed in Section III, the ISO must then pay out the congestion rental in order to make it approximately revenue neutral. This leads to complications if the implied operating point for the issued financial rights do not happen to correspond to a secure dispatch for the system.

C.2 Contingency constraints

In the case where contingency constraints are binding, an exact calculation of the sensitivity of optimal welfare implies a payment involving the sum over base and contingency states of the product of net injection or withdrawal in a state times an appropriate state dependent LMP [1, appendix B]. This result is also proved as Theorem 4 in the Appendix. In simple cases, the payment can be evaluated with (1) using the flows on the lines calculated for the contingency cases. This payment scheme is also revenue neutral for the ISO.

C.3 Proposed property right

As a definition of a property right for transmission, we suggest replacing the question marks in Table I with (1). An approximation in the case of contingency constraints is to define the property right for transmission to be the right to receive the revenue stream specified by (1) based on the pre-contingency flows, even when contingency constraints are binding.

In the case of a lossless line, the revenue stream would equal the energy transported multiplied by the LMP difference. The approximate payment is easier to administer than payment based on the contingency flows and LMPs, since the pre-contingency flows are measured or estimated already in typical systems.

One advantage of the revenue stream that we describe is that an arbitrary collection of elements, including transmission lines, generators, and consumers, can be considered as one unit, paying the natural generalization of (1) that involves the flows at the borders between the unit and the rest of the system. This observation also motivates the name “border flow rights.” The total payment to the unit is the same as the sum of the net payments to the individual elements considered separately so that the payment scheme is neutral regarding the aggregation of generation and transmission elements.

High voltage direct current transmission also fits the model we describe since it is often modelled as a paired generator and demand. This payment is consistent with the revenue stream we propose.

V. EXAMPLE ILLUSTRATING PROPOSED REVENUE STREAM

We consider a simple two bus system, with buses k and ℓ, and a corridor of three lines joining the buses, lines $e = 1,2$, and 3, each having the same admittance, with the absolute value of the imaginary part of the admittance equal to $B_e = 0.5$ units, $e = 1,2,3$. However, the lines have different capacities of $\overline{f}_1 = 50$ MW, $\overline{f}_2 = 60$ MW, and $\overline{f}_3 = 70$ MW, respectively. For simplicity, we assume that these capacities apply in both normal and emergency conditions and that, for each line, the capacity is the same for flow in each direction. (We ignore line resistance for convenience, but this can be included in the model.) The situation is illustrated in Figure 1.

There is a generator at bus k that, during “typical” hours, offers its energy at \$20/MWh and, during “exceptional” hours offers at \$60/MWh.[15] For clarity in Figure 1, we only illustrate the offer at bus k for typical hours. There is 150 MW of demand at bus k on-peak. There is also a generator at bus ℓ that offers its energy at \$30/MWh during all hours and seasons. We ignore capacity constraints on the generators. There is 150 MW of demand at bus ℓ on-peak.

[15]For example, typical hours could be during typical hydro inflow seasons, while exceptional hours are during drought seasons.

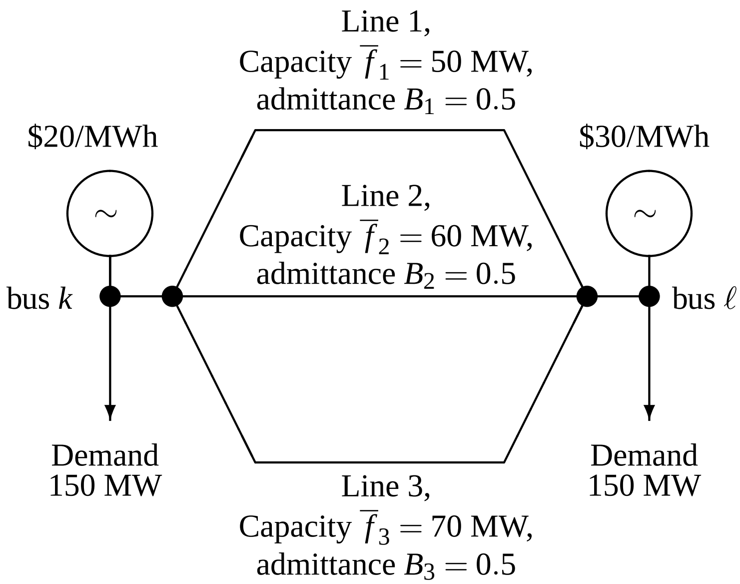

Fig. 1. Two bus, three line network with offers shown for typical hours on-peak.

In Section V-A, we discuss the solution of OBSCED and the remuneration schemes for typical hours, while in Section V-B, we discuss the situation for exceptional hours. In Section V-C, we summarize and compare the remuneration schemes.

A. Typical hours

In Section V-A.1, we discuss the OBSCED problem for typical hours and its solution. Then in sections V-A.2–V-A.6 we discuss revenue streams for remuneration of transmission lines based on, respectively, FTRs, flowgate rights, (1) using contingency flows and LMPs, (1) using the pre-contingency flows and LMPs, and the incremental contribution to welfare.

A.1 OBSCED problem and solution

The OBSCED problem is to minimize the cost of the accepted offer quantities while meeting demand and satisfying the security constraints. (Since the demand is constant, the (revealed) welfare is a constant minus the costs.) The contingency constraints require that, in the event of an outage of any transmission element, the flow on the remaining lines are within ratings. Since there are three transmission elements, there are three contingencies and a total of six contingency constraints.

Let us write q_k and q_ℓ for the generations at buses k and ℓ, respectively. The system energy balance constraint requires that $q_k + q_\ell = 300$.

We write f_e for the base-case flow on line e in the direction from k to ℓ if all lines are in service. Because of the lossless assumption, the power flow from the line into bus k is $-f_e$, whereas the power flow from the line into bus ℓ is f_e.

The non-contingency inequality constraints require that:

$$\forall e = 1,2,3, f_e \leq \overline{f}_e.$$

In the development in the Appendix, we consider the base-case voltage magnitudes and phases and explicitly represent the base-case power flow equality and inequality constraints in terms of them. For this simple system, however, we can invert the power flow equality constraints and obtain the following expressions for the non-contingency inequality constraints:

$$\forall e = 1,2,3, \frac{B_e}{\sum_\tau B_\tau}(q_k - 150) \leq \overline{f}_e.$$

Let us also write f_e^ω for the flow on line e if there were a contingency on line ω, for $e, \omega = 1,2,3$. Again, $-f_e^\omega$ and f_e^ω are, respectively, the contingent flow from the line into bus k and the contingent flow from the line into bus ℓ, in the event of a contingency on line ω. Naturally, $f_\omega^\omega = 0, \omega = 1,2,3$.

The contingent flows are implicit functions of q_k and q_ℓ and must be limited so that:

$$\forall e = 1,2,3, \forall \omega \neq e, f_e^\omega \leq \overline{f}_e.$$

In the Appendix, we consider the contingency-case voltage magnitudes and phases and explicitly represent the contingency-case power flow equality and inequality constraints in terms of them. As with the base-case constraints, however, we can invert the contingency-case power flow equality constraints and obtain the following expressions for the contingency inequality constraints:

$$\forall e = 1,2,3, \forall \omega \neq e, \forall \tau \neq e, \omega, \frac{B_e}{B_e + B_\tau}(q_k - 150) \leq \overline{f}_e.$$

For this OBSCED problem, the non-contingency inequality constraints are never binding and so we will not consider the non-contingency constraints explicitly. Moreover, because line 1 has the lowest capacity, the most binding contingency inequality constraints will always involve the flow on line $e = 1$ in the event of an outage of either line $\omega = 2$ or 3. We will not explicitly consider the four other contingency inequality constraints since they are never binding, so that in this example we can use the index of the outaged line to index the potentially binding contingency constraints. Summarizing the effects of the constraints, the secure capability to transmit from bus k to bus ℓ is 100 MW.

Omitting the constraints that are never binding, the OBSCED problem for typical hours can be written as:

$$\min_{q_k, q_\ell} \left\{ 30q_k + 20q_\ell \,\middle|\, q_k + q_\ell = 300, \frac{B_1}{B_1 + B_2}(q_k - 150) \leq \overline{f}_1, \frac{B_1}{B_1 + B_3}(q_k - 150) \leq \overline{f}_1 \right\}.$$

The solution of this problem is $q_k^\star = 250$ and $q_\ell^\star = 50$, with $33\frac{1}{3}$ MW flowing on each line from k to ℓ pre-contingency and 50 MW flowing on each remaining line from k to ℓ in the event of any contingency.[16]

The LMPs at k and ℓ are $p_k = 20$ \$/MWh and $p_\ell = 30$ \$/MWh, respectively. The two inequality constraints are identical, so that there is redundancy in the constraints: the sum of the Lagrange multipliers on these constraints is $\eta^\star = 20$ \$/MWh.

A.2 Financial transmission rights payment

In this section we assume that transmission rights have been allocated under a financial transmission rights (FTRs) nomination system. The FTRs that can be nominated depend on the order of nomination of the FTRs. Moreover, in the case of only one line between bus k and ℓ, the capacity to securely transmit energy would be zero, so we assume that initial construction involved two lines between k and ℓ and then an expansion added a third line. We assume that the owner of the two initially constructed lines would nominate the maximum FTR capability from k to ℓ, in accordance with the direction of flow during typical hours. We assume that the owner of the expansion line then nominates the maximum incremental FTR capability from k to ℓ that is simultaneously feasible with the initial FTR nomination.[17] The situation is illustrated in table II.

Since the LMP difference between buses k and ℓ is \$10/MWh, the payment to each line owner is equal to the FTR nomination times \$10/MWh.

[16] Inverting the power flow equality constraints makes solving this OBSCED problem easy, but obscures the linear dependence of the explicit form of the constraints on the parameters $B_1, \overline{f}_1, B_2, \overline{f}_2, B_2$, and $\overline{f}_3$. The theoretical derivation in the Appendix uses the explicit form of the equality and inequality constraints; however, the numerical values of the sensitivities calculated in sections V-A.3–V-A.6 do not depend on the method of solution of the OBSCED problem.

[17] As discussed above, the FTR process might allow for seasonal nominations. However, we will ignore this possibility here.

TABLE II

FTR NOMINATIONS UNDER VARIOUS ASSUMPTIONS ABOUT ORDER OF CONSTRUCTION OF LINES.

Initial construction		Expansion	
Lines built	FTR nomination	Line built	FTR nomination
1,2	50 MW	3	50 MW
1,3	50 MW	2	50 MW
2,3	60 MW	1	40 MW

A.3 Flowgate rights payment

The only congested flowgate in the system is line 1, with a Lagrange multipler of $\eta^\star = 20$ \$/MWh. Using a flowgate based payment scheme based on the contingent flow on line 1, the revenue to line 1 would be:

$$\eta^\star \overline{f}_1 = 1000 \text{ \$/MWh}. \tag{2}$$

The payment to lines 2 and 3 would be zero. Note that although the payment to these lines is zero, they contribute to welfare in the sense that if line 2 or 3 were to be out of service, the welfare would be reduced since the capacity to transfer cheap power to the consumer would be reduced. This is the fundamental reason why the owners of lines in a flowgate rights system would not be incented to maintain capacity that is, nevertheless, valuable to the network. (In Section V-A.6, we explicitly calculate the reduction in welfare if line 2 or 3 were out of service.)[18]

A.4 Payment based on contingency flows

Using the payment scheme proposed in Gribik, Shirmohammadi, *et al.* [1, appendix B], the revenue stream to each line is based on the sensitivity of optimal welfare to capacity and to admittance, which in this case is also equal to the flow on the line in the contingency case multiplied by the price difference between bus k and bus ℓ. That is, the payment is based on (1) where the flows are taken to be contingency flows. The sensitivities can be calculated from the solutions of the OBSCED problem [59], [60].

For line 1, the sensitivity of optimal welfare to capacity is $\eta^\star$. The sensitivity of optimal welfare to admittance is equal to $-\frac{B_2}{(B_1+B_2)^2}\eta^\star q_k^\star$. The revenue stream for line 1 is therefore:

$$\eta^\star \overline{f}_1 - \frac{B_2}{(B_1+B_2)^2}\eta^\star q_k^\star B_1 = 500 \text{ \$/h}.$$

This is equal to the revenue stream based on the LMPs and the contingent flow on line 1 (on outage of line 3):

$$p_k(-f_1^3) + p_\ell f_1^3 = 500 \text{ \$/h}.$$

For lines 2 and 3, the payment depends on the "sharing" of the Lagrange multiplier $\eta^\star$ between the two binding constraints. At one extreme, suppose that the Lagrange multiplier on the constraint $\frac{B_1}{B_1+B_2}q_k \leq \overline{f}_1$ is assumed to be equal to $\eta^\star$, while the Lagrange multiplier on the other constraint is assumed to be 0.[19]

In this case the sensitivity of optimal welfare to the capacity of line 2 is zero and the sensitivity of optimal welfare to the admittance of line 2 is $\frac{B_1}{(B_1+B_2)^2}\eta^\star q_k^\star$. The revenue stream for line 2 is:

$$\frac{B_1}{(B_1+B_2)^2}\eta^\star q_k^\star B_2 = 500 \text{ \$/h}.$$

[18] In some flowgate rights mechanisms, such as in the Electric Reliability Council of Texas (ERCOT) zonal balancing market, proxy limits based on pre-contingency flows are used [58]. For example, a pre-contingency limit of $\tilde{f}_1 = 33\frac{1}{3}$ MW might be set for line 1. A correspondingly higher Lagrange multiplier of $\tilde{\eta} = 30$ \$/MWh would then produce the same revenue stream as (2).

[19] In fact, we already adopted this assumption for calculating the revenue stream to line 1. However, for line 1, the revenue stream does not depend on how the Lagrange multiplier is shared between the two constraints.

This is equal to the revenue stream based on the LMPs and the contingent flow on line 2 (on outage of line 3):

$$p_k(-f_2^3) + p_\ell f_2^3 = 500 \text{ \$/h}.$$

The sensitivity of optimal welfare to capacity of line 3 is zero and the sensitivity of optimal welfare to admittance of line 3 is zero and so the revenue stream is equal to zero. This is equal to the contingent flow on line 3 on outage of line 3, which is zero, times the marginal cost price difference.

At the other extreme, the Lagrange multiplier on the constraint $\frac{B_1}{B_1+B_2}q_k \leq \overline{f}_1$ could be assumed equal to zero while the Lagrange multiplier on the other constraint would be equal to $\eta^\star$. In this case, the revenue stream to line 2 would be zero while the revenue stream to line 3 would be 500 \$/h.

If we "share" the Lagrange multiplier equally between the two constraints then the revenue stream for both lines 2 and 3 would be 250 \$/h.

A.5 Payment based on pre-contingency flows

In this section, we consider the approximate payment based on (1) and using the pre-contingency flows. The flow on each line pre-contingency is $f_e = 33\frac{1}{3}$ MW, $e = 1, 2, 3$, so that the revenue stream to each line is:

$$p_k(-f_e) + p_\ell f_e = 333\frac{1}{3} \text{ \$/h}, e = 1, 2, 3.$$

A.6 Incremental payment

We also consider payment to each line based on its incremental contribution to welfare. We can also think of this as resulting from a Vickrey auction [61] or a Groves-Ledyard mechanism [62].

If line 1 were to be removed from the system then there would be 60 MW of secure transmission capability from bus k to bus ℓ. That is, the presence of line 1 provides 40 MW of incremental capability to the system, which increases the welfare by 400 \$/h by allowing 40 MW of generation from bus k at price \$20/MWh to displace 40 MW of generation at bus ℓ at price \$30/MWh. The payment to line 1 would be 400 \$/h under this payment scheme.

Similarly, lines 2 and 3 each provide 50 MW of incremental capability, with a corresponding contribution to welfare of 500 \$/h. The payment to lines 2 and 3 would be 500 \$/h. Lines 2 and 3 provide more incremental capability to the system than line 1.

The sum of the three incremental payments exceed the total congestion rental for the system of 1000 \$/h. That is, using the incremental payments would violate the ISO revenue adequacy requirement. One *ad hoc* approach to this issue is to scale the payments down so that they exactly match the congestion rental. Scaling all payments by the same factor to make the payments revenue neutral would yields payments of 285.7 \$/h, 357.1 \$/h, and 357.1 \$/h, respectively.

B. Exceptional hours

In Section V-B.1, we discuss the solution of the OBSCED problem for exceptional hours. Then in sections V-B.2–V-B.6 we discuss revenue streams during these hours for remuneration of transmission lines based on, respectively, FTRs, flowgate rights, (1) using contingency flows, (1) using the pre-contingency flows, and the incremental contribution to welfare.

B.1 OBSCED problem and solution

In principle, we could re-solve the OBSCED problem with the changed offers from the generator at bus k during exceptional hours. The problem again has only two inequality constraints that can bind. In this case, the two constraints correspond to the limit on flow on line 1 in the direction from bus ℓ to bus k on contingency of, respectively, either line 2 or line 3.

However, because of the symmetry in the problem between typical and exceptional hours, we can write down the solution more directly based on the solution for the typical hours. In the following discussion, we will abuse notation somewhat by using the same symbols for both exceptional hours and typical hours. In exceptional hours, power will flow from bus ℓ to bus k and the solution of the problem is $q_k^\star = 50$ and $q_\ell^\star = 250$, with $33\frac{1}{3}$ MW flowing on each line from ℓ to k pre-contingency and 50 MW flowing on each remaining line from ℓ to k in the event of any contingency.

The LMPs at k and ℓ are $p_k = 60$ \$/MWh and $p_\ell = 30$ \$/MWh, respectively, during exceptional hours. A key observation is that, in exceptional hours, the LMP difference between bus k and bus ℓ is three times in magnitude larger than in typical hours and of opposite sign. All the revenue streams can be calculated from this observation. The two inequality constraints are again identical, so that there is redundancy in the constraints: the sum of the Lagrange multipliers on these constraints is $\eta^\star = 60$ \$/MWh.

B.2 Financial transmission rights payment

We will assume that the FTR nominations are the same as in Section V-A.2 as shown in Table II. That is, we will assume that the occurrence of typical and exceptional hours is not correlated to any seasonal re-nomination of FTRs. Since the LMP difference between buses k and ℓ is $-\$30$/MWh, the line owners owe payments to the ISO equal to the FTR nomination times \$30/MWh. It is important to understand that the lines do contribute to (revealed) welfare, since they allow for transport of energy from buses with lower marginal offer cost to buses with higher marginal offer cost; however, under the FTR scheme, the line owners owe payments to the ISO. In this extreme case, the ISO will generate considerable net income, since it receives *twice* the congestion rental in the system.

B.3 Flowgate rights payment

The only congested flowgate in the system is again line 1, with a Lagrange multipler of $\eta^\star = 60$ \$/MWh. Using a flowgate based payment scheme based on the contingent flow on line 1, the revenue to line 1 would be:

$$\eta^\star \overline{f}_1 = 3000 \text{ \$/MWh}.$$

The payment to lines 2 and 3 would again be zero.

B.4 Payment based on contingency flows

Using the payment scheme proposed in Gribik, Shirmohammadi, *et al.* [1, appendix B], the revenue stream for line 1 is therefore:

$$\eta^\star \overline{f}_1 - \frac{B_2}{(B_1 + B_2)^2} \eta^\star q_\ell^\star B_1 = 1500 \text{ \$/h}.$$

This is again equal to the revenue stream based on the LMPs and the contingent flow on line 1 (on outage of line 3):

$$p_k(f_1^3) + p_\ell(-f_1^3) = 1500 \text{ \$/h},$$

where f_1^3 is now the flow on line 1 on contingency of line 3 in the direction from bus ℓ to bus k.

For lines 2 and 3, the payment again depends on the "sharing" of the Lagrange multiplier $\eta^\star$ between the two binding constraints. If we again "share" the Lagrange multiplier equally between the two constraints then the revenue stream for both lines 2 and 3 would be 750 \$/h.

B.5 Payment based on pre-contingency flows

In this section, we consider the approximate payment based on (1) and using the pre-contingency flows. The flow on each line pre-contingency is $f_e = 33\frac{1}{3}$ MW, $e = 1, 2, 3$, so that the revenue stream to each line is:

$$p_k(f_e) + p_\ell(-f_e) = 1000 \text{ \$/h}, e = 1, 2, 3,$$

where f_e is now the pre-contingency flow on line e in the direction from bus ℓ to bus k.

TABLE III
REVENUE STREAMS, IN $/H, UNDER VARIOUS PAYMENT SCHEMES DURING TYPICAL HOURS.

Line	1	2	3
FTR: initially build 1,2 then 3	250	250	500
FTR: initially build 1,3 then 2	250	500	250
FTR: initially build 2,3 then 1	400	300	300
Flowgate	1000	0	0
Based on (1) and contingency flows	500	250	250
Based on (1) and pre-contingency flows	$333\frac{1}{3}$	$333\frac{1}{3}$	$333\frac{1}{3}$
Incremental	400	500	500
Scaled incremental	285.7	357.1	357.1

TABLE IV
REVENUE STREAMS, IN $/H, UNDER VARIOUS PAYMENT SCHEMES DURING EXCEPTIONAL HOURS.

Line	1	2	3
FTR: initially build 1,2 then 3	−750	−750	−1500
FTR: initially build 1,3 then 2	−750	−1500	−750
FTR: initially build 2,3 then 1	−1200	−900	−900
Flowgate	1000	0	0
Based on (1) and contingency flows	1500	750	750
Based on (1) and pre-contingency flows	1000	1000	1000
Incremental	1200	1500	1500
Scaled incremental	857.1	1071.4	1071.4

B.6 Incremental payment

Since line 1 provides 40 MW of incremental capability to the system, which increases the welfare by 1200 $/h by allowing 40 MW of generation from bus ℓ at price $30/MWh to displace 40 MW of generation at bus k at price $60/MWh. The payment to line 1 would be 1200 $/h under this payment scheme.

Similarly, lines 2 and 3 each provide 50 MW of incremental capability, with a corresponding contribution to welfare of 1500 $/h. The payment to lines 2 and 3 would be 1500 $/h. Lines 2 and 3 provide more incremental capability to the system than line 1.

The sum of the three incremental payments again exceed the total congestion rental for the system of 3000 $/h. Scaling all payments by the same factor to make the payments revenue neutral would yields payments of 857.2 $/h, 1071.4 $/h, and 1071.4 $/h, respectively.

C. Summary and comparison of incentives

Tables III and IV show the revenue streams under the various payment schemes for typical hours and exceptional hours, respectively. For the FTR payment, we have considered each of the three possible orderings of construction that were presented in Table II. Moreover, for each pair of lines owned by a single entity, we have shared the total FTR payment equally between each line to illustrate it in Tables III and IV. For the payment scheme based on (1) and using the contingent flows, we have chosen to share the revenues equally between lines 2 and 3.

To consider the incentives provided by each payment scheme to transmission owners and investors, first consider the options typically open to a builder of a new transmission line. Such a builder is faced with building a relatively large, lumpy contribution to the system. The efficient incentive for such a decision is the incremental change to welfare, which is positive both during typical and exceptional hours reflecting the incremental value of transmission to move energy in both directions. Unfortunately, paying all lines the incremental change to welfare is not revenue adequate if, for example, the system is contingency constrained, and presents a revenue adequacy problem for the ISO. Since transmission systems are generally contingency constrained, we can expect that in general there will be a revenue shortfall with such a payment scheme, even in the absence of economies of scale in transmission construction costs.[20]

A scaled version of incremental welfare might be used to enforce revenue adequacy; however, the incentive provided is no longer as closely related to the incremental contribution to welfare. In summary, payment schemes based directly on incremental change to welfare are problematic although, by definition, they efficiently incent lumpy investment.

We now consider payment based on FTRs. We first observe that the payment depends on the ordering of nomination of FTRs, which institutionalizes an asymmetry between payments to lines. Moreover, and more seriously, Table IV shows that payments to FTR holders are negative for exceptional hours, despite the value provided by the lines to the system. The reason for the negative revenue stream is the difference between the dispatch for the SFT and the actual dispatch [6, section 6]. Whenever there is a difference between the implied operating point of the SFT dispatch and the actual dispatch, the congestion rental may exceed the FTR obligations [6, section 5]. In extreme cases, such as the exceptional hours in the example system, the difference between the congestion rental and the FTR obligations is double the congestion rental itself. Unless the patterns of flow were highly correlated with seasonal nominations of FTRs, the incentives provided by FTRs will not induce efficient expansion of transmission.

We now consider payment based on flowgate rights. Under flowgate rights, line 1 receives a positive payment in both typical and exceptional hours, which avoids the problem discussed for FTRs. However, lines 2 and 3 are indifferent to being in-service or out-of-service with such a flowgate rights payment scheme. This is very problematic because the owner of these lines have no incentive to maintain their transmission capacity since it benefits other parties.[21]

To analyze the payment schemes based on (1) consider the options open to an owner of an existing line to *change* the electrical properties of the line. Reconductoring and series capacitors can change the thermal capacity and the admittance, respectively.[22] Moreover, there are various reconductoring options and various values of series capacitors that could be used to achieve a fine grained change in the thermal capacity or the admittance. Although such options for changing the electrical characteristics of a line are likely to have economies of scale, the total change in transmission capability may be a relatively small fraction of the existing capability in a corridor. Consequently, a marginal signal is likely to be useful in these cases. Moreover, the marginal value of the line is positive for flows in both directions so that the revenue stream is positive both in typical and exceptional hours. As with payment under flowgate rights, this avoids the drawback of the possibility of negative payments to FTR holders when the actual dispatch differs significantly from the implied operating point of the SFT dispatch. However, unlike flowgate rights, under (1), revenue streams accrue to all lines.

Since, under (1), benefits accrue to all lines whose flows are affected, it would be necessary to form coalitions of beneficiaries to enable welfare maximizing funding of transmission expansion. This presents a free-rider problem and is more cumbersome than having all the benefits accrue to a single party as in FTRs. However, transmission expansion currently typically involves the formation of coalitions of beneficiaries. Moreover, transmission upgrades often involve simultaneous upgrading of transmission elements in several locations, already necessitating the formation of coalitions [6, section 2]. As Bushnell and Stoft point out, an important role for regulators "is to oversee the coalition process and prevent barriers to investment along

[20] Revenue inadequacy would be avoided if the incremental payment were made only to new transmission construction. However, this institutionalizes a permanent asymmetry between payment for old and new transmission.

[21] In principle, we could consider all three lines together as a unit and share the net proceeds, by redistributing the congestion rental. The border flow rights mechanism does precisely this on the basis of the flows on the lines.

[22] In many urban areas with limited corridor space, such upgrades may be the only options. See [63] for a summary of estimated costs.

transmission paths" [6, section 5.2]. Moreover, as Rosellón points out, "the regulator must then take measures to vertically separate the electricity industry, so that expansion projects may be undertaken by any economic agent" [37, section 5].

It would be interesting to test empirically whether (1) based on the pre-contingency flows is a workable approximation to (1) based on the contingency flows when averaged over various operating conditions. We hypothesize that it might be. As mentioned in Section IV-C, it would be easier to administer a payment based on pre-contingency flows than one based on contingency flows. In the discussion in sections VI and following, either the exact payment scheme based on contingency flows or the approximation based on pre-contingency flows could be used.

VI. Contracts for differences of differences

The proposal (1) for remuneration of a transmission owner, whether based on pre-contingency or contingency flows, is independent of any financial contract definition. It depends only on line characteristics and the results of the OBSCED. In this section, we describe how the revenue stream can be used to fund a risk hedging financial instrument for transmission customers. The financial instrument is similar to "basis spreads" in current electricity markets [10]. In Section VI-A, we provide a description of the basic mechanism, while in Section VI-B we provide an interpretation in terms of contract paths. Finally, in Section VI-C, we consider option rights.

A. Description

First consider a generator and a consumer at a bus ℓ. Suppose that the generator and consumer have signed a CFD to hedge the LMP at bus ℓ. The CFD provides a side payment from consumer to generator equal to a contract quantity q times the difference between a strike price for energy and the LMP at bus ℓ. The variation of LMP at ℓ is hedged for production by the generator equal to q and also hedged for consumer demand equal to q, since the sum of the LMP payments and the payment due to the CFD is always equal to q times the strike price. The CFD allows the generator and consumer, who have equal and opposite exposures to the variation of LMP at a bus, to both costlessly hedge their price risks for the quantity q.

Now suppose that the generator is located at a different bus, bus k. We now assume that the generator at k and the consumer at ℓ have signed a CFD to hedge the LMP at bus k for a contract quantity q. As is well known, a CFD based on the LMP at bus k is insufficient to hedge the price differences between buses k and ℓ. In a market with point-to-point FTRs, the consumer could hedge the price difference by purchasing an appropriate FTR of quantity q with point of injection k and point of withdrawal ℓ.

In contrast to an FTR-only system, we propose an alternative hedging instrument here, called a "contract for differences of differences" (CFDD). The CFDD is a contract between:
- the consumer (or generator or both), and
- the owner of the transmission line or lines joining bus k and ℓ.

The CFDD provides for a side payment to the transmission line owner equal to a contract quantity times the difference between:
- a strike price for transmission services, and
- the *difference* between the LMPs at bus k and ℓ.

The CFDD is so-called because it pays based on the *difference* between a strike price and LMP *differences* between two buses. Given a contract quantity of q, the CFDD would hedge the variation in LMP differences since the net payment for transmission services would always be q times the strike price.[23]

A transmission owner and its transmission customer have equal and opposite exposures to the variation of LMP differences. As with the CFD, the CFDD allows them both to hedge their price risk for the quantity q. That is, in addition to defining an underlying revenue stream in Table I by (1), we would also suggest

[23] In an FTR-based system it would also be possible for FTR holders to sell CFDDs based on their FTRs. I am indebted to Karsten Neuhoff of Cambridge University for suggesting this. This provides an alternative for over-the-counter trading of financial rights, but does not remove the risk to the ISO of shortfall under outage conditions.

TABLE V

PROPOSED IMPLEMENTATIONS OF ENERGY AND AC TRANSMISSION MARKETS.

Asset:	Product:	Underlying revenue stream:	Financial instrument:
Generation	Energy	Energy × LMP	Contract for differences
Transmission	Transport	Energy × LMP differences	Contracts for differences of differences

replacing FTRs by CFDDs as shown in Table V. With these changes to the transmission property rights definition, there is a symmetry between energy and transmission markets: just as generators and demand have opposite exposures to risks of LMP variation that can be hedged with CFDs, transmission owners and customers have opposite exposure to risks of LMP *difference* variation that can be hedged with CFDDs.

In the case of an FTR-only system, revenue adequacy for the ISO requires the FTRs to satisfy the SFT test. In contrast, with the proposed transmission property right and the use of CFDDs, revenue adequacy associated with issuing financial transmission rights is devolved to transmission line owners. If transmission owners over-sell compared to their actual flows then they are responsible for the shortfall. If they under-sell compared to their actual flows then they will receive a volatile revenue stream.

Furthermore, during a transmission outage, the transmission owner will still be liable for the CFDD payment, providing a powerful incentive to the transmission line owner to make the line available and in-service whenever congestion in the network makes LMP differences large.[24] The risk of ill-timed transmission outages is transferred from the ISO and transmission customers, who have little fundamental control of transmission outages, to the transmission owners themselves, who have more control and an ability to focus on long-term reliability issues.[25] This is analogous to transmission owners funding shortfalls during maintenance outages as in New York.

To summarize, since the CFDD is a purely financial contract there is no need for any derating policy administered by the ISO. The transmission owner is financially responsible for its outages. The OBSCED can take place independently of financial positions of transmission owners and the purchasers of transmission rights. As in the case of CFDs, however, creditworthiness requirements may be appropriate for transmission owners that take on more risk than is covered by likely anticipated flows. Moreover, trading of CFDDs can also be facilitated by an exchange. To calculate the amount of CFDD payment that is covered by the revenue stream, a version of the simultaneous feasibility test can be conducted as will be discussed in Section VII-C.

B. Contract paths

In the example in Figure 1, buses k and ℓ are directly connected by transmission. In general, a generator may want to hedge a transaction of quantity q against variation in the LMP differences between two buses k and ℓ that are not directly connected by a transmission line. In this case, a sequence of CFDDs can be assembled from bus k through intermediate lines to bus ℓ. A CFDD is purchased on each intermediate line (or collection of lines) for quantity q, just as in the contract path paradigm. The sum of the revenue streams from the CFDDs will exactly hedge the transaction from bus k to bus ℓ. That is, a “contract path” of CFDDs can be used to hedge the LMP difference.

Moreover, if there are multiple parallel paths from bus k to bus ℓ then any one of the contract paths, or an appropriate combination of several of the contract paths, can be used to hedge the LMP difference. This means that different paths can effectively compete to supply CFDDs to transmission customers.

[24]While a CFDD could have provisions to become inactive when a transmission line is out of service, this would reduce the hedging value. We assume that CFDs and CFDDs have no provisions to be inactive during outage conditions.

[25]This issue is particularly problematic for FTR and flowgate mechanisms, since lines other than the binding constraints can reduce transfer capability if out of service, but such lines receive no payment and therefore have no incentive to be available during critical times if they are not beneficiaries.

Although this mechanism is reminiscent of the contract path paradigm for transmission services as traditionally implemented by North American utilities, it is important to emphasize that the CFDD mechanism respects Kirchhoff's laws and security constraints, since physical flows on the lines are always determined by the ISO as the result of a offer-based, security-constrained economic dispatch. Unlike the contract path paradigm, which does not support *bona fide* competition in transmission provision, the CFDD mechanism does support price competition between sellers of CFDDs on parallel paths.

C. Option rights

The CFDDs described so far have been so-called "obligation" rights, which have negative payoff if the sign of the LMP difference reverses. However, the financial nature of these rights provides significant flexibility to define "option" rights that have zero payoff when the sign of an LMP difference reverses as in the "exceptional" hours for the example system. In fact, as illustrated for the example system, the revenue stream (1) to a line will usually be positive, irrespective of the sign of the LMP difference, since the direction of flow is usually reversed when the LMP price difference is reversed.

Consequently, the line owner can sign an option CFDD for flow in each direction. This matches the underlying physical capability of the line, which allows flow in either direction and contrasts with the straightforward and most usual definition of an FTR where, as in the example system, LMP reversals produce negative payoffs to the FTR holder. The financial character of CFDDs means that more flexible financial products can be defined that parallel the flexibility of CFDs. The drawback of FTRs, described in Section IV-A, of requiring a nomination that applies throughout an FTR auction contract period, is not problematic for the border flow right and for the associated financial right of CFDDs. The drawback of FTRs, mentioned in Section III, of the difficulty of representing options and other financial instruments in the SFT, is not problematic for CFDDs.

Nevertheless, there are cases where the border flow right would produce a negative revenue stream. In particular, if the marginal contribution to welfare of the line is negative then the revenue stream may be negative.[26] The line can avoid such a negative revenue stream by disconnecting itself from the network and can usually be expected to make an incremental improvement in welfare by doing so.[27]

VII. Trading of CFDDs

The CFDD mechanism allows for trading of financial transmission rights by entities other than the ISO because the simultaneous feasibility test to protect the revenue adequacy of the ISO is not part of the CFDD mechanism. In principle, the trading can even be completely decentralized. Decentralized trading will be discussed in Section VII-A.

Nevertheless, it may be helpful for transmission customers to be able to purchase contract paths of CFDDs that are assembled through a centralized exchange and for transmission owners to sell their CFDDs through such an exchange. There are at least two ways that this could be done, as will be described in sections VII-B and VII-C. In either case, there is no need for the ISO to be involved in the exchange. That is, the exchange can focus on the longer-term issues of transmission rights and creditworthiness, while the ISO remains focused on day-to-day operational issues. Moreover, such an exchange could trade CFDs and CFDDs simultaneously, providing long-term financial hedging instruments that would facilitate *both* generation and transmission capital formation.

We will describe each of these three alternatives for trading and provide examples to illustrate, basing

[26] Because (1) based on the pre-contingency flows is only an approximation to the marginal value of the line, the revenue stream could be positive even if the marginal contribution to welfare were negative, and vice versa.

[27] See [44, section 5.1 and figure 5] for an example. A line in that example is not at capacity, but the sensitivity of optimal welfare to its admittance is negative. Welfare could presumably be improved if the line were to disconnect from the system. However, this is not always the case as shown in the example described in [57, box 1 and page 31]. In such cases, the ISO should have the right to "commit" a line that increases welfare by being connected. The ISO could compensate the line in the same manner that "make-whole" payments are provided to generators that are committed but do not cover their offer costs from LMP payments [64]. Conversely, we will see in the discussion of merchant construction in Section VIII that we should allow the ISO to "de-commit" a line if its being in-service decreases welfare.

payments on (1) using pre-contingency flows. The trading alternatives are then summarized in Section VII-D.

A. Decentralized trading

A.1 Description

No central reconfiguration or intervention of the CFDDs are necessarily required, so long as transmission owners can bear the risk that payments to transmission customers may exceed the revenue from the OBSCED if CFDDs are "over-sold." Furthermore, portions of the capacity can be traded for differing durations to support both long-term and short-term hedging needs.

A.2 Example with on-peak demand

Consider the system shown in Figure 1, with on-peak demand at bus ℓ of 150 MW. Suppose that the three lines were owned by different entities (or, more generally, that each line stands for a transmission system owned by a different entity, with all three systems interconnected at bus k and also interconnected at bus ℓ.) We focus on typical hours for the example system; however, a similar analysis applies for exceptional hours.

From the analysis in Section V-A, the flow on each line is $33\frac{1}{3}$ MW when flow between bus k and bus ℓ is at its security-constrained maximum. Because of the purely financial nature of CFDDs, any entity could offer to sell any quantity of CFDD. However, we will not consider speculative trades and assume that each line owner offers to sell $33\frac{1}{3}$ MW of CFDD. If the demand at ℓ were seeking to hedge price volatility on all of its demand of 150 MW then it would, among other things, seek to purchase all 100 MW of offered CFDDs. Together with a CFD for 100 MW with the generator at bus k, the CFDD would completely hedge the volatility of the price of importing the 100 MW. (A CFD for 50 MW with the generator at bus ℓ would hedge its remaining demand.)

A.3 Example with on- and off-peak demand

Suppose that in the example of Figure 1, the demand were 150 MW on-peak and 50 MW off-peak. CFDDs can, in principle, be offered both on- and off-peak (and "seasonally.") If both on- and off-peak CFDDs are available then the demand might not purchase CFDDs for off-peak delivery but only purchase 100 MW of CFDDs for on-peak delivery.

A.4 Discussion

Decentralized trading may be adequate for systems with few participants, but is likely to be unwieldy when there are multiple participants desiring to hedge transmission. In the next section, we will consider the first of two centralized trading alternatives.

B. Network flow based clearing

B.1 Description

Suppose that individual line owners specify offers for strike prices and quantities of CFDDs referenced between the ends of their lines. If transmission customers also bid their willingness-to-pay to purchase contract paths of CFDDs between injection and withdrawal points then a simple "network flow" based algorithm can be used to optimally match the bids and offers [65].

B.2 Example

Suppose that lines $e = 1, 2, 3$ each offer $33\frac{1}{3}$ MW of CFDDs at prices of, respectively, \$1/MWh, \$2/MWh, and \$3/MWh, for all hours of a contract period. Suppose that the consumer at bus ℓ bids for 150 MW of CFDDs at \$4/MWh for all hours of a contract period. All 100 MW of offered CFDDs are sold and the clearing strike price is \$4/MWh.

TABLE VI
REVENUE STREAMS FOR 50 MW DEMAND.

Revenue stream description	Amount
Consumer at ℓ pays ISO at LMP	$50 \times 20 = \$1000/\text{h}$
Consumer at ℓ pays transmission owner e based on CFDD	$33\frac{1}{3} \times (4-(0-0)) = \$133\frac{1}{3}/\text{h}$
ISO pays transmission owner e at LMP difference	$16\frac{2}{3} \times (0-0) = \$0/\text{h}$
ISO pays generator at bus k at LMP	$50 \times 20 = \$1000/\text{h}$
ISO pays generator at bus ℓ at LMP	$0 \times 20 = \$0/\text{h}$

TABLE VII
REVENUE STREAMS FOR 150 MW DEMAND.

Revenue stream description	Amount
Consumer at ℓ pays ISO at LMP	$150 \times 30 = \$4500/\text{h}$
Consumer at ℓ pays transmission owner e based on CFDD	$33\frac{1}{3} \times (4-(30-20)) = -\$200/\text{h}$
ISO pays transmission owner e at LMP difference	$33\frac{1}{3} \times (30-20) = \$333\frac{1}{3}/\text{h}$
ISO pays generator at bus k at LMP	$100 \times 20 = \$2000/\text{h}$
ISO pays generator at bus ℓ at LMP	$50 \times 30 = \$1500/\text{h}$

We now consider the revenue stream under OBSCED under two demand conditions and assuming that the generator at bus k offers energy at \$20/MWh and the generator at bus ℓ offers energy at \$30/MWh. We initially assume that there are no CFDs.

Off-peak demand of 50 MW. In this case, OBSCED yields 50 MW of generation at bus k, 0 MW of generation at bus ℓ, and LMPs of $p_k = p_\ell =$ \$20/MWh with no transmission congestion. The revenue streams are shown in Table VI. By construction, the ISO net income is zero. The consumer net payment is \$1400/h or an average price, including transmission payments, of \$28/MWh. On net, each transmission owner e is paid $\$133\frac{1}{3}$/h. The generator at bus k is paid \$1000/h, while the generator at bus ℓ is paid nothing.

On-peak demand of 150 MW. In this case, OBSCED yields 100 MW of generation at bus k, 50 MW of generation at bus ℓ, and LMPs of $p_k =$ \$20/MWh and $p_\ell =$ \$30/MWh with transmission congestion. The revenue streams are shown in Table VII. By construction, the ISO net income is again zero. The consumer net payment is \$3900/h or an average price, including transmission payments, of \$26/MWh.[28] On net, each transmission owner e is paid $\$133\frac{1}{3}$/h. The generator at bus k is paid \$2000/h, while the generator at bus ℓ is paid \$1500/h.

B.3 Discussion

In the previous example, the LMP difference is either zero or the actual flow is equal to the contract quantity. Consequently, the net revenue stream to each transmission owner e is constant irrespective of demand level, indicating that transmission owner e has hedged the risk of volatile LMP difference payments by selling CFDDs. This is a powerful incentive for transmission owners to sell CFDDs. The strike price, together with the distribution of the demand, will determine the financial position of the transmission owner relative to the expected revenue from the unhedged revenue stream (1).

[28]The average price is lower than for the off-peak demand of 50 MW because we have included the transmission payments on a per MWh basis. Note, however, that some of the transmission payments are effectively sunk costs at the time of dispatch so that the marginal cost is higher on-peak than off-peak. In particular, for deviations from the contract quantity, energy is priced at the LMP of \$20/MWh off-peak and at \$30/MWh on-peak.

The revenue stream paid by the consumer at ℓ is also more constant than it would be in the absence of the CFDDs. Without CFDDs, the consumer would pay $20/MWh off-peak and $30/MWh on-peak. Hedging of transmission risk is a powerful incentive for the consumer to purchase CFDDs.

Moreover, as discussed in Section VI, transmission customers and transmission owners have opposite tastes for exposure to LMP difference fluctuations and they both hedge their risk by signing CFDDs. Analogously to CFDs, transmission customers and transmission owners can both costlessly hedge their risk by signing CFDDs. For the transmission customer, all transmission price risk is hedged for the contract quantity. For the transmission owner, hedging is imperfect since the actual flow will typically vary when there is a non-zero price difference across the line.

Furthermore, the consumer could also hedge its exposure to energy price variation by signing CFDs with the generators at buses k and ℓ. The clearing auction could easily be expanded to include offers for generation as well as offers for transmission service. (We will see an example of this in Section VII-C.6.)

In this example, the simple form of the network allowed determination of the flow on the line under the condition of non-zero LMP differences. However, a drawback of the network flow solution is that it is unlikely to correspond to an actual dispatch for any given line except for such simple networks. In more realistic situations, the net payment to the transmission owner will vary because non-zero LMP differences will occur at varying levels of flow on the line. Consequently, the revenue streams due to CFDDs may not match the payment from (1), even on average. Nevertheless, the net revenue stream will be less volatile with CFDDs than without. In the next section, we discuss an alternative trading mechanism where the match between the revenue streams due to the CFDDs and to the payment from (1) may be closer on average.

C. Simultaneous feasibility test clearing

C.1 Description

Instead of clearing based on the network flow solution, line owners may want to sign CFDDs only up to what they are sure will be the actual flow on their lines in contingency-constrained conditions for an assumed test system. In this case, a modified OBSCED similar to an FTR auction would enable CFDDs to be assembled. Transmission owners would effectively be offering their capacity, possibly as "price takers," in which case the prices would be set by the transmission customer bids.

The solution of the auction specifies the CFDD contract quantity and strike price for each transmission element. The transmission owner is then paid the contract quantity times the strike price, with the contract quantity times the resulting LMP differences from the OBSCED refunded by the transmission owner to its transmission customers. While this scheme is collectively revenue adequate assuming all lines are in service (using the same argument that shows that the ISO is revenue adequate under conventional FTRs), individual lines may be revenue inadequate if the flows and binding constraints under actual dispatch differ significantly from the solution of the auction or if lines are out of service. Mitigation of the risk of revenue inadequacy for individual lines could be accomplished through ownership of several lines, similarly to the way in which ownership of a portfolio of generators mitigates the risk that revenue is inadequate for individual generators when out-of-service.

C.2 Example with a single bid for CFDDs

We again consider the system of Figure 1 and again focus on typical hours. Assume that each transmission owner e offers its capacity as a price taker. Also suppose that the consumer at ℓ bids for 150 MW of CFDDs from k to ℓ at $4/MWh for all hours in a contract period. Performing OBSCED, there would again be 100 MW of CFDD at a strike price of $4/MWh.

C.3 Example with multiple bids for CFDDs

If there are multiple bids for CFDDs and multiple owners of transmission then the allocation of contracts to transmission owners is not unique. To see this, suppose that in the system of Figure 1 there are two successful bidders for the 100 MW of CFDDs between k and ℓ and that each bidder receives 50 MW of CFFDs. The assignment of contracts between lines $e = 1,2,3$ and bidders is arbitrary. In particular, one of the bidders might sign CFDDs for 30 MW with line 1 and for 20 MW with line 2, or it could sign any other combination adding up to 50 MW.

A natural way to resolve this arbitrariness in a DC power flow model would be to assign contracts based on power transfer distribution factors, so that in this case each bidder would receive $16\frac{2}{3}$ MW of CFDD from each line.

C.4 Example with previously allocated capacity

Suppose that 50 MW of CFDDs from k to ℓ had already been signed. For concreteness, suppose that there were $16\frac{2}{3}$ MW of previously allocated CFDDs with each of the lines $e = 1,2,3$. These CFDDs might have been arranged in a previous auction or as part of a long-term contract signed outside the centralized exchange.

An auction for the remaining 50 MW capacity can be performed by modeling the previously allocated CFDDs as a 50 MW injection at k and 50 MW withdrawal at ℓ. The new bids could then be cleared as previously.

C.5 Example with new transmission capacity offer

Suppose there was a proposal for construction of a fourth transmission line in parallel with the three existing lines in Figure 1. Suppose that 40 MW of transmission capacity was offered at a price of $3/MWh, with the new line having the same admittance as the three other lines. The additional line will increase the capability from k to ℓ to 120 MW. Assuming that no CFDDs had been signed on the system previously, and that the consumer at ℓ again bids for 150 MW of CFDDs from k to ℓ at a price of $4/MWh, then 120 MW of CFDDs are cleared at a strike price of $4/MWh.[29]

C.6 Example with transmission and energy offers and bids

We now consider an auction involving both forward transmission and forward energy offers and bids in the system in Figure 1. Suppose that energy was offered by the generator at bus k at 20 $/MWh and by the generator at bus ℓ at 30 $/MWh for all hours in a contract period. Furthermore, suppose that the consumer at bus ℓ bid for 50 MW of delivered power at bus ℓ at 28 $/MWh and for 100 MW of transmission service at 4 $/MWh. Furthermore, suppose all lines offer in their capacity as price takers. In this case, 50 MW of generated power is cleared at bus k at 20 $/MWh, 50 MW of delivered power is cleared at bus ℓ at 24 $/MWh, and, in addition, 50 MW of transmission service is cleared at 4 $/MWh. The demand at bus ℓ has acquired financial contracts for:

- 50 MW of power delivered at bus ℓ, and
- 50 MW of power generated at bus k and 50 MW of transmission from bus k to bus ℓ.

This establishes forward financial positions for both energy and transmission.

D. Summary

The discussion in Section VII-A shows that the CFDDs can be traded in a *decentralized* fashion, albeit with downside risk to the transmission owner if the contract quantity exceeds the actual flow on the line when the LMP price difference equals or is above the strike price. This risk may be acceptable in well-established systems having predictable flows when there is congestion; however, trading through an exchange can be arranged to reduce the risk.

In both proposals for *centralized* trading of transmission rights, the auctions arrange for financial hedges to transmission customers, but without necessitating the participation of the ISO. The actual dispatch is determined by the OBSCED, with no reference to the strike price of the CFFDs.[30]

[29] In general, we need to explicitly consider the decision of whether or not the offered line is built by incorporating an integer variable into the auction formulation. In this example, however, we can determine that the new line should be built. Note that the pre-existing transmission will receive a different revenue stream once the new transmission is built. There are two reasons for this change: the flow on each of the pre-existing lines will change and the LMP differences will be no greater.

[30] In the case of merchant transmission, it may be appropriate to allow a non-zero price for transmission offered into the OBSCED and possibly

VIII. MERCHANT CONSTRUCTION

A. Discussion

Merchant generators hoping to build new construction can be expected to desire to sign long-term contracts (both CFDs and CFDDs) to hedge themselves against LMP variation and to lock in prices in advance that their presence in the spot market may (temporarily) depress. Liquid forward long-term energy markets are an important part of encouraging merchant generation investment.

Similarly, merchant transmission providers can utilize forward markets to enable them to sign long-term contracts to hedge LMP variation and also lock in LMP differences. Because of lumpiness and economies of scale in transmission construction, it is conceivable that a transmission addition will significantly reduce LMP differences between the ends of the line at least during the first years of operation of a line until demand grows. The ability to sign contracts that are based on forward nodal energy price differences would allow such merchant transmission investment to be profitable despite temporarily depressing the LMP difference.

To date, only merchant high-voltage direct current (DC) transmission, which acts as a simultaneous buyer and seller of energy and receives a revenue stream given by (1), has appeared in any markets in the world. The introduction of border flow rights as a property right definition for *all* transmission would allow merchant AC transmission to also function as a simultaneous buyer and seller of energy, as implied by the revenue stream (1). The combination of border flow rights and CFDDs would help to enable the financing of merchant AC transmission through forward financial contracts for energy. To summarize, CFDDs, built on border flow rights, allow forward financial energy and transmission markets to support the development of merchant AC and DC transmission.

The value of transmission in providing for flow in both directions is compensated directly under border flow rights. This feature of border flow rights avoids a drawback of current FTR formulations where deviation between the nomination of the FTR and the actual patterns of dispatch can reduce or negate the payment to the FTR holder. Moreover, there is considerable flexibility for a transmission provider to offer option CFDDs and other financial products that considerably generalize point-to-point obligation FTRs.

As mentioned in the introduction, Bushnell and Stoft show that, under somewhat restrictive assumptions, any transmission investment that is detrimental to the grid will result in FTRs that have negative value to the builder [17], [18], [19]. Such a result does not extend to our proposed revenue stream as defined in (1). For example, consider again the construction of a fourth line in parallel with the three lines shown in Figure 1. If the line had the same admittance as the others, but had a capacity of only 10 MW then the welfare would be reduced by the presence of this line, even though it would be paid a positive amount under (1). A natural solution to this issue is presented in [64], where the ISO has flexibility to "commit" or "de-commit" transmission lines in a way that is analogous to commitment of generation. In particular, if the ISO has flexibility to disconnect a line if its presence reduces welfare, then the 10 MW line would be disconnected any time that the demand at bus ℓ exceeded 30 MW.[31]

Moreover, the benefit due to an expansion of the capacity of line 1 will accrue to all three lines. Indeed, by design, for price-taking marginal expansions of line 1, the marginal benefit of added capacity of line 1 is shared amongst all three lines. Consequently, even in the absence of lumpiness, an efficient level of investment requires financing by coalitions of beneficiaries. (See Theorem 5 and Corollary 6 in the Appendix for precise statement.)

While coalition funding is somewhat cumbersome and presents free-rider problems, transmission expansion currently involves such mechanisms in many jurisdictions. The border flow rights mechanism formalizes a property right for such expansion. To facilitate this process, there is a need for regulatory process that supports a third party expansion process, as in Chile and Argentina [57, page 30]. (See also [6, section 5.2] and [37, section 5].)

into the CFDD auction. See O'Neill *et al.* [64] for related discussion.

[31] Anecdotally, it appears that some lower voltage lines in the North American electricity system sometimes lower welfare when they are in-service because their capacity determines a binding contingency constraint. We have not explored the full implications of ISO flexibility to remove all such lines from service.

B. Example

Consider again the two node, three line network shown in Figure 1. We again focus on typical hours. Consider an expansion of the capacity of line 1 by 1.5 MW, with no change in admittance. This expansion increases the capability to import power from bus k to bus ℓ by 3 MW, increasing welfare by $(3\text{ MW}) \times (\$30/\text{MWh} - \$20/\text{MWh})$, or \$30/h, due to the decrease in generation at bus ℓ and the increase in generation at bus k. This change does not affect the prices and so is a "price-taking" marginal expansion.

The flows on each line increase by 1 MW and each line receives an additional payment of \$10/h. That is, the total increase in welfare of \$30/h due to the transmission expansion is paid out to the lines. Although perhaps somewhat cumbersome, a coalition of the owners of the three lines could finance the expansion of line 1 if the cost of expansion were compensated by the total increase in welfare of \$30/h.

IX. Conclusion

In this paper, we have proposed a property right for transmission based on the approach of Gribik, Shirmohammadi, *et al.* [2], by defining an underlying revenue stream that accrues to the owner of a transmission line. Under the proposed border flow rights model, the owner of a transmission line is paid at the locational marginal price for net energy that it delivers to the rest of the system, therefore guaranteeing revenue neutrality for ISO. (See Theorems 1 and 3.) Border flow rights using pre-contingency flows provide an approximation to efficient marginal incentives for transmission expansion by coalitions of beneficiaries. (See Theorem 5 and Corollary 6.)

Based on the property right for transmission, we have proposed a financial right for hedging LMP differences, called a contract for differences of differences, and provided examples of its use. The CFDD is based on the underlying revenue stream in the border flow right. Unlike previous FTR formulations, we first define a property right in terms of an underlying revenue stream that is independent of FTR nominations and then define a financial right that is built on the underlying property right.

Analogously to contracts for differences, contracts for differences of differences can be traded without an ISO. Nevertheless, exchange trading of CFDDs has several advantages over completely decentralized trading. Furthermore, both transmission and energy can be traded forward in one exchange, avoiding the bifurcation in current markets. The ISO could be, but does not have to be, involved in the exchange.

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References

[1] Paul R. Gribik, Dariush Shirmohammadi, Joseph S. Graves, and James G. Kritikson, "Long-term rights for transmission expansions," Unpublished manuscript, 2002.

[2] Paul R. Gribik, Dariush Shirmohammadi, Joseph S. Graves, and James G. Kritikson, "Transmission rights and transmission expansions," *IEEE Transactions on Power Systems*, vol. 20, no. 4, pp. 1728–1737, November 2005.

[3] National Electricity Code Administrator (Australia), "Entrepreneurial interconnectors: Safe harbour provisions," Report of working group on inter-regional hedges and entrepreneurial interconnectors. Available at www.neca.com.au, November 1998.

[4] M. C. Caramanis, R. E. Bohn, and F. C. Schweppe, "Optimal spot pricing: Practice and theory," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-101, no. 9, pp. 3234–3245, September 1982.

[5] I. J. Pérez-Arriaga, F. J. Rubio, J. F. Puerta, J. Arceluz, and J. Marin, "Marginal pricing of transmission services: An analysis of cost recovery," *IEEE Transactions on Power Systems*, vol. 10, no. 1, pp. 546–553, February 1995.

[6] James B. Bushnell and Steven E. Stoft, "Improving private incentives for electric grid investment," *Resource and Energy Economics*, vol. 19, pp. 85–108, 1997.

[7] Richard P. O'Neill, Emily S. Bartholomew, Benjamin F. Hobbs, and Ross Baldick, "A complete real-time electricity market design," Submitted to *Journal of Regulatory Economics*, 2007.

[8] William W. Hogan, "Contract networks for electric power transmission," *Journal of Regulatory Economics*, vol. 4, no. 3, pp. 211–242, 1992.

[9] Steven Stoft, *Power System Economics: Designing Markets for Electricity*, IEEE Press and Wiley Interscience and John Wiley & Sons, Inc., Piscataway, NJ, 2002.

[10] Intercontinential Exchange, "Product guide," Available from www.theice.com, October 2006.

[11] Bert Willems, "Modeling Cournot competition in an electricity market with transmission constraints," *The Energy Journal*, vol. 23, no. 3, pp. 95–125, 2002.

[12] William Hogan, "Flowgate rights and wrongs," Tech. Rep., Center for Business and Government, The John F. Kennedy School of Government, Harvard University, August 2000.

[13] William Hogan, "Financial transmission right formulations," John F. Kennedy School of Government, Harvard University, March 2002.

[14] Hung-po Chao and Stephen Peck, "A market mechanism for electric power transmission," *Journal of Regulatory Economics*, vol. 10, no. 1, pp. 25–59, July 1996.

[15] Hung-po Chao and Stephen Peck, "An institutional design for an electricity contract market with central dispatch," *The Energy Journal*, vol. 18, no. 1, pp. 85–110, 1997.

[16] Shmuel S. Oren, "Economic inefficiency of passive transmission rights in congested systems with competitive transmission," *The Energy Journal*, vol. 18, no. 1, pp. 63–83, 1997.

[17] James B. Bushnell and Steven E. Stoft, "Grid investment under a contract networks regime," *Journal of Regulatory Economics*, vol. 10, no. 1, pp. 61–79, 1996.

[18] James Bushnell and Steven Stoft, "Grid investment: Can a market do the job?," *The Electricity Journal*, January/February 1996.

[19] James Bushnell and Steven Stoft, "Transmission and generation investment in a competitive electric power industry pwp-030," Tech. Rep., POWER University of California Energy Institute Berkeley, January 1996.

[20] P. Wei, Y. Ni, and F. F. Wu, "Decentralized approach for congestion management and congestion price discovering," *IEE Proceedings–Generation, Transmission, Distribution*, vol. 149, no. 6, pp. 645–652, November 2002.

[21] William W. Hogan, "Market-based transmission investments and competitive electricity markets," Tech. Rep., Center for Business and Government, The John F. Kennedy School of Government, Harvard University, 1999.

[22] Paul Joskow and Jean Tirole, "Merchant transmission investment," Unpublished manuscript, Department of Economics, M.I.T., Cambridge, MA, February 2003.

[23] Stephen Littlechild, "Transmission regulation, merchant investment, and the experience of SNI and Murraylink in the Australian national electricity market," Presented to Harvard Electricity Policy Group, Harvard University, available from ksgwww.harvard.edu/hepg/index.html, June 2003.

[24] Stephen Littlechild, "Regulated and merchant interconnectors in Australia: SNI and Murraylink in the n national electricity market," Cambridge Working Papers in Economics CWPE 0410, available from ksgwww.harvard.edu/hepg/index.html, January 2004.

[25] Stephen C. Littlechild and Carlos J. Skerk, "Regulation of transmission expansion in Argentina part I: State ownership, reform, and the fourth line," Cambridge Working Papers in Economics CWPE 0464, available from ksgwww.harvard.edu/hepg/index.html, November 2004.

[26] Stephen C. Littlechild and Carlos J. Skerk, "Regulation of transmission expansion in Argentina part II: Developments since the fourth line," Cambridge Working Papers in Economics CWPE 0465, available from ksgwww.harvard.edu/hepg/index.html, November 2004.

[27] Paul Joskow, "Transmission policy in the United States," Tech. Rep. Related Publication 04-26, AEI-Brookings Joint Center for Regulatory Studies, October 2004.

[28] Benjamin F. Hobbs, "Network models of spatial oligopoly with an application to deregulation of electricity generation," *Operations Research*, vol. 34, no. 5, pp. 395–409, May–June 1986.

[29] William W. Hogan, "A market power model with strategic interaction in electricity markets," *The Energy Journal*, vol. 18, no. 4, pp. 107–141, 1997.

[30] Judith B. Cardell, Carrie Cullen Hitt, and William W. Hogan, "Market power and strategic interaction in electricity networks," *Resource and Energy Economics*, vol. 19, no. 1–2, pp. 109–137, March 1997.

[31] Steven Stoft, "Financial transmission rights meet Cournot: How TCCs curb market power," *The Energy Journal*, vol. 20, pp. 1–23, 1999.

[32] Carolyn A. Berry, Benjamin Hobbs, William A. Meroney, Richard P. O'Neill, and William R. Stewart, Jr., "Analyzing strategic bidding behavior in transmission networks," in *IEEE Tutorial on Game Theory Applications in Power Systems*, H. Singh, Ed., pp. 7–32. IEEE, 1999.

[33] Severin Borenstein, James Bushnell, and Steven Stoft, "The competitive effects of transmission capacity in a deregulated electricity industry," *RAND Journal of Economics*, vol. 31, no. 2, pp. 294–325, Summer 2000.

[34] Benjamin Hobbs, Carolyn Metzler, and Jong-Shi Pang, "Strategic gaming analysis for electric power networks: An MPEC approach," *IEEE Transactions on Power Systems*, vol. 15, no. 2, pp. 638–645, May 2000.

[35] Paul L. Joskow and Jean Tirole, "Transmission rights and market power on electric power markets," *RAND Journal of Economics*, vol. 31, no. 3, pp. 450–487, Autumn 2000.

[36] Benjamin F. Hobbs, "LCP models of Nash-Cournot competition in bilateral and poolco-based power markets," *IEEE Transactions on Power Systems*, vol. 16, no. 2, May 2001.

[37] Juan Rosellón, "Different approaches towards electricity transmission expansion," *Review of Network Economics*, vol. 1, no. 3, pp. 85–108, September 2003.

[38] In-Koo Cho, "Competitive equilibrium in a radial network," *RAND Journal of Economics*, vol. 34, no. 3, pp. 438–460, Autumn 2003.

[39] Richard Gilbert, Karsten Neuhoff, and David M. Newbery, "Allocating transmission to mitigate market power in electricity markets," *RAND Journal of Economics*, vol. 35, no. 4, pp. 691–709, Winter 2004.

[40] William W. Hogan, "An efficient electricity pool market model," unpublished manuscript, 1994.

[41] O. Alsaç, J. M. Bright, S. Brignone, M. Prais, C. Silva, B. Stott, and N. Vempati, "The rights to fight price volatility," *IEEE Power and Energy Magazine*, vol. 2, no. 4, pp. 47–57, July–August 2004.

[42] Hung-Po Chao, Stephen Peck, Shmuel Oren, and Robert Wilson, "Flow-based transmission rights and congestion management," *The Electricity Journal*, vol. 13, no. 8, pp. 38–58, 2000.

[43] Richard O'Neill, Udi Helman, Ross Baldick, William Stewart, and Michael Rothkopf, "Contingent transmission rights in the standard market design," *IEEE Transactions on Power Systems*, vol. 18, no. 4, pp. 1331–1337, November 2003.

[44] Felix Wu, Pravin Varaiya, Pablo Spiller, and Shmuel Oren, "Folk theorems on transmission access: Proofs and counterexamples," *Journal of Regulatory Economics*, vol. 10, no. 1, pp. 5–23, 1996.

[45] Richard P. O'Neill, Udi Helman, Benjamin F. Hobbs, Jr. William R. Stewart, and Michael H. Rothkopf, "A joint energy and transmission rights auction: Proposal and properties," *IEEE Transactions on Power Systems*, vol. 17, no. 4, pp. 1058–1067, November 2002.

[46] Potomac Economics, Ltd, *2004 State of the Market Report New York ISO*, October 2004.
[47] Edward J. Anderson and Huifu Xu, "Optimal supply functions in electricity markets with option contracts and non-smooth costs," Australian Graduate School of Management, The University of New South Wales, Sydney, NSW, 2005.
[48] Susan L. Pope, "TCC awards for transmission expansions," Presentation prepared for PJM Regional Transmission Planning Stakeholder Process Meeting, available from http://www.pjm.com/services/trans/meeting/downloads/20020412_tcc_awards.pdf, April 2002.
[49] William Hogan, "Financial transmission right incentives: Applications beyond hedging," John F. Kennedy School of Government, Harvard University, May 2002.
[50] Tarjei Kristiansen and Juan Rosellón, "A merchant mechanism for electricity transmission expansion," *Journal of Regulatory Economics*, vol. 29, no. 2, pp. 167–193, March 2006.
[51] Tarjei Kristiansen, "Allocation of long-term financial transmission rights for transmission expansion," Unpublished manuscript, 2005.
[52] Arthur R. Bergen and Vijay Vittal, *Power Systems Analysis*, Prentice-Hall, Upper Saddle River, NJ, second edition, 2000.
[53] C. Dechamps and E. Jamoulle, "Interactive computer program for planning the expansion of meshed transmission networks," *International Journal of Electrical Power and Energy Systems*, vol. 2, no. 2, pp. 103–108, April 1980.
[54] Mario V.F. Pereira and Leontina M.V.G. Pinto, "Application of sensitivity analysis of load supplying capability to interactive transmission expansion planning," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-104, no. 2, pp. 381–389, February 1985.
[55] Rubén Darío Cruz, Gerardo Latorre, and Ross Baldick, "AC sensitivities to transmission expansion," Submitted to *IEEE Transactions on Power Systems*, 2006.
[56] Krishnan Dixit and Ross Baldick, "An empirical study of the economies of scale in AC transmission line construction costs," Unpublished manuscript, December 2003.
[57] Shmuel Oren, Pablo Spiller, et al., "Nodal prices and transmission rights: A critical appraisal," *The Electricity Journal*, pp. 24–35, April 1995.
[58] Ross Baldick, "Shift factors in ERCOT congestion pricing," Technical report. Available from www.ece.utexas.edu/ baldick/papers/shiftfactors.pdf, March 2003.
[59] Anthony V. Fiacco, *Introduction to Sensitivity and Stability Analysis in Nonlinear Programming*, Academic Press, New York, 1983.
[60] Paul R. Gribik, Dariush Shirmohammadi, Shangyou Hao, and Chifong L. Thomas, "Optimal power flow sensitivity analysis," *IEEE Transactions on Power Systems*, vol. 5, no. 3, pp. 969–976, August 1990.
[61] William Vickrey, "Counterspeculation, auctions, and competitive sealed tenders," *Journal of Finance*, vol. 16, March 1961.
[62] Theodore Groves and John Ledyard, "Optimal allocation of public goods: A solution to the "free rider" problem," *Econometrica*, vol. 45, no. 4, pp. 783–809, May 1977.
[63] Ross Baldick, "Inventory of the capabilities of transmission assets," Unpublished manuscript, 2005.
[64] Richard P. O'Neill, Ross Baldick, Udi Helman, Michael H. Rothkopf, and William R. Stewart, Jr., "Dispatchable transmission in RTO markets," *IEEE Transactions on Power Systems*, vol. 20, no. 1, pp. 171–179, February 2005.
[65] Eugene L. Lawler, *Combinatorial Optimization: Networks and Matroids*, Holt, Rinehart and Winston, New York, 1976.
[66] David G. Luenberger, *Linear and Nonlinear Programming*, Addison-Wesley Publishing Company, Reading, MA, second edition, 1984.

APPENDIX

This appendix first discusses the equivalence of sensitivity based payment and payment based on (1). Then incentives for price taking marginal expansion are discussed.

I. EQUIVALENCE OF SENSITIVITY BASED PAYMENT AND (1)

We formulate the security-constrained (revealed) welfare maximization problem. Our formulation explicitly represents line and other transmission element parameters and limits that appear *linearly* in the power flow equality and inequality constraints when the base-case and contingency-case constraints are represented explicitly. We illustrate the formulation with reference to the example problem shown in Figure 1 for typical hours.

Our notation is, in part, based on that in [13]. In particular, let $y \in \mathbb{R}^{n_y}$ be the vector of net power loads at all the buses and let $b : \mathbb{R}^{n_y} \to \mathbb{R}$ be the net (revealed) benefits of consumption minus costs of production at the buses. We assume that there may also be a vector of controls $u \in \mathbb{R}^{n_u}$ that the ISO can adjust, potentially within limits. (Depending on the formulation, y may include both real and reactive power net loads, or just real power net power loads, or might be expanded to include any collection of quantities that are conserved nodally, including reserves. In the following, we will refer to y as the "net loads," with the understanding that it might encompass more than just real power.)

We differ from the formulation in [13] in some respects. In particular, we will explicitly represent voltage magnitudes $V \in \mathbb{R}^{n_V}$ and phases $\theta \in \mathbb{R}^{n_\theta}$ at all the buses (including the slack bus). We also consider a vector, $\chi \in \mathbb{R}^{n_\chi}$, of *parameters* and *limits* of transmission system elements.

In section A-A, we first consider the case ignoring contingency constraints and then incorporate contingency constraints in section A-B.

A. No contingency constraints

In this section, we consider the case where only constraints on base-case operations are modelled.

A.1 Net loads, objective, and other variables for example system during typical hours

The generation in the example in Figure 1 would appear in the vector y as negative net loads, $-q_k$ and $-q_\ell$, respectively. In particular, $y \in \mathbb{R}^2$, with:

$$y = \begin{bmatrix} -q_k + 150 \\ -q_\ell + 150 \end{bmatrix},$$

and $b : \mathbb{R}^2 \to \mathbb{R}$ with:

$$\forall y \in \mathbb{R}^2, b(y) = -20(150 - y_1) - 30(150 - y_2).$$

If we use a DC power flow, we can ignore the vector V. There are no controls u. There is an angle for each bus, so that $\theta = \begin{bmatrix} \theta_k \\ \theta_\ell \end{bmatrix} \in \mathbb{R}^2$.

A.2 Constraints

In addition to the objective of the welfare maximization problem, there are equality and inequality constraints on base-case flows. The equality constraints arise from Kirchhoff's laws and relate the vector of net loads to the voltage magnitudes, voltage phases, controls, and parameters and limits. We assume a very general form for this functional relationship. In particular, we assume that there is a matrix valued function $S : \mathbb{R}^{n_V} \times \mathbb{R}^{n_\theta} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_y \times n_\chi}$, such that the net loads satisfy:

$$y = S(V, \theta, u)\chi. \tag{3}$$

That is, the parameters χ enter linearly into the specification of the power flow equations, while the effect of voltage magnitudes, voltage phases, and controls is, in principle, arbitrary.

For example, consider a transmission line e, with parameters that are specified by a sub-vector $\chi_e = \begin{bmatrix} G_e \\ B_e \\ \overline{f}_e \end{bmatrix}$ of χ. The parameters G_e, B_e, and $\overline{f}_e$ are, respectively, the absolute value of the real and imaginary parts of the series admittance of the line and the capacity of the line. (We will discuss $\overline{f}_e$ when we define the inequality constraints.)

Suppose that the element e joins buses k and ℓ and that the k-th and the ℓ-th entries of y specify real power balance at buses k and ℓ respectively. For concreteness, partition χ into $\chi = \begin{bmatrix} \chi_e \\ \chi_R \end{bmatrix}$, where χ_R specifies the parameters of the rest of the system. Similarly, partition S into $S = \begin{bmatrix} S_e & S_R \end{bmatrix}$ and let S_{ke} and $S_{\ell e}$ be the k-th and ℓ-th rows of S_e, respectively, and let S_{kR} and $S_{\ell R}$ be the k-th and ℓ-th rows of S_R, respectively.

Then, using the AC power flow formulation,

$$\begin{aligned} S_{ke} &= \begin{bmatrix} -V_\ell V_k \cos(\theta_k - \theta_\ell) & -V_\ell V_k \sin(\theta_k - \theta_\ell) & 0 \end{bmatrix}, \\ S_{\ell e} &= \begin{bmatrix} -V_\ell V_k \cos(\theta_\ell - \theta_k) & -V_\ell V_k \sin(\theta_\ell - \theta_k) & 0 \end{bmatrix}, \end{aligned}$$

with all of the other rows of S_e equal to the zero function, so that:

$$\begin{aligned} S_{ke}\chi_e &= -V_\ell V_k [G_e \cos(\theta_k - \theta_\ell) + B_e \sin(\theta_k - \theta_\ell)], \\ S_{\ell e}\chi_e &= -V_\ell V_k [G_e \cos(\theta_\ell - \theta_k) + B_e \sin(\theta_\ell - \theta_k)]. \end{aligned}$$

Therefore, (3) reproduces the appropriate terms in the real power flow balance equations, since power balance at buses k and ℓ requires that:

$$\begin{aligned} y_k &= -V_\ell V_k[G_e\cos(\theta_k-\theta_\ell)+B_e\sin(\theta_k-\theta_\ell)]+S_{kR}(V,\theta,u)\chi_R, \\ y_\ell &= -V_\ell V_k[G_e\cos(\theta_\ell-\theta_k)+B_e\sin(\theta_\ell-\theta_k)]+S_{\ell R}(v,\theta,u)\chi_R. \end{aligned}$$

We assume that there is also a matrix valued function $F:\mathbb{R}^{n_V}\times\mathbb{R}^{n_\theta}\times\mathbb{R}^{n_u}\to\mathbb{R}^{n_F\times n_\chi}$ such that:

$$F(V,\theta,u)\chi\le\mathbf{0}, \tag{4}$$

specifies the inequality constraints in the system. Moreover, we assume that F is block diagonal, with each diagonal block specifying inequality constraints relating to a different transmission element. In particular, using the example transmission element e above, we assume that F partitions into:

$$F=\begin{bmatrix} F_e & \mathbf{0} \\ \mathbf{0} & F_R \end{bmatrix},$$

where F_e specifies the constraints for element e and F_R specifies the constraints for the rest of the system. Given a real power limit of $\overline{f}_e$ on this line, F_e could be specified as:

$$F_e=\begin{bmatrix} V_\ell V_k\cos(\theta_k-\theta_\ell) & V_\ell V_k\sin(\theta_k-\theta_\ell) & -1 \\ V_\ell V_k\cos(\theta_\ell-\theta_k) & V_\ell V_k\sin(\theta_\ell-\theta_k) & -1 \end{bmatrix},$$

with flow constraints on e then given by $F_e(V,\theta,u)\chi_e\le\mathbf{0}$. That is:

$$\begin{aligned} V_\ell V_k[G_e\cos(\theta_k-\theta_\ell)+B_e\sin(\theta_k-\theta_\ell)]-\overline{f}_e &\le 0, \\ V_\ell V_k[G_e\cos(\theta_\ell-\theta_k)+B_e\sin(\theta_\ell-\theta_k)]-\overline{f}_e &\le 0. \end{aligned}$$

Other constraints can be represented similarly.[32]

A.3 Constraints for example system

The specification of:

$$\begin{aligned} \chi &= \begin{bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{bmatrix}\in\mathbb{R}^6, \\ S &= \begin{bmatrix} S_1 & S_2 & S_3 \end{bmatrix}:\mathbb{R}^2\to\mathbb{R}^{2\times 6}, \text{ and,} \\ F &= \begin{bmatrix} F_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & F_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & F_3 \end{bmatrix}:\mathbb{R}^2\to\mathbb{R}^{3\times 6}, \end{aligned}$$

for the example system, using the DC power flow formulation, are:

$$\begin{aligned} \chi_e &= \begin{bmatrix} B_e \\ \overline{f}_e \end{bmatrix}\in\mathbb{R}^2, e=1,2,3, \\ \forall\theta\in\mathbb{R}^2, S_e(\theta) &= \begin{bmatrix} -(\theta_k-\theta_\ell) & 0 \\ -(\theta_\ell-\theta_k) & 0 \end{bmatrix}, e=1,2,3, \text{ and,} \\ \forall\theta\in\mathbb{R}^2, F_e(\theta) &= \begin{bmatrix} (\theta_k-\theta_\ell) & -1 \end{bmatrix}, e=1,2,3. \end{aligned}$$

[32] For example, a current limit can be parametrized in this way if the magnitude of the admittance is included as a parameter.

A.4 Welfare optimization problem

To summarize, the welfare optimization problem is formulated as:

$$\max_{y\in\mathbb{R}^{n_y}, V\in\mathbb{R}^{n_V}, \theta\in\mathbb{R}^{n_\theta}, u\in\mathbb{R}^{n_u}} \{b(y) | y = S(V,\theta,u)\chi, F(V,\theta,u)\chi \leq \mathbf{0}\}, \tag{5}$$

which we assume has a solution $y^\star, V^\star, \theta^\star, u^\star$. As χ varies, the maximum of the welfare optimization problem (5) will vary. We will denote the maximum of problem (5) by $\mathcal{W}(\chi)$. This formulation differs from (4) of [13] in that we have explicitly kept the dependence of the constraints on voltage magnitudes and phases. The reason for doing this is to maintain a form for the constraint functions that is linear in χ. This will enable us to prove the main results very directly.

A.5 Lagrangian

The Lagrangian of the welfare problem is:

$$\mathcal{L}(y,V,\theta,u,p,\eta) = b(y) - p^\dagger(y - S(V,\theta,u)\chi) - \eta^\dagger F(V,\theta,u)\chi,$$

where p and η are dual variables and superscript † means transpose. Let $p^\star$ and $\eta^\star$ be the vectors of Lagrange multipliers corresponding to the solution $y^\star, V^\star, \theta^\star, u^\star$. We note that $p^\star$ is the vector of LMPs and that we can partition $\eta^\star$ into $\eta^\star = \begin{bmatrix} \eta_e^\star \\ \eta_R^\star \end{bmatrix}$, where $\eta_e^\star$ are the Lagrange multipliers corresponding to constraints on operation of element e.

A.6 Revenue stream based on border flow right and LMPs

We note that $S_e(V^\star,\theta^\star,u^\star)\chi_e$ represents the net border flows from element e into the rest of the system. Paying for that border flow at LMPs results in a revenue stream of $[p^\star]^\dagger S_e(V^\star,\theta^\star,u^\star)\chi_e$ to element e. This is the general case of (1). Summing this expression over all elements e yields the total revenue to transmission elements of:

$$\mathcal{R}(\chi) = [p^\star]^\dagger S(V^\star,\theta^\star,u^\star)\chi.$$

Since $y^\star = S(V^\star,\theta^\star,u^\star)\chi$ and net loads pay according to the prices $p^\star$, we observe that the net load payment $[p^\star]^\dagger y^\star = [p^\star]^\dagger S(V^\star,\theta^\star,u^\star)\chi = \mathcal{R}(\chi)$. We have proved:

Theorem 1: Under the revenue stream based on the border flow right with pre-contingency flows and LMPs, the ISO is *exactly* revenue neutral under all dispatch conditions. □

A.7 Payment based on sensitivity of optimal welfare

Assuming appropriate second-order, linear independence, and non-degeneracy conditions [59, theorem 2.4.4], the sensitivity of optimal welfare to χ_e evaluated at the solution is given by:

$$\begin{aligned}
\frac{\partial \mathcal{W}}{\partial \chi_e} &= \frac{\partial \mathcal{L}}{\partial \chi_e}, \\
&= [p^\star]^\dagger S_e(V^\star,\theta^\star,u^\star) - [\eta^\star]^\dagger \begin{bmatrix} F_e(V^\star,\theta^\star,u^\star) \\ \mathbf{0} \end{bmatrix}, \\
&= [p^\star]^\dagger S_e(V^\star,\theta^\star,u^\star) - [\eta_e^\star]^\dagger F_e(V^\star,\theta^\star,u^\star).
\end{aligned}$$

Revenue to the owner of element e based on the sensitivity of optimal welfare is given by:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \chi_e}\chi_e &= [p^\star]^\dagger S_e(V^\star,\theta^\star,u^\star)\chi_e - [\eta_e^\star]^\dagger F_e(V^\star,\theta^\star,u^\star)\chi_e, \\
&= [p^\star]^\dagger S_e(V^\star,\theta^\star,u^\star)\chi_e - [\eta_e^\star]^\dagger \begin{bmatrix} F_e(V^\star,\theta^\star,u^\star) & \mathbf{0} \end{bmatrix} \chi, \\
&= [p^\star]^\dagger S_e(V^\star,\theta^\star,u^\star)\chi_e,
\end{aligned}$$

by complementary slackness [66]. That is, we have proved:

Theorem 2: Suppose that there are no contingency constraints. Under appropriate second-order, linear independence, and non-degeneracy conditions, payment based on sensitivity of optimal welfare is the same as payment based on the border flow right of net injections times LMPs. □

B. Contingency constraints

B.1 Variables

Following [13], we distinguish contingencies by a superscript ω. We assume that each contingency $\omega \in \Omega$ has an associated probability Π^{ω}, which may be vanishingly small. The probability of the base-case is $\Pi^0 = 1 - \sum_{\omega \in \Omega} \Pi^{\omega}$. In each contingency, we consider that the vector of net loads may change from y to $y + \Delta y^{\omega}$, with associated benefit changed by $\Delta b(y, \Delta y^{\omega})$. If only transmission constraints are modelled and there are no losses then we can omit Δy^{ω} and Δb. Assuming risk neutrality, the objective of the revealed welfare maximization problem is now $\Pi^0 b(y) + \sum_{\omega} \Pi^{\omega} \Delta b(y, \Delta y^{\omega})$.

B.2 Constraints and problem

Under each contingency, there is a specification of Kirchhoff's laws S^{ω} and constraints F^{ω}. The solution of the network equations and controls under contingency ω is specified by $V^{\omega}, \theta^{\omega}, u^{\omega}$. The welfare optimization problem becomes:

$$\max_{\substack{y \in \mathbb{R}^{n_y}, \Delta y^{\omega} \in \mathbb{R}^{n_y}, \forall \omega \in \Omega, \\ V \in \mathbb{R}^{n_V}, V^{\omega} \in \mathbb{R}^{n_V}, \forall \omega \in \Omega, \\ \theta \in \mathbb{R}^{n_\theta}, \theta^{\omega} \in \mathbb{R}^{n_\theta}, \forall \omega \in \Omega, \\ u \in \mathbb{R}^{n_u}, u^{\omega} \in \mathbb{R}^{n_u}, \forall \omega \in \Omega}} \left\{ \Pi^0 b(y) + \sum_{\omega \in \Omega} \Pi^{\omega} \Delta b(y, \Delta y^{\omega}) \,\middle|\, \begin{array}{r} y = S(V, \theta, u)\chi, y + \Delta y^{\omega} = S^{\omega}(V^{\omega}, \theta^{\omega}, u^{\omega})\chi, \forall \omega \in \Omega, \\ F(V, \theta, u)\chi \leq \mathbf{0}, F^{\omega}(V^{\omega}, \theta^{\omega}, u^{\omega})\chi \leq \mathbf{0}, \forall \omega \in \Omega \end{array} \right\}.$$

We assume that there is a solution $y^{\star}, \Delta y^{\omega\star}, \forall \omega \in \Omega, V^{\star}, V^{\omega\star}, \forall \omega \in \Omega, \theta^{\star}, \theta^{\omega\star}, \forall \omega \in \Omega, u^{\star}, u^{\omega\star}, \forall \omega \in \Omega$.

B.3 Contingency constraints for example problem

The contingency constraints for the example problem are specified by:

$$\begin{array}{rcl}
\Omega & = & \{1,2,3\}, \\
S^{\omega} & = & \left[\begin{array}{ccc} S_1^{\omega} & S_2^{\omega} & S_3^{\omega} \end{array}\right] : \mathbb{R}^2 \to \mathbb{R}^{2\times 6}, \\
\text{where: } \forall \theta^{\omega} \in \mathbb{R}^2, S_e^{\omega}(\theta^{\omega}) & = & \left[\begin{array}{cc} -(\theta_k^{\omega} - \theta_\ell^{\omega}) & 0 \\ -(\theta_\ell^{\omega} - \theta_k^{\omega}) & 0 \end{array}\right], \omega = 1,2,3, e \neq \omega, \\
\forall \theta^{\omega} \in \mathbb{R}^2, S_{\omega}^{\omega}(\theta^{\omega}) & = & \mathbf{0}, \omega = 1,2,3, \\
F^1 & = & \left[\begin{array}{ccc} \mathbf{0} & F_2^1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & F_3^1 \end{array}\right] : \mathbb{R}^2 \to \mathbb{R}^{2\times 6}, \\
F^2 & = & \left[\begin{array}{ccc} F_1^2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & F_3^2 \end{array}\right] : \mathbb{R}^2 \to \mathbb{R}^{2\times 6}, \\
F^3 & = & \left[\begin{array}{ccc} F_1^3 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & F_2^3 & \mathbf{0} \end{array}\right] : \mathbb{R}^2 \to \mathbb{R}^{2\times 6}, \\
\text{where: } \forall \theta^{\omega} \in \mathbb{R}^2, F_e^{\omega}(\theta) & = & \left[\begin{array}{cc} (\theta_k^{\omega} - \theta_\ell^{\omega}) & -1 \end{array}\right], \omega = 1,2,3, e \neq \omega.
\end{array}$$

The only binding inequality constraints correspond to $\omega = 2,3$ and $e = 1$. Since only transmission contingencies are modelled and since the system is lossless, we can omit Δb and the vectors Δy^{ω}.

B.4 Lagrangian

The Lagrangian is now:

$$\begin{aligned}
&\mathcal{L}(y, \Delta y^{\omega}, \omega \in \Omega, V, V^{\omega}, \omega \in \Omega, \theta, \theta^{\omega}, \omega \in \Omega, u, u^{\omega}, \omega \in \Omega, p^0, p^{\omega}, \omega \in \Omega, \eta^0, \eta^{\omega}, \omega \in \Omega) \\
&\quad = \Pi^0 b(y) + \sum_{\omega\in\Omega} \Pi^{\omega} \Delta b(y, \Delta y^{\omega}) \\
&\qquad - [p^0]^{\dagger}(y - S(V,\theta,u)\chi) - \sum_{\omega\in\Omega} [p^{\omega}]^{\dagger}(y + \Delta y^{\omega} - S^{\omega}(V^{\omega},\theta^{\omega},u^{\omega})\chi) \\
&\qquad - [\eta^0]^{\dagger} F(V,\theta,u)\chi - \sum_{\omega\in\Omega} [\eta^{\omega}]^{\dagger} F^{\omega}(V^{\omega},\theta^{\omega},u^{\omega})\chi,
\end{aligned}$$

where we denote the dual variables corresponding to the base-case constraints by a superscript 0 to distinguish them from the symbol that we will use for the LMPs.

Again we assume that there are corresponding Lagrange multipliers $p^{0\star}, p^{\omega\star}, \forall\omega \in \Omega, \eta^{0\star}, \eta^{\omega\star}, \forall\omega \in \Omega$. As in the non-contingency constrained case, we will focus on a particular transmission element and assume that the functions $F, F^{\omega}, \omega \in \Omega, S$, and $S^{\omega}, \omega \in \Omega$, and the Lagrange multipliers $\eta^{0\star}$ and $\eta^{\omega\star}$ have been partitioned as before.

B.5 Nodal prices

The LMPs (for net loads y in the base-case) are now given by $p^{\star} = p^{0\star} + \sum_{\omega\in\Omega} p^{\omega\star}$. We can interpret these as the sum of prices corresponding to energy and to binding constraints in the base and contingency cases.

If revenues to transmission lines are based on the sum over base and contingency states of the product of net injection or withdrawal in a state times the state dependent LMP then we again have that:

Theorem 3: Under the revenue stream based on the border flow right with contingency flows and state dependent LMPs, the ISO is *exactly* revenue neutral under all dispatch conditions. □

B.6 Payment based on sensitivity of optimal welfare

Again assuming appropriate second-order, linear independence, and non-degeneracy conditions [59, theorem 2.4.4], the sensitivity of optimal welfare $\mathcal{W}$ to χ_e is given by:

$$\begin{aligned}
\frac{\partial \mathcal{W}}{\partial \chi_e} &= \frac{\partial \mathcal{L}}{\partial \chi_e} \\
&= [p^{0\star}]^{\dagger} S_e(V^{\star},\theta^{\star},u^{\star}) + \sum_{\omega\in\Omega} [p^{\omega\star}]^{\dagger} S_e^{\omega}(V^{\omega\star},\theta^{\omega\star},u^{\omega\star}) \\
&\quad - [\eta_e^{0\star}]^{\dagger} F_e(V^{\star},\theta^{\star},u^{\star}) - \sum_{\omega\in\Omega} [\eta^{\omega\star}]^{\dagger} F_e^{\omega}(V^{\omega\star},\theta^{\omega\star},u^{\omega\star}).
\end{aligned}$$

Again by complementary slackness, payment to the owner of element e based on the sensitivity of optimal welfare is given by:

$$\frac{\partial \mathcal{L}}{\partial \chi_e}\chi_e = \left[[p^{0\star}]^{\dagger} S_e(V^{\star},\theta^{\star},u^{\star}) + \sum_{\omega\in\Omega} [p^{\omega\star}]^{\dagger} S_e^{\omega}(V^{\omega\star},\theta^{\omega\star},u^{\omega\star}) \right] \chi_e. \tag{6}$$

This payment is the sum over base and contingency cases of injections in each case weighted by corresponding Lagrange multipliers. We have proved:

Theorem 4: Under appropriate second-order, linear independence, and non-degeneracy conditions, payment based on sensitivity of optimal welfare is the same as payment based on the border flow right of the sum over base and contingency states of net injections in each case weighted by corresponding Lagrange multipliers. □

This result appears to simplify and generalize the analysis in Gribik, Shirmohammadi, *et al.* [2]. In the example, the two constraints were both binding but not linearly independent. In this case, we must divide the revenue appropriately to equate sensitivity based revenue and revenue based on border flow rights.

B.7 Comparison of sensitivity based payment to (1)

The sensitivity based payment (6) is not the same as the expression corresponding to (1) using pre-contingency flows, which in this case would be:

$$[p^{0\star} + \sum_{\omega\in\Omega} p^{\omega\star}]^{\dagger} S_e(V^{\star},\theta^{\star},u^{\star})\chi_e. \tag{7}$$

The two expressions (6) and (7) differ for element e to the extent that the base-case and contingency flows differ on element e. The payment based on pre-contingency flows (7) is an approximation to the exact expression (6), which provides efficient marginal incentives for transmission expansion under conditions established in Theorem 5 below.

The pre-contingency and post-contingency flows can be considerably different. For example, for a line e that is the binding contingency for a constraint, the contingency case flow on e is zero, while the base-case flow on e is non-zero. For other lines, however, the base-case and contingency-case flows differ by only a fraction of the base-case flow on the contingency constrained line. It is an empirical question as to whether (6) and (7) differ significantly in practice and we hypothesize that (7) may typically be a useful approximation to (6).

II. Incentives for marginal transmission expansion by price taker

Theorem 5: Consider an infinitesimal expansion of transmission. Suppose that the sensitivity of prices to variation in the transmission parameters is zero. Then the change in optimal welfare due to this expansion equals the change in revenue to the the lines in the system.

Proof: Let the infinitesimal expansion of transmission be denoted $d\chi$. The change in optimal welfare due to this expansion is: $\frac{\partial \mathcal{W}}{\partial \chi} d\chi$. The revenue to the lines in the system is $\frac{\partial \mathcal{W}}{\partial \chi}\chi$. The change in revenue to the lines in the system due to the infinitesimal expansion is $\frac{\partial \mathcal{W}}{\partial \chi} d\chi + \chi^{\dagger}\frac{\partial^2 \mathcal{W}}{\partial \chi^2} d\chi$. Under the assumption that the sensitivity of prices to transmission parameters is zero, the second term in the revenue equals zero. □

Corollary 6: Price-taking expansion by a profit maximizing coalition of transmission owners results in capacity that maximizes the welfare of operation of the system minus the cost of the expansion. □