# Contingent Transmission Rights in the Standard Market Design

Richard O'Neill, Udi Helman, Ross Baldick, William Stewart, Michael Rothkopf

*Abstract*—**We define transmission rights that are compatible with the Federal Energy Regulatory Commission's proposed Standard Market Design (SMD) and that provide flexibility in the points of receipt or delivery of energy contracts. The contingent transmission rights introduced here provide a viable, flexible method for defining SMD-compatible rights for transmission customers having current (pre-SMD) transmission rights that cover multiple points. These contingent rights can be bought and sold in the transmission rights auctions under SMD.**

*Index Terms*—**Contingent transmission rights, Standard Market Design.**

## I. Introduction and Summary

In current electricity markets, a number of transmission customers hold transmission rights (or contracts or entitlements) that allow them to choose among multiple points of injection or sources and multiple points of withdrawal or sinks. Transmission capacity is reserved to support any of these alternatives, which are exercised *contingent* on prevailing system conditions such as availability and cost of energy at particular locations.

In the transition to a single transmission tariff under the Federal Energy Regulatory Commission's (FERC's) proposed Standard Market Design (SMD), a market design in which congestion management is based on locational marginal pricing (LMP) and financial transmission rights (FTRs), market participants have expressed the desire to maintain transmission rights with these same properties; that is, to be awarded, or provided the opportunity to acquire, transmission rights that provide an equivalent hedge against future locational congestion costs.[1] The purpose of this paper is to show that there are more efficient ways to provide such a right, within the framework of the SMD tariff, than simply reserving simultaneously transmission capacity for each contingent power flow (as is more or less the approach used in service under Order 888).

In particular, we show how to define transmission rights that have all of the needed capacity to meet a specific contingent contract for energy. We also show that these rights may cost less to acquire than the separate purchase of the individual rights, depending on the extent of the overlap of the transmission needs of the various alternatives in the contingent energy contract. It should be noted that the contingent transmission rights described here are consistent with the SMD tariff, but that the approach is a general one and could be implemented in any market where point to point and/or flowgate rights are auctioned.

In this paper, we define types of contingent financial transmission rights (henceforth just "contingent transmission rights" or "contingent rights") that can provide a method for converting existing rights and specifying these types of rights in the future SMD markets. Contingent rights protect the holder of a contingent contract for energy against future congestion charges and give the holder the option to choose among multiple sources and multiple sinks–thus allowing for generation portfolio choice and also providing a hedge against generation outages. However, as with all financial rights, the contingent rights we describe do not constrain the actual physical dispatch, which is determined by a bid-based economic dispatch. (See [6] for more details.)

Contingent rights are similar to the "network service" under Order 888, which allows for multiple points of receipt and delivery and provides the option to make injections and withdrawals at those points contingent on changing operating conditions. We suggest that the contingent rights under the proposed SMD tariff can in effect replace the prior network service. Here we show how this conversion can be accomplished while maintaining the simultaneous feasibility of all rights, and we demonstrate that contingent transmission rights are at least as expensive as any of the underlying constituent point-to-point obligation or option rights, but can be less expensive than buying the individual rights that hedge all the contingent transactions.

The paper presents examples of how to design contingent transmission rights and then shows how to generalize these rights in SMD markets. These transmission rights can be allocated to existing customers or load serving entities or purchased in an SMD auction. Like other financial transmission rights, these rights can be cashed out or exchanged at any point in the sequence of periodic (monthly/weekly, etc.) reconfiguration auctions for transmission rights (in which participation is voluntary).

We assume a fixed transmission network topology, but we recognize that under outage conditions it is necessary to implement derating rules or other pricing mechanisms. The

[1] See the papers by FERC and intervenors in Electricity Market Design and Structure, Docket No. RM01-12-000. These are available on the Commission website at http://www.ferc.gov. See in particular, "Rulemaking Comment of RTO West Investor-Owned Utilities and Bonneville Power Administration, et al. under RT01-35 et al."

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formulation we develop encompasses *N*-1 security, but we do not consider the ramifications of system deratings that would be necessary subsequent to an outage actually occurring. That is, outage considerations are beyond the scope of this paper.

The paper is organized as follows. Section II gives some basic definitions that may not be familiar to the general reader or for which the definition in this paper may differ from other definitions. Section III offers several simple examples on three-node networks. Section IV is a more technical statement of the SMD auction that presents the mathematical formulation of the contingent contract. Section V concludes.

## II. Preliminary Definitions

**A. Flowgates.** In this paper, we adopt a general definition of flowgate for use in the markets envisioned in the SMD tariff by identifying all transmission elements (lines, transformers, etc.), or linear combinations of such elements, as "flowgates" and letting market participants choose which of the flowgates have commercial significance. The NERC-defined flowgates–the locations or set of locations in the electricity network that are specifically monitored for security purposes–are thus a subset of our set.

The more general definition of flowgates that we use has at least two purposes. First, it helps to clarify that both point-to-point rights and flowgate rights can be constructed in the linear case from the set of elemental flowgates, as defined below. Second, in practical terms, it avoids the problem of having the market operator identify the significant flowgates for market participants to hedge and, in some formulations, insuring the holders of rights on those flowgates against additional congestion charges. In the model presented here, the market operator does not insure any flowgates, although we do not preclude third parties from providing such a service.[2]

In general, each transmission element has two "elemental" flowgates: one in each direction. An elemental flowgate has capacity, in MW, in a single (pre-specified) direction. This construct allows all elemental flowgate capacity prices to be nonnegative. In the examples that follow, we designate the direction of flow on an elemental flowgate by the order of the nodes in the labeling. For example, if A and B are two nodes in the network connected by a line (transmission element), then "AB" is the elemental flowgate for flow from A to B and "BA" is the elemental flowgate for flow from B to A, along that element. (When the system ultimately clears, total flow over each elemental flowgate is determined by the sum of the flows on the transmission element's paired elemental flowgates, and, in the actual power flow, the flow on one of the paired elemental flowgates will be zero.

**B. Distribution Factors.** A power transfer distribution factor (PTDF) is the amount of flow (+) or "counterflow" (-) on an elemental flowgate induced by a unit injected at a node and a unit withdrawn at another node. This construct can be generalized to multiple injection nodes and multiple withdrawal nodes. The PTDFs are determined by Kirchhoff's laws from the impedances of individual transmission elements in the transmission system [9].

In general, the PTDFs vary with flows on the lines. Under some circumstances, however, they may be relatively constant [1]. Constancy will be assumed here for expositional purposes.

As an example, consider Figure 1, which shows a three node network with nodes A, B, and C. All lines in the network have the same impedance and a thermal capacity in each direction of 100 MW. In Figure 1, for the elemental flowgates AC and CA, the PTDFs are 1/3 and -1/3, respectively, for injection at A and withdrawal at B.

**C. Hedges.** A hedge is a financial instrument or asset that protects its owner against a potential financial loss. Hedges can be "imperfect," also called partial or incomplete, or "perfect," also called complete. A perfect hedge is an instrument that insures that a forward contract will not incur additional expenses nor collect additional revenues when executed. The contingent transmission rights presented in this paper are *not* perfect hedges since they protect the holder from paying additional charges if the contract is executed, but they may produce additional revenues. We will call a hedge with no downside risk a "no-regrets" hedge.

**D. Auction model.** We assume that auctions will be used to set uniform clearing prices for the purchase of financial transmission rights. Holders of such rights are then, in turn, paid based on nodal energy prices determined by a bid-based economic dispatch. We will focus on the acquisition of financial rights and consider dispatch only briefly in sections III.A and III.B to explain the nature of a no-regrets hedge. We assume, however, that the network model is the same for the financial transmission rights auction and for the bid-based economic dispatch.

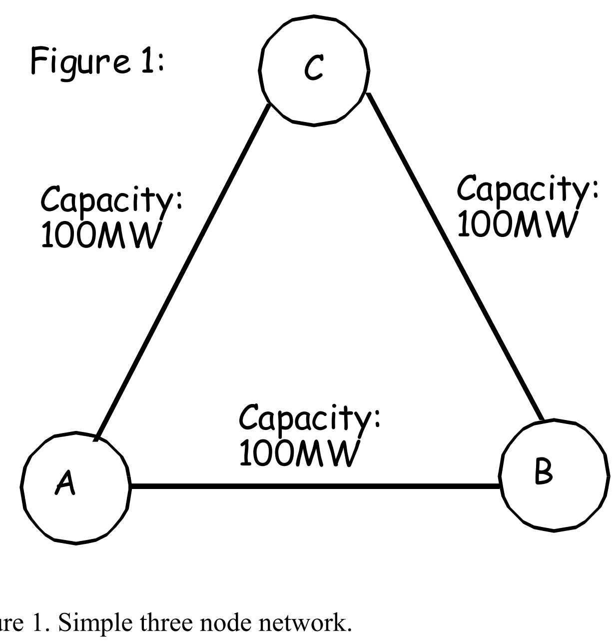


Figure 1. Simple three node network.

[2] For further discussion, see [2] and [5].

## III. Contingent Transmission Rights and Three Node Examples

This section shows how to construct financial transmission rights specified as obligations and as options. We will refer to these as FTR obligations and FTR options, respectively. The basic procedure is illustrated through the use of examples based on the simple three node network shown in Figure 1. There are three transmission elements in the system, each of which is a transmission line, that are each split into two elemental flowgates. Hence, there are six elemental flowgates, AB, BA, AC, CA, BC, and CB.[3]

The SMD tariff will offer two basic types of transmission rights: point-to-point rights and flowgate rights. For example, a unit of FTR obligation from A to B (FTR AB) perfectly hedges the marginal congestion charges (but not marginal loss charges) associated with an energy transaction that puts a unit (e.g., one MW) of energy into the system at A and withdraws it at B. An FTR obligation can be constructed as a point-to-point right or as a portfolio of elemental flowgate rights using the PTDFs. In the simple linear examples that follow, the point-to-point and portfolio of flowgates formulations are equivalent. (See also [2]). The PTDFs for each of the elemental flowgates are shown in Table 1 for the two FTR obligation rights (for one MW of flow) from A to B and from C to B, respectively.

To show how the individual flowgate rights (in this case, each line in the simple network) are composed into the FTR, the paper adopts the portfolio of flowgate rights description. In actual (nonlinear) power systems, this equivalence may hold only approximately.

In sections III.A and III.B, we describe the payments to transmission rights holders in the bid-based dispatch. Then in section III.C we describe the payments in the auction where the rights are acquired.

**A. Contingent Transmission Rights as Obligations.** Suppose that a transmission customer would like a transmission right that hedges a contingent energy obligation contract against congestion charges in subsequent transmission markets. A simple example of a contingent energy obligation contract is that the seller will inject one unit at *either* node A or node C (but not both) and withdraw it at B.

A portfolio of elemental flowgate rights for a transmission contract that hedges this transaction is called a "contingent obligation" for A to B or C to B, and its PTDFs are presented in the fourth column of Table 1. The obligation aspect of the portfolio comes from the inclusion of the "counterflow" flowgate rights. When an energy contract is executed (power injection at A or C and withdrawal at B), the transmission right in column 4 creates a no-regrets hedge. In contrast, holding an FTR AB is not a no-regrets hedge if the energy is injected at C and withdrawn at B. Similarly, holding an FTR CB is not a no-regrets hedge if the energy is injected at A and withdrawn at B.

The contingent transmission right is constructed by taking the maximum PTDF on each elemental flowgate (across each row in Table 1) that could occur under each alternative transmission need associated with the contingent energy contract hedged by the right. The contingent obligation can be thought of as:

(1) buying the *maximum* amount of capacity on each elemental flowgate that could be required under each alternative by a contingent energy contract; and

(2) selling the *minimum* amount of counterflow that could be produced by under each alternative of a contingent energy contract.

For example, in row 1 of Table 1, on elemental flowgate AB, the contingent right PTDF is required that corresponds to the flow if the A to B energy option is exercised, since that takes up more capacity on AB than the C to B energy option. However, in row 2, on elemental flowgate BA, the contingent right PTDF can only reflect the lesser amount of counterflow available from either energy option, since that is the most that the system operator can count on for simultaneous feasibility with other transmission rights.

The properties of the contingent obligation are illustrated briefly. For purposes of comparison, consider first a market participant that holds *separate* transmission rights for each alternate point of injection for a contingent energy contract for injecting at A or C and withdrawing at B. That is, suppose that the market participant holds two transmission rights, one from A to B and one from C to B. The PTDFs for these rights are shown in Columns 2 (FTR AB) and 3 (FTR CB) in Table 1.

**Table 1:** PTDFs for a Contingent Transmission Obligation Right with Two Possible Contingencies.

| Elemental Flowgate Name | FTR Obligation A to B | FTR Obligation C to B | Contingent Transmission Obligation for A to B or C to B Energy Obligation Contract |
|---|---|---|---|
| AB | .667 | .333 | .667 |
| BA | -.667 | -.333 | -.333 |
| CA | -.333 | .333 | .333 |
| AC | .333 | -.333 | .333 |
| CB | .333 | .667 | .667 |
| BC | -.333 | -.667 | -.333 |

Assume that, at the time of dispatch, all the elemental flowgate prices are zero except AB, which has a marginal flowgate price of \$15. In this case, the nodal price difference from A to B is $\$10 = p_B - p_A = \$15 \times .667$, where $p_i$ is the price at node $i$). If the market participant exports one unit

[3] Participants in the transmission rights auction may choose to bid on linear combinations of these elemental rights. For example, a participant could designate a "flowgate right" equal to (0.5)AB and (0.5)CB.

(MW) from A to B he pays $10 in congestion costs and simultaneously receives $10 for holding the FTR from A to B. If the A to B transaction is not executed then the FTR holder receives $10. If the market participant ships one unit (MW) from C to B then he pays $5 in congestion costs and simultaneously receives $5 (= $15 × .33) for holding the FTR from C to B. If the C to B transaction is not executed, the holder is paid $5.

Now, rather than the two separate rights, suppose that the market participant holds a single contingent transmission obligation for A to B or C to B. The PTDFs for this right are specified in Column 4 of Table 1. If the A to B transaction is executed, the holder pays $10 in congestion costs and simultaneously receives $10 for holding the contingent transmission obligation for A to B or C to B. If the C to B transaction is executed, the holder pays $5 in congestion costs and simultaneously receives $10 for holding the contingent transmission obligation for A to B or C to B. The transaction either "perfectly" hedges the contingent energy obligation contract when the energy is supplied from node A or nets a payment of $5 when the energy is supplied from node C. Hence, the contingent right is a "no regrets" hedge. Here, the contingent right pays less ($10) than holding both rights ($15).

**B. Contingent Transmission Rights as Options.**

Now suppose that a transmission customer would like a transmission right that hedges a contingent energy option right. For example, a contingent transmission option might be needed to provide a financial hedge against transmission costs for the following energy contract: the buyer may call upon the seller to inject one unit (MW) at either node A or node C for withdrawal at B, or he may (has the option to) not call for this energy and no energy will be injected or withdrawn. A portfolio of flowgate rights for a transmission contract that hedges this contingent energy contract option transaction is illustrated in Table 2.

This hedge is not perfect since it hedges against paying additional charges but may produce additional revenues whether or not the contract is executed. If the energy contract is executed, this transmission contract creates a no regrets hedge. Again, the PTDFs are constructed by taking the maximum PTDF on each elemental flowgate (across each row in Table 2) that could occur under each possible alternative transmission need corresponding to each contingent energy option. This can be thought of as buying the maximum amount of capacity on each flowgate that will be needed by the contingent energy option contract. However, unlike obligation rights, no counterflow is sold as part of the option right.

**C. Auction Examples.**

To show how the prices for acquiring each right are determined, we conduct two sample auctions for the three-node network described above. Hypothetical bids to acquire particular transmission rights are used to develop prices on all other rights. These rights then entitle the holder to collect congestion revenues from the energy market as described in sections III.A and III.B.

**Table 2:** PTDFs for a Contingent Transmission Option with Two Possible Contingent Energy Contracts.

| Elemental Flowgate Name | FTR Option A to B | FTR Option C to B | Contingent Transmission Option for A to B or C to B Energy Option Contract |
|---|---|---|---|
| **AB** | .667 | .333 | .667 |
| **BA** | 0 | 0 | 0 |
| **CA** | 0 | .333 | .333 |
| **AC** | .333 | 0 | .333 |
| **CB** | .333 | .667 | .667 |
| **BC** | 0 | 0 | 0 |

The auction examples will demonstrate two properties of the contingent rights. Property I is that a contingent transmission *option* right is always at least as expensive as any of the individual rights that are a part of it, but is never more expensive than the sum total of the individual rights. This is because each individual constituent transmission option aggregated into the contingent transmission option provides no counterflow, and, as shown in column 4 of Table 2, the contingent transmission option contract requires the maximum capacity on each elemental flowgate associated with any constituent option. Property II is that a contingent transmission *obligation* right may be more expensive than the set of individual rights that cover the same points of injection and withdrawal. As shown below, this is because obligations may contribute to counterflow. [4]

**(a) Auction 1.** Auction 1 has a marginal bid for an FTR CB of $15/MW. This bid produces a dual price of $22.50 on elemental flowgate CB (and a dual price of $0 on all other flowgates). From this dual price, prices for all other combinations of rights (obligations and options), can be derived by multiplying their associated PTDFs by the dual prices on the flowgates. In the calculations below, all flowgates with zero prices will be ignored. The resulting flowgate prices are shown in figure 2.

[4] This auction example can accommodate counterflow obligation FTRs, in which the buyer gets paid (i.e., bids a negative value) to purchase an FTR with a "negative" value (i.e., that requires the holder to pay rather than collect associated congestion charges). To bid rationally for such rights, the buyer must assume that the payment from the previous holder of the rights or the auctioneer is greater than the obligation payments that holding the right will entail. In this simple example, this type of bidding can substantially increase the volume of the auction and is not considered generally, but is introduced for purposes of illustrating the properties of contingent obligation rights.

**FTR Obligations.** The price of an FTR from A to B is \$7.5 (= \$22.5 × .33). The price of an FTR from C to B is \$15 (=\$22.5 × .67). The price of a contingent FTR obligation from A or C to B is \$15 (= \$22.5 × .67). The contingent FTR obligation from A or C to B is equal to or more expensive than either an FTR from A to B or an FTR from C to B, and happens to be equal to or less expensive than holding both, \$22.5 (= \$7.5 + \$15).

This initial example appears to illustrate Property I. However, as discussed above, Property I applies generally only to contingent FTR options. To show this, we introduce another set of rights, including a counterflow obligation FTR (see footnote 4). The PTDFs for these rights can be constructed in the same fashion as shown in Table 1 and they are priced as in the prior example.

The price of an FTR from C to A is \$7.5 (=\$22.5 × .33). The price of an FTR from B to A is -\$7.5 (= \$22.5 × -.33) (the holder is paid to commit for the counterflow transaction from B to A). The price of an FTR from B or C to A is \$7.5. The contingent FTR from B or C to A is equal to or more expensive than either an FTR from C to A or an FTR from B to A, but is more expensive than buying both, since \$7.5 + \$-7.5 = 0. This illustrates Property II.

**FTR Options.** Similarly, the price of an FTR option from C to A is \$7.5 (= \$22.5 × .33). The price of an FTR option from B to A is 0 (= \$22.5 × 0). The price of an FTR option from B or C to A is \$7.5. The contingent FTR option from B or C to A is equal to or more expensive than either an FTR option from C to A or an FTR option from B to A, but is equal to or less expensive than holding both, \$7.5 (Property I). Property I is a general result for contingent transmission options.

**(b) Auction 2.** We now add another bidder, which results in the auction producing two elemental flowgates with positive prices. Along with the bid price for an FTR from C to B of \$15, we additionally have a bid price for an FTR from A to B of \$10. These bids produce prices on the elemental flowgates AB and CB of \$5 and \$20, respectively.

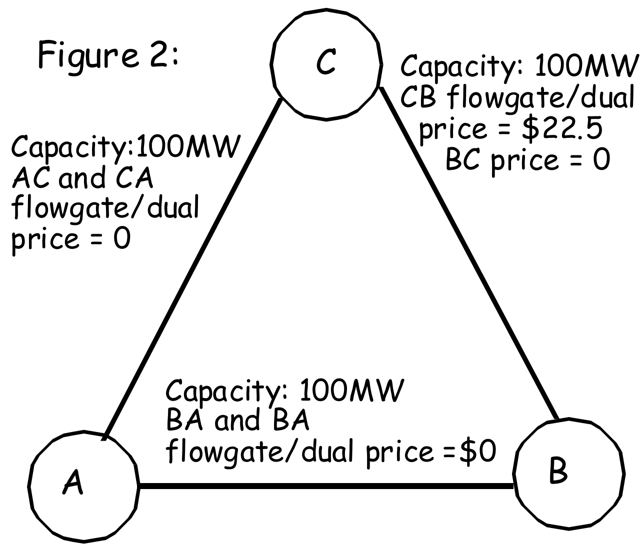


Figure 2. Flowgate prices for auction examples.

**FTR Obligations.** The price of an FTR obligation from A to B is \$10 (=\$5 × .67 + \$20 × .33). The price of an FTR obligation from C to B is \$15 (=\$5 × .33 + \$20 × .67). The price of a contingent FTR from A or C to B is \$16.67. This example again shows the case of a contingent obligation that is cheaper than holding two separate rights, but it is emphasized that this is not a general property of such rights (Property II).

**FTR Options.** The price of an FTR option from A to B is \$10 (=\$5 × .67 + \$20 × .33). The price of an FTR option from C to B is \$15 (=\$5 × .33 + \$20 × .67). The price of an FTR option from A or C to B is \$16.67 (=\$5 × .67 + \$20 × .67). The contingent FTR option from A or C to B is equal to or more expensive than either an FTR option from A to B or an FTR option from C to B, but is equal or less expensive than holding both, \$25 This verifies Property I.

The generality of Property 1 can be illustrated by considering other possible option rights, such as those from B or C to A. The price of an FTR option from C to A is \$6.67 (=\$20 × .33). The price of an FTR option from B to A is 0. The price of an FTR option from B or C to A is \$6.67 (=\$20 × .33). Hence, again the contingent FTR option from B or C to A is equal to or more expensive than either an FTR option from C to A or an FTR option from B to A, but is equal to or less expensive than holding both, which would cost \$6.67.

## IV. Contingent Options Rights in the General Linear Auction Model

In this section, we show how the contingent rights can be entered into an auction for energy and transmission rights similar to the one being proposed for the Standard Market Design. We have earlier represented this auction mathematically, but without contingent transmission rights, in [6][7]; other important references on the modeling of transmission rights are, [2][3][4][5][8]. Readers not interested in a general mathematical presentation can skip to the summary that follows; those interested in a fuller account of the issues in transmission rights specification, auction design, and solution procedures should refer to these papers.

**A. A Simplified General Model.**

***(a) The Non-Linear Case.*** The following simplified general model of the auction was suggested in [6] and incorporates features earlier proposed in [2] and [3]. We first present a version of the auction model with nonlinear transmission constraints and a nonlinear bid function for energy and point-to-point transmission rights. This is a nodal model, but we skip the indexing of bidders for rights, nodes, and transmission network elements for purposes of brevity. The model is:

$$\begin{aligned} &Max \;\; bt + B(g) \\ &Ag - y = 0 \qquad (\pi) \qquad (1) \\ &\beta t + K(y) \le f \qquad (\mu) \qquad (2) \\ &0 \le t_l \le t \le t_u \qquad (\rho_l, \rho_u) \qquad (3) \\ &0 \le g_l \le g \le g_u \qquad (\psi_l, \psi_u) \qquad (4) \end{aligned}$$

The objective of the auction is to maximize the surplus or benefits from the auction bids. Unless otherwise stated, each variable or parameter in lower case is a vector, each function is in upper case, and a parameter in upper case is a matrix.

The constraint equations (1) are commonly called the energy or power balance constraint (for each node). This equation translates the bids for energy and point-to-point transmission rights into net injections (at each node). The constraint equations (2) are the set of constraints associated with transmission network constraints, such as Kirchhoff's Laws, and transfer capacities. The transmission function $K(y)$ translates injections, $y$, into flows, $K(y)$, on transmission elements. We let the flow in each direction be a separate constraint. The vector of constraints on transmission elements such as thermal, reactive power, and stability constraints are represented by $f$.

The Lagrange multipliers on the auction constraints are $\pi$, $\mu$, $\rho_l$, $\rho_u$, $\psi_l$, and $\psi_u$. The Lagrange multipliers $\pi$ are the marginal prices for energy at each node. The Lagrange multipliers $\mu$ are the marginal prices on transmission network constraints.

The decision variable for the quantity of flowgate rights is $t$; the decision variable for the quantity for energy injections and withdrawals and the injections and withdrawals associated with point-to-point rights is $g$. The bid parameters $b$ are the price bids on the flowgate rights while $B(g)$ defines the bid function for energy injections and withdrawal and point-to-point rights. Each flowgate bid could be for a single flowgate or for multiple flowgates. Each energy bid could be for a single point of injection and a single point of withdrawal or for multiple points of injection to, for example, represent the distribution factors for a hub.

Other bid parameters are in the set of auction constraints (1) to (4). The parameters associated with bids for $g$ include $A$, the net injections at each node for each unit of bid, and $g_l$, and $g_u$, the lower and upper bounds (quantities) on the bids.

A bid for a point-to-point right from node "Y" to node "Z" is specified by elements in $A$ indicating 1 for an injection at Y and -1 for a withdrawal at Z. A minimum desired right of 5 MW and a maximum desired right of 50 MW is specified by $g_l$ = 5 MW and $g_u$ = 50 MW. Awarded point-to-point rights are paid for at nodal price differences as specified by the Lagrange multipliers, $\pi$.

A bid for flowgate rights is specified by a column of the matrix $\beta$ that designates the proportion of each elemental flowgate to be included in the bid per unit of the corresponding variable $t$. The entries of $t_l$, and $t_u$ specify the lower and upper bounds (quantities) on the bids.

Awarded flowgate rights are paid for at the prices specified by the Lagrange multipliers $\mu$. In particular, if bidder $k$ desired the proportions of flowgates specified by the vector $\beta_k$, and was awarded $t_k$, then it would pay a total of $\mu\beta_k t_k$ for the awarded rights, $\beta_k t_k$.

For example, consider a flowgate right bid $\beta_k$ for elemental flowgate "XY." The appropriate vector $\beta_k$ has zeros everywhere except for the entry corresponding to XY in constraint (2), for which $\beta_{k(\mathrm{XY})} = 1$. If, furthermore, $t_{lk}$ = 5 MW, and $t_{uk}$ = 50 MW then this bid specifies that each 1 MW increment of the bid is on flowgate XY, that the minimum quantity desired is 5 MW, and that the maximum quantity desired is 50 MW.

As another example, suppose that the bidder desired that of each 1 MW bid increment of $t_k$, 0.5 MW would be on flowgate WY and 0.5 MW on flowgate XY. In this case, $\beta_{k(\mathrm{WY})} = 0.5$ and $\beta_{k(\mathrm{XY})} = 0.5$, with the rest of the entries of $\beta_k$ equal to zero.

Note that the entries in $\beta$ are not PTDFs, although a purchaser of flowgate rights may be using PTDFs evaluated at certain operating points to determine the appropriate values of the entries of $\beta$ to choose in a bid as a hedge for a particular transaction. "Unbalanced" FTRs and bids for losses can also be accommodated in this formulation.

***(b) The Linear Case and Auction Properties.*** For computational purposes, energy and transmission auctions are often approximated and solved as linear models. To convert the non-linear model above to a linear model, we require a linear bid function, linearization of the transmission constraints using the DC load flow approximation (with the resulting matrix $D$ of PTDFs) and have a constraint for flow in each direction.

In addition, unlike the previous non-linear model, $B$ is now a vector of bids for $g$. The resulting linear auction model is:

$$\begin{aligned} &Max \;\; bt + Bg \\ &Ag - y = 0 \qquad (\pi) \\ &\beta t + Dy \le f \qquad (\mu) \\ &0 \le t_l \le t \le t_u \qquad (\rho_l, \rho_u) \\ &0 \le g_l \le g \le g_u \qquad (\psi_l, \psi_u) \end{aligned}$$

By complementary slackness, $(-\pi + \mu D)\, y = 0$. If $y \ne 0$ then $\pi = \mu D$. By duality, if all offered transmission capacity were offered into the auction by rights holders with no capacity being directly provided by the ISO so that $f = 0$ then all surplus is exchanged among the bidders as rents. That is, revenues collected equal the revenues paid (revenue neutrality). No money is left on the table and no subsidies are needed.

**B. Specifying the Contingent Rights.**

This section states mathematically how to specify the contingent rights in the auction model above. The general contingent options or obligations right is represented as follows: let $\beta_1,..., \beta_M$ be the vectors of flowgate options or obligations for each alternative of a contingent energy contract $m = 1, ..., M$. We write $\beta_C$ to represent the appropriate contingent transmission option or obligation right. The vector $\beta_C$ is defined by:

$$\beta_{Ci} = \max_{m = 1,...,M} \{\beta_{mi}\},$$

where $\beta_{mi}$ is the value of the $i^{th}$ element of the $m^{th}$ column vector $\beta_m$.

For options, since $\mu \geq 0$ and $\beta_C \geq 0$, therefore $\mu\beta_C \geq \mu\beta_m$ for $m = 1, ..., M$. The contingent option right is equal to or more expensive than any individual option right that encompasses the same set of injection and withdrawal points. Further, the contingent option right is equal to or less expensive than the sum of each individual option right, $\beta_C \leq \Sigma_m\ \beta_m$. This transmission right reserves or hedges the contingent energy obligation or option contract that is defined by $\beta_1$,,or $\beta_2$,or ,..., or $\beta_M$.[5] This formulation accepts any combination of option and obligation contingent transmission rights.

The extent to which a specified $\beta_m$ can replicate an FTR obligation or option depends on whether the PTDFs change as a function of the operating point. The consensus is that under constant network topology and normal operating conditions, the PTDFs are essentially constant as a function of the operating point. As mentioned in the introduction, non-normal, outage, or emergency conditions often have or should have special rules for compensation.

## V. Summary

The contingent transmission rights introduced here provide a viable, flexible method for defining SMD-compatible rights for transmission customers having current (pre-SMD) transmission rights that cover multiple points of injection and delivery. These rights can be bought and sold in the transmission rights auctions under SMD.

In a linear model, the contingent transmission rights introduced here are feasible and revenue adequate (do not need subsidies). Contingent transmission option and obligation rights are at least as expensive as any single point-to-point obligation or option right. Contingent transmission options rights are equal to or less expensive than buying individual rights that make up all the contingencies. On the other hand, contingent transmission obligation rights may or may not be equal to or less expensive than buying individual obligation rights.

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[5] This contract is different from a hub contract.

## References

[1] Baldick, Ross. 2003. "Variation of distribution factors with loading," to appear in *IEEE Transactions on Power Systems*.

[2] Chao, Hung-Po, Stephen Peck, Shmuel Oren and Robert Wilson. 2000. "Flow-based Transmission Rights and Congestion Management," *The Electricity Journal*, 13, 8, 38-58.

[3] Harvey, Scott M., William W. Hogan, and Susan L. Pope. 1997. "Transmission Capacity Reservations and Transmission Congestion Contracts," John F. Kennedy School of Government, Harvard University (Revised March 8).

[4] Hogan, William W. 2000. "Flowgate Rights and Wrongs." Center for Business and Government, John F. Kennedy School of Government, Harvard University, Cambridge, MA (August 20).

[5] Hogan, William W. 2002. "Financial Transmission Right Formulations," Center for Business and Government, John F. Kennedy School of Government, Harvard University, Cambridge, MA (March 31.)

[6] O'Neill, Richard P., Udi Helman, Benjamin F. Hobbs, William R. Stewart, Jr., and Michael H. Rothkopf, 2001, "The Joint Energy and Transmission Rights Auction: A General Framework for RTO Market Designs," OMTR, FERC, Washington, DC.

[7] O'Neill, R.P., U. Helman, B.F. Hobbs, W.R. Stewart, Jr., and M.H. Rothkopf, 2002, "A Joint Energy and Transmission Rights Auction: Proposal and Properties," *IEEE Trans. Power Systems.*, vol. 17, 1058-1067.

[8] Ruff, Larry E., 2001. "Flowgates, Contingency-Constrained Dispatch, and Transmission Rights," The Electricity Journal, 14, 1, January/February, 34-55.

[9] Wood, Allen J. and Wollenberg, Bruce F., 1996. *Power Generation, Operation, and Control*, Wiley, New York, Second Edition.

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