Stability of supply function equilibria: Implications for daily versus hourly bids in a poolco market

Ross Baldick
Department of Electrical and Computer Engineering
The University of Texas at Austin

William Hogan
Center for Business and Government
John F. Kennedy School of Government
Harvard University

February 28, 2002

Abstract

We consider a supply function model of a poolco electricity market where demand varies significantly over a time horizon such as a day and also has a small responsiveness to price. We show that a requirement that bids into the poolco be consistent over the time horizon has a significant influence on the market outcome. In particular, although there are many equilibria yielding prices at peak that are close to Cournot prices, such equilibria are typically unstable and consequently are unlikely to be observed in practice. The only stable equilibria involve prices that are relatively closer to competitive prices. We demonstrate this result both theoretically under somewhat restrictive assumptions and also numerically using both a three firm example system and a five firm example system having generation capacity constraints. This result contrasts with markets where bids can be changed on an hourly basis, where Cournot prices are possible outcomes. The stability analysis has important policy implications for the design of day-ahead electricity markets.

1 Introduction

We consider an electricity market where generating firms bid to supply energy and we analyze the effect of requiring that the bid remains fixed throughout a time horizon, for example, a day. This assumption matches several markets, such as:

- England and Wales, until 2001,
- markets in Eastern United States such as Pennsylvania–New Jersey–Maryland (PJM).

Typical bid-based pools usually require a schedule of prices as a function of quantity (and we will show graphs as price to quantity plots). However, we will analyze such a market using functions from price to quantity to represent the bids. We consider an equilibrium in such supply functions.

Klemperer and Meyer [1] provided the seminal analysis of supply function equilibrium (SFE). They demonstrated conditions for the equilibrium to be unique. Subsequently, Green and Newbery [2] applied Klemperer and Meyer's SFE analysis to the electricity market of England and Wales. Several other studies have since used SFE analysis to consider the England and Wales market and other electricity markets [3, 4, 5]. The basic assumption in these studies is that the bid function remains constant over an extended horizon, such as a day. This assumption was essentially satisfied in the England and Wales market until 2001.

An apparent attraction of SFE analysis over other techniques, such as Cournot or Bertrand analysis, is that the SFE explicitly represents the functional form of the bid requirements in a pool. This contrasts with quantity or price bids in Cournot or Bertrand analysis, since the actual function that is bid into the pool is only implicit in the results of Cournot or Bertrand analysis. Nevertheless, we will see that the apparent match of SFE analysis to bid rules is misleading when the market rules allow for bids to be modified over the time horizon, as in the (now defunct) California Power Exchange, or if there are only a relatively small number of "pricing periods" over the time horizon. We will consider the implications when SFE analysis is not applicable to a bid-based pool.

The organization of this paper is as follows. In section 2, we present the formulation, which is essentially standard from the literature. Then section 3 discusses SFE analysis applied to electricity markets, highlighting the issue of the wide range of possible SFEs. In section 4, we consider how stability affects the range of observed equilibria and then in section 6 discuss the significance of the stability analysis. We conclude in section 7 with some policy implications. This paper is an abridged version of [6]. Fuller explanations, technical details, proofs of results, and many more numerical simulations are reported in [6].

2 Formulation

We follow the SFE and electricity market literature in the following formulation [2, 3, 4, 5, 6]. We consider a market consisting of n firms, indexed $i = 1, \ldots, n$, and restrict our model to the case where the generation marginal costs C'_i for firm $i = 1, \ldots, n$ are affine:

$$\forall q_i \in \mathbb{R}_+, C'_i(q_i) = c_i q_i + a_i,$$

and, moreover, require $c_i \geq 0, i = 1, \ldots, n$, so that the total variable cost function C_i for firm i is quadratic and convex.

We assume that each firm bids a supply function into the bid-based pool. The bid supply function of firm i remains fixed over the time horizon, is denoted by S_i, and is required to be a non-decreasing function from price to quantity produced. As noted in the introduction, typical bid-based pools require a function from quantity to price, which is the inverse of the supply function. We will graph supply functions with price on the vertical axis and quantity on the horizontal axis to be consistent with this convention.

In some cases, we will represent capacity constraints for firms, so that the marginal cost function is only valid for firm i for $q_i \in [0, \overline{q}_i]$, where $\overline{q}_i$ is the capacity of firm i. In some cases, we will also consider price caps. We represent price caps by requiring that for firm i, $S_i(\overline{p}) = \overline{q}_i$, where $\overline{p}$ is the price cap. That is, the price cap is implemented by assuming that a firm must bid in all its capacity if the price reaches the price cap.

Demand is assumed to be a continuous function of the price p and of the (normalized) time t over the time horizon. The normalized time ranges from 0 to 1 and we assume that the chronological aspect of the variation of demand has been abstracted into a load-duration characteristic. That is, demand is given by:

$$\forall p, \forall t \in [0,1], D(p,t) = N(t) - \gamma p,$$

where:

- N is a continuous load-duration characteristic that specifies the variation of demand over the time horizon and
- demand responds to price variations according to the demand slope γ.

Figure 1 shows a continuous load-duration characteristic. By convention, load-duration characteristics are drawn with the peak conditions corresponding to $t = 0$ and the minimum demand conditions corresponding to $t = 1$.

The price, $P(t)$, at each time $t \in [0,1]$ is determined by the market clearing conditions of demand equaling supply:

$$D(t, P(t)) = N(t) - \gamma P(t) = \sum_{i=1}^{n} S_i(P(t)),$$

assuming a solution exists for this equation. The profit per unit time to firm i is its revenue minus its costs:

$$\pi_{it} = S_i(P(t))P(t) - C_i(S_i(P(t))).$$

The total profit π_i to firm i is the integral of its profit per unit time over the time horizon:

$$\begin{aligned}\pi_i(S_i, S_{-i}) &= \int_{t=0}^{1} \pi_{it} dt, \\ &= \int_{t=0}^{1} [S_i(P(t))P(t) - C_i(S_i(P(t)))]dt,\end{aligned}$$

where $S_{-i} = (S_j)_{j \neq i}$ are the supply functions of the other firms. That is, the profit of firm i depends not only on its own supply function but also on the supply functions of the other firms since the price at each time is determined by the supply-demand cross.

A collection of supply functions $S^\star = (S_i^\star)_{i=1,\ldots,n}$ is a Nash supply function equilibrium (SFE) if no firm can be made better off by unilaterally changing its bid. That is, $S^\star$ is an SFE if:

$$\forall i = 1, \ldots, n, S_i^\star \in \underset{S_i}{\operatorname{argmax}} \{\pi_i(S_i, S_{-i}^\star)\},$$

where $S_{-i}^\star = (S_j^\star)_{j \neq i}$ are the supply functions of the other firms.

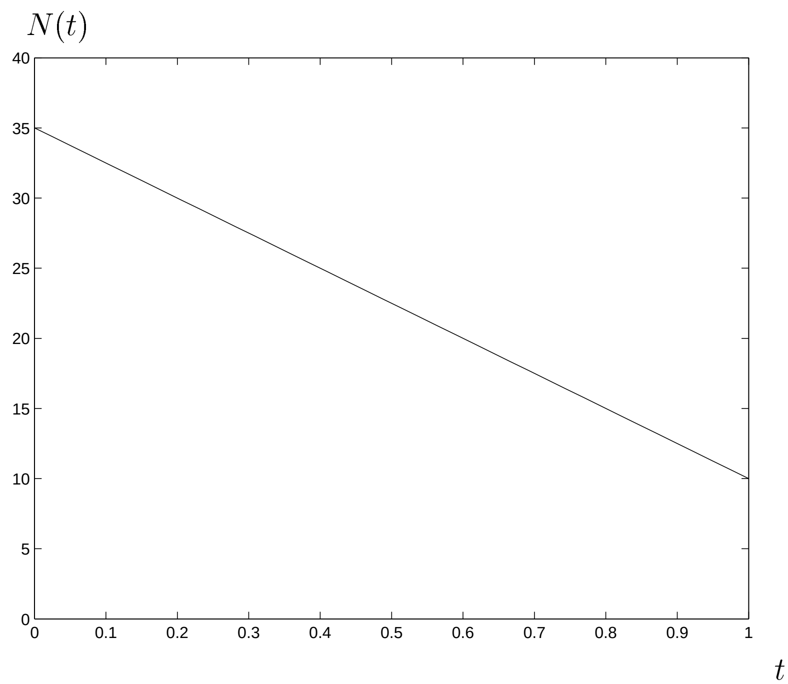

Figure 1: Example load-duration characteristic.

3 Basic analysis

Klemperer and Meyer [1] expressed the equilibrium conditions described in section 2 as a coupled differential equation. For the affine marginal cost case, there is an affine solution to this differential equation. Klemperer and Meyer also showed conditions under which this affine solution would be the only SFE. Translated into the electricity market context, the conditions are that the load-duration characteristic is unbounded. Unfortunately, in the practical case that the load-duration characteristic is bounded there are multiple SFEs, some of which are more competitive than the affine SFE and some of which are less competitive than the affine SFE.

The coupled differential equation derived by Klemperer and Meyer is not in the standard form of a non-linear vector differential equation. It can be transformed into a standard non-linear vector differential equation, as shown in [5]. The solution of the differential equation characterizes some, but not all, possible SFEs.

Unfortunately, the differential equation approach does not represent capacity constraints. Moreover, solutions of the differential equation can turn out to not satisfy the requirement that the supply functions be non-decreasing. If a solution of the differential equation is not non-decreasing at a price that is a solution of the market clearing conditions then the solution cannot be an SFE. In summary, the solutions of the differential equation are not in one to one correspondence with SFEs.

Nevertheless, as Green and Newbery show in [2] for the symmetric case of all firms having the same costs, there can be a wide range of solutions of the differential equation that satisfy the non-decreasing requirement and so are SFEs. That is, in these cases, there is a wide range of possible SFEs.

To understand this wide range of SFEs, consider the peak demand condition as specified by the load-duration characteristic at $t = 0$ and consider the Cournot equilibrium at this demand level. We write $p^{\text{Cournot}}_{\text{peak demand}}$ for the Cournot price at this demand level and consider the corresponding quantities q_i^{Cournot}. If we use this price and these quantities as an "initial condition" in the differential equation and integrate "backwards" from $p = p^{\text{Cournot}}_{\text{peak demand}}$ towards $p = 0$ then we obtain a solution of the differential equation that satisfies the non-decreasing constraints. That is, we obtain an SFE. We call this SFE the "least competitive SFE." At times other than $t = 0$, the prices resulting from least competitive SFE bids are lower than the prices that would occur under Cournot competition at each time [2].

Now consider competitive behavior at peak demand conditions and write $p^{\text{comp}}_{\text{peak demand}}$ for the competitive price at peak demand. Let q_i^{comp} be the corresponding quantities and again use this price and these quantities as an "initial condition" in the differential equations. We again obtain a solution that satisfies the non-decreasing constraints, which is therefore an SFE and which we call the "most competitive SFE." At times other than $t = 0$, the prices resulting from most competitive SFE bids are higher than the prices that would occur under competitive behavior at each time [2].

Green and Newbery illustrate this wide range of SFEs. Figure 2 is an adaptation of their illustration using a symmetric three firm example based on cost data from [7]. (We will return to this example in section 5.) Since the firms are symmetric then, for each SFE, each firm has the same supply function. The dashed functions show the relationship between quantity and price if, respectively, the Cournot and the competitive outcomes occurred at

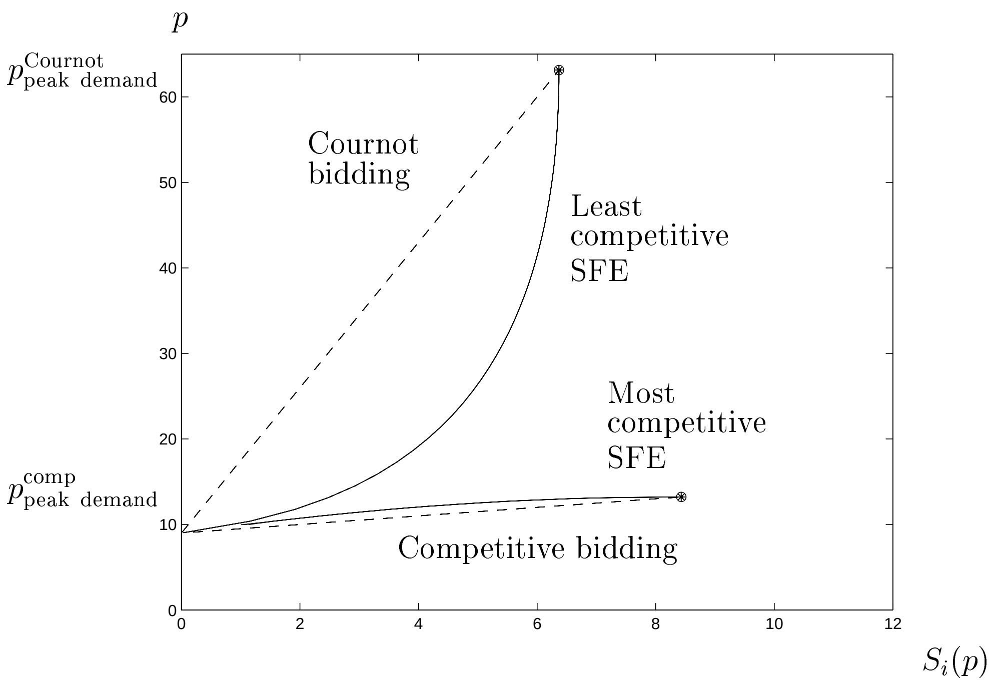

Figure 2: Least and most competitive symmetric SFEs shown solid, together with Cournot bidding and competitive bidding, shown dashed.
Source: This figure is based on [2, Figure 3], but uses cost data for the symmetric three firm system based on [7].

each time over the time horizon. For future reference, we call these "Cournot bidding" and "competitive bidding," respectively.

The wide range of supply function equilibria weakens the usefulness of SFE analysis because it is unclear which equilibrium will be observed in practice or if the electricity market will converge to any equilibrium. Various authors have singled out one of the SFEs. For example, Green and Newbery [2] use the least competitive SFE to analyze the England and Wales market. However, using "reasonable" values of the demand slope, they calculate prices well above those observed. More recently, Green [3], Rudkevich [4], Baldick and Kahn [5], and others use the affine SFE. The prices calculated using affine SFE analysis for the England and Wales market, for example, are typically closer to those observed; however, it should be remarked that the results depend strongly on the assumed demand slope.

Under some conditions, the range of SFEs is smaller. For example, Green and Newbery show that if there are binding generator capacity constraints at peak then there is only one SFE [2]. There are other issues besides capacity constraints that might potentially limit the range of the observed equilibria, such as:

- price caps and
- instability of equilibria.

In the next section, we consider stability and then return in section 5 to generator capacity constraints and price caps using a numerical framework.

4 Stability analysis

There are various timescales in an electric power system (and in an electricity market) and corresponding notions of stability:

- responses of automatic controls and stability of the electromechanical system [8],
- stability of interaction between the electromechanical system and short-term electricity markets [9], and
- stability of economic equilibria.

Alvarado has analyzed the stability of electricity market equilibria involving quantity bids [10]. Here we will consider the stability of SFE.

We will prove that all of the SFEs between the least and most competitive, except for the affine SFE, are unstable. The significance of this observation, to be discussed in detail in section 6, is that unstable equilibria are unlikely to be observed in practice. The conditions for unstable equilibrium are somewhat restrictive, since they only apply to SFEs that are found using the differential equation approach. However, in section 5, we will use a numerical framework to investigate the range of observed equilibria under less restrictive assumptions, including the imposition of:

- generator capacity constraints and
- price caps.

We first must define what we mean by an unstable equilibrium. An SFE $S^\star = (S_i^\star)_{i=1,\ldots,n}$ is unstable if a small perturbation $S^\epsilon = (S_i^\epsilon)_{i=1,\ldots,n}$ to $S^\star$ results in responses $\tilde{S} = (\tilde{S}_i)_{i=1,\ldots,n}$ by firms that deviate even more from $S^\star$. Formally, first let $\|\bullet\|$ be a norm on equivalence classes of SFEs such that if $\|S - S^\star\| = 0$ then the resulting prices for S are the same as the resulting prices for $S^\star$. The reason for considering such a norm is that we want to be able to define "small" perturbations, and we recognize that only perturbations to the supply functions that actually change the observed prices are relevant. With this norm, we say that $S^\star$ is an unstable equilibrium if for every $\epsilon > 0$ there exist supply functions $S^\epsilon = (S_i^\epsilon)_{i=1,\ldots,n}$ such that:

- $\|S^\epsilon - S^\star\| < \epsilon$ and
- if, for each i, $\tilde{S}_i$ is any optimal response to $S_j^\epsilon, j \neq i$ and we define $\tilde{S} = (\tilde{S}_i)_{i=1,\ldots,n}$ then $\left\|\tilde{S} - S^\star\right\| > \|S^\epsilon - S^\star\|$.

We have the following stability theorem [6]:

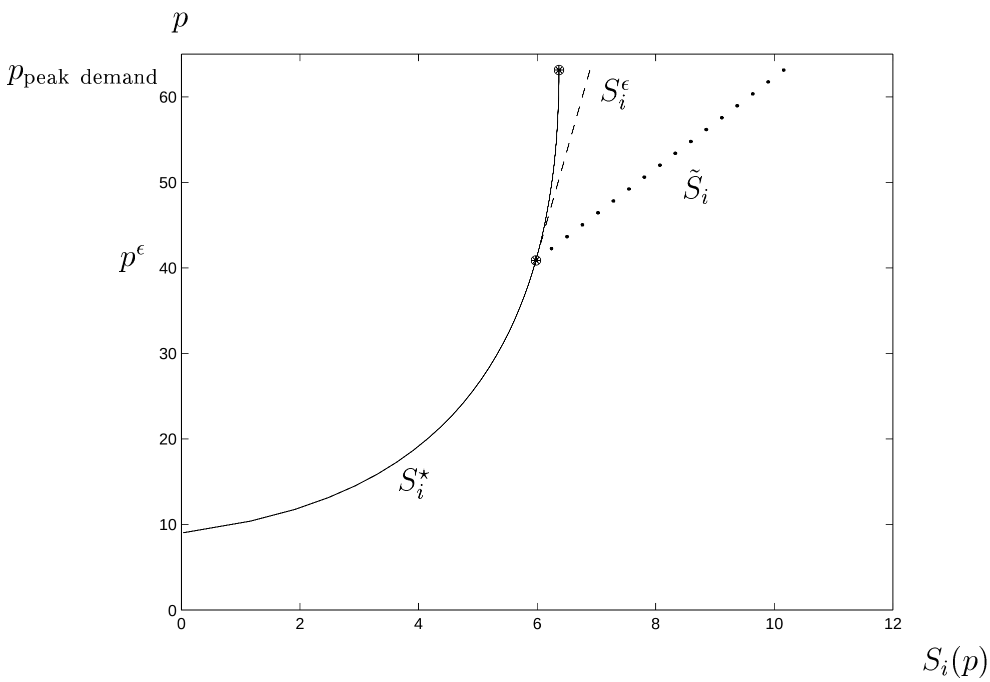

Figure 3: Illustration of proof of stability theorem.

Theorem *Suppose that there are no capacity constraints on the production of the firms and also suppose that $S^{\star}$ is an SFE given by the solution of the differential equation form of the equilibrium conditions. (That is, the solution $S^{\star}$ of the differential equations is non-decreasing over the range of prices over the time horizon.) Moreover, suppose that the resulting supply functions are all strictly concave or are all strictly convex (as functions of price.) Then the SFE $S^{\star}$ is unstable.*

Outline of proof: We consider a price p^{ϵ} that is close to the price, $p_{\text{peak demand}}$, at peak demand and define a perturbation S_i^{ϵ} of S_i by setting $S_i^{\epsilon}(p)$ equal to $S_i(p)$ for $0 \leq p \leq p^{\epsilon}$. For prices $p > p^{\epsilon}$, however, we define $S_i^{\epsilon}(p)$ so that it is affine and tangential to S_i at $p = p^{\epsilon}$. This situation is illustrated in figure 3 and shows that S_i^{ϵ} "bends away" from S_i for prices $p > p^{\epsilon}$.

If each firm $i = 1, \ldots, n$ adopts this perturbed supply function S_i^{ϵ} then we can then show that the optimal response by firm i to $(S_j^{\epsilon})_{j \neq i}$, which we will call $\tilde{S}_i$, involves an even larger bend away from S_i. This is also illustrated in figure 3. That is, $\left\|\tilde{S} - S^{\star}\right\| > \|S^{\epsilon} - S^{\star}\|$ and the SFE $S^{\star}$ is unstable. $\square$

5 Numerical analysis

In this section we use an iterative numerical framework to analyze SFEs. We begin the iterations with a starting function and then at each successive iteration update each firm's supply function by seeking that firm's best response to the supply functions of the other firms from the previous iteration. That is, we iterate in the function space of allowed supply functions to seek an equilibrium. We consider the convergence of the sequence of iterates. Details of the computational framework including discussion of the parameterization of the supply functions and various caveats about the analysis are contained in [6].

The stability theorem in the previous section is most easily demonstrated numerically with respect to a symmetric system since symmetric non-affine SFEs all satisfy the hypotheses of the theorem. We consider a symmetric three firm example in section 5.1. We then consider an asymmetric five firm system in section 5.2.

5.1 Symmetric three firm system.

This system has cost functions based on Day and Bunn [7]. The cost functions are the same for each firm and there are no capacity constraints and no price caps represented. This is the same system as was used for figure 2. As well as the most and least competitive SFEs shown in figure 2, there is a range of possible SFEs between them as illustrated in figure 4, which shows various equilibrium supply functions for one of the firms. (Again, since the firms are symmetric, each SFE involves the same function for each firm.) The dashed line in figure 4 shows the affine SFE.

We used each SFE illustrated in figure 4 as the starting function for the numerical framework. Since these starting functions are themselves SFEs, it is reasonable to expect that at each iteration each firm will not change its supply function, since the supply function from the previous iteration remains an optimal response. Moreover, the profit for each firm should stay constant at each iteration.

Figure 5 illustrates the profit at each iteration using the SFEs from figure 4 as starting functions. Figure 5 confirms that the profit stays constant at each iteration. (The dashed curve represents the profits versus iteration with the affine SFE used as starting function.)

Now we consider slight perturbations of the SFE. In particular, we use the construction from the theorem to construct perturbed functions S_i^ϵ that bend slightly away from S_i for prices $p > p^\epsilon$. For each SFE, we chose the corresponding p^ϵ to be just slightly below the price at peak demand for that SFE. These perturbed functions are shown in figure 6. The nearly vertical dotted line shows the vicinity of the prices and quantities at peak demand for the SFEs. At prices significantly below this level, each supply function in figure 6 matches the corresponding supply function in figure 4. At higher prices, each supply function in figure 6 is affine.

The results from using the supply functions in figure 6 as starting functions are shown in figure 7. Figure 7 is dramatically different from figure 5. For all starting functions except the ones closest to the affine starting function, the sequence of profits drifts away from the profit corresponding to the starting function. This means that all but the SFEs that are closest to the affine starting function are numerically unstable.

This numerical analysis with the symmetric three firm system confirms the theoretical

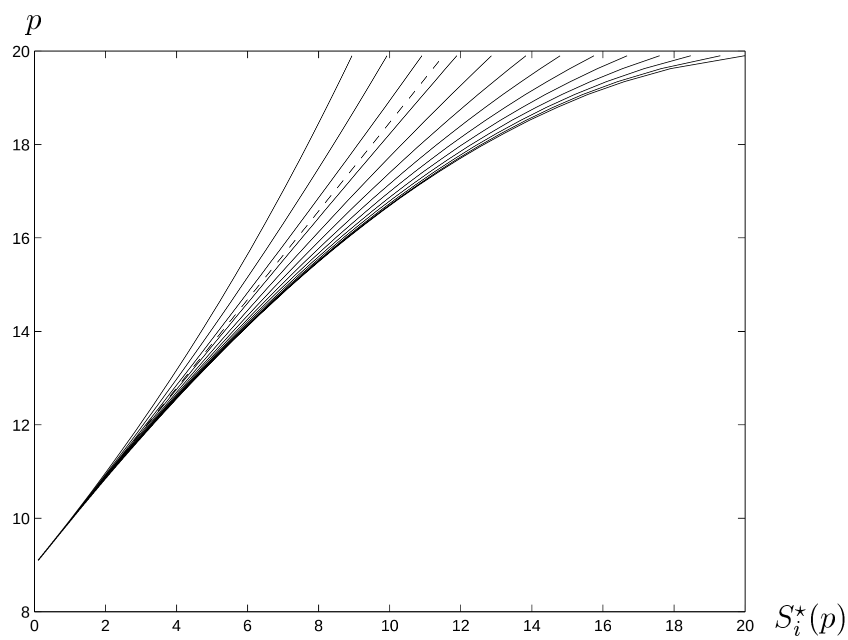

Figure 4: Starting functions for symmetric three firm example.

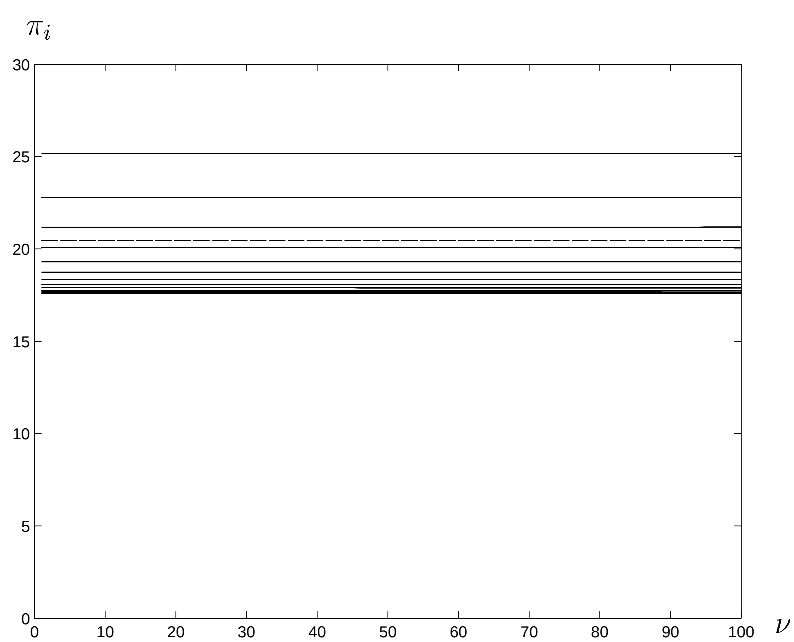

Figure 5: Profits versus iteration for SFE starting functions.

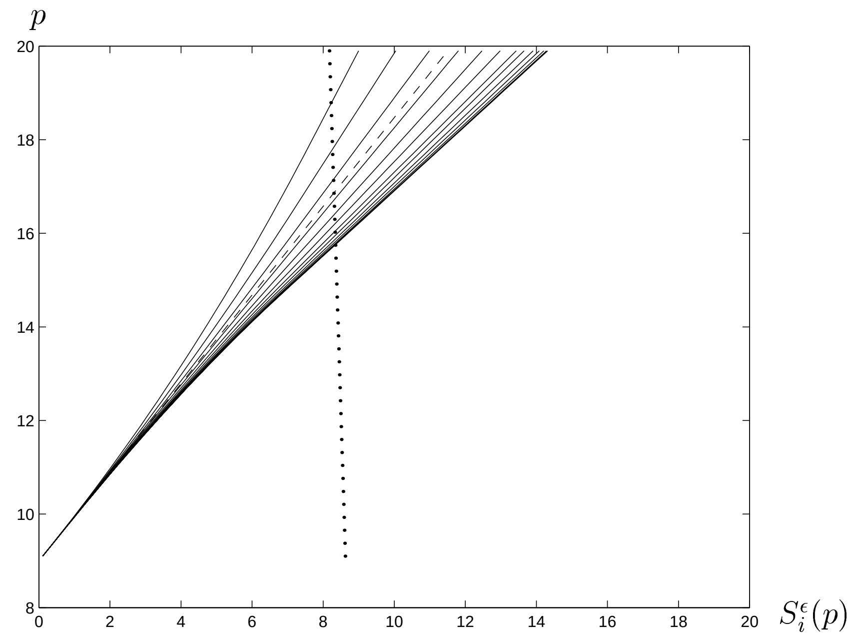

Figure 6: Perturbed starting functions constructed as in proof of stability theorem.

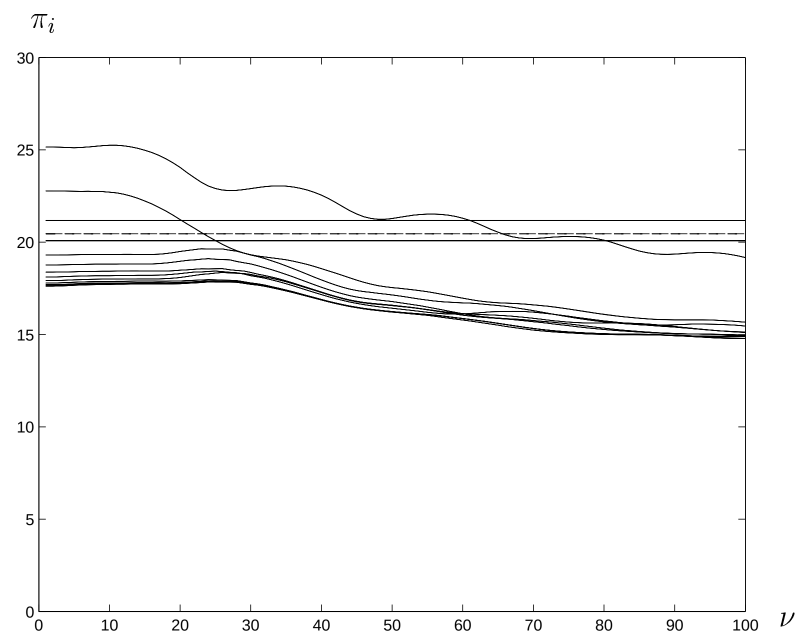

Figure 7: Profits versus iteration for perturbed SFE starting functions.

stability analysis. In particular, the simulations confirm that there are multiple equilibria, but that for a starting function that is a slight perturbation from an equilibrium, the sequence of iterates typically drifts away from equilibrium. As discussed in more detail in [6], when the starting function is a perturbation of an unstable equilibrium, the numerical results depend critically on simulation assumptions, such as the way that the function space of bid functions is parameterized. From a numerical perspective, equilibria that are significantly different from the affine equilibrium are not stable.

5.2 Asymmetric five firm system.

In this section, we consider cost functions that are based on the five non-nuclear firms in England and Wales circa 2000. The cost functions for these firms are presented in [6]. These firms have capacity constraints and we choose the load-duration characteristic so that almost all generation is required on-peak. While economic dispatch at peak demand would result in marginal costs of 27 pounds per MWh, the Cournot prices at peak are 80 pounds per MWh.

As with the three firm symmetric system, we again iterate in the function space of allowed supply functions to seek the equilibrium. In this case, however, it turns out that the construction of the most and least competitive SFEs fails. Instead we use several *ad hoc* starting functions, including functions similar to the Cournot bidding and competitive bidding functions that were shown dashed in figure 2. The results from the various starting functions all turn out to be similar. That is, there is apparently only a relatively small range of observed equilibria. Moreover, all equilibria have peak prices well below Cournot prices.

Figure 8 shows the price-duration curve at iteration 100 for one particular starting function when there is no price cap. The price at peak is around 43 pounds per MWh, which is far below the Cournot price at peak. Prices off-peak are even lower. Results for other starting functions are similar.

Figure 9 shows the price duration curve at iteration 100 for one particular starting function when a price cap of 40 pounds per MWh is imposed. The price cap affects the prices not only at peak but also at off-peak conditions. Again, results for other starting functions are similar.

For other values of the price cap, the price at peak varies with the price cap when the price cap is binding. Figure 10 shows the price at peak versus the price cap for two different starting functions for each value of price cap. For a given price cap, the range of peak prices between the two starting functions is relatively small, confirming that there is only a relatively small range of observed equilibria.

6 Significance of stability theorem

In this section we discuss the significance of the stability theorem and of its numerical confirmation. We first observe that if there is no requirement to bid consistently over the time horizon or if the time horizon consists of only a few pricing periods, such as in:

- the (now defunct) California Power Exchange, or
- any market without a mandatory pool,

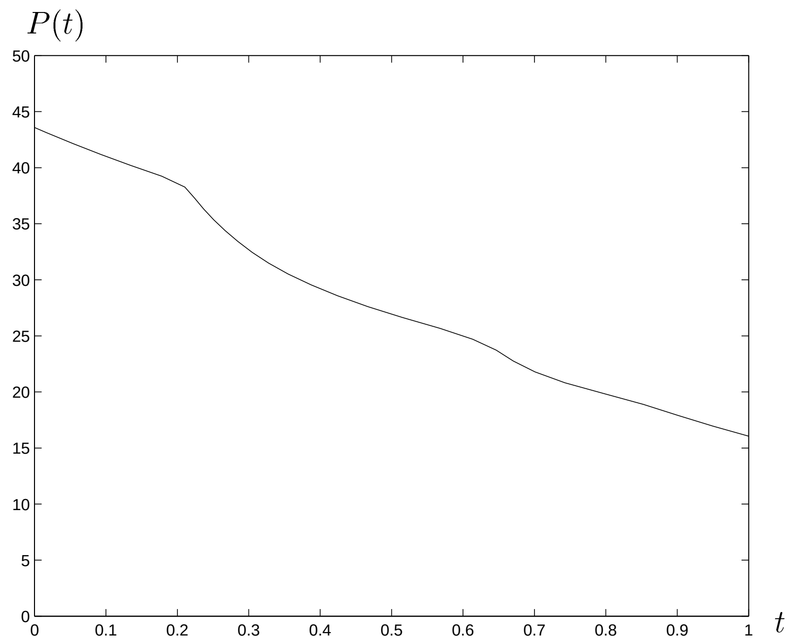

Figure 8: Price-duration curve at iteration 100 with no price cap.

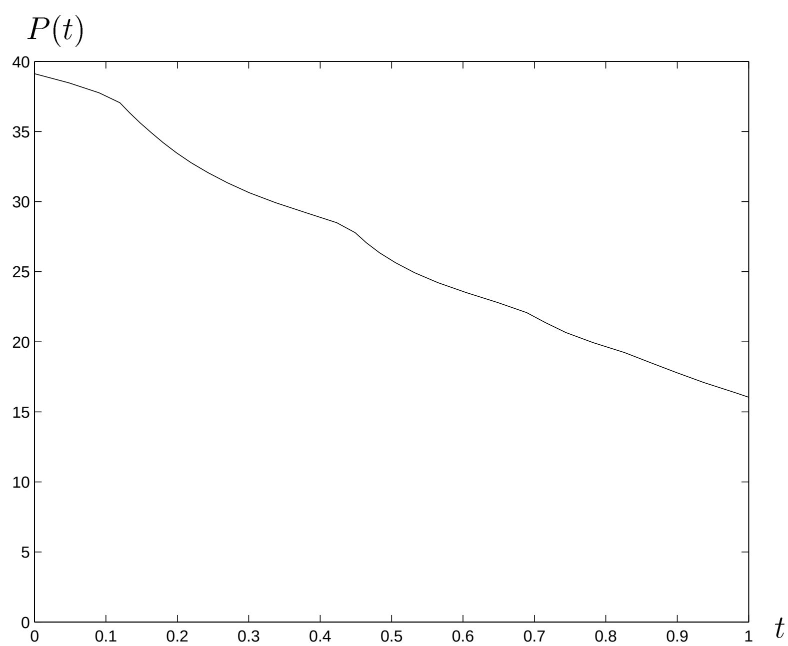

Figure 9: Price-duration curve at iteration 100 with price cap of 40 pounds per MWh.

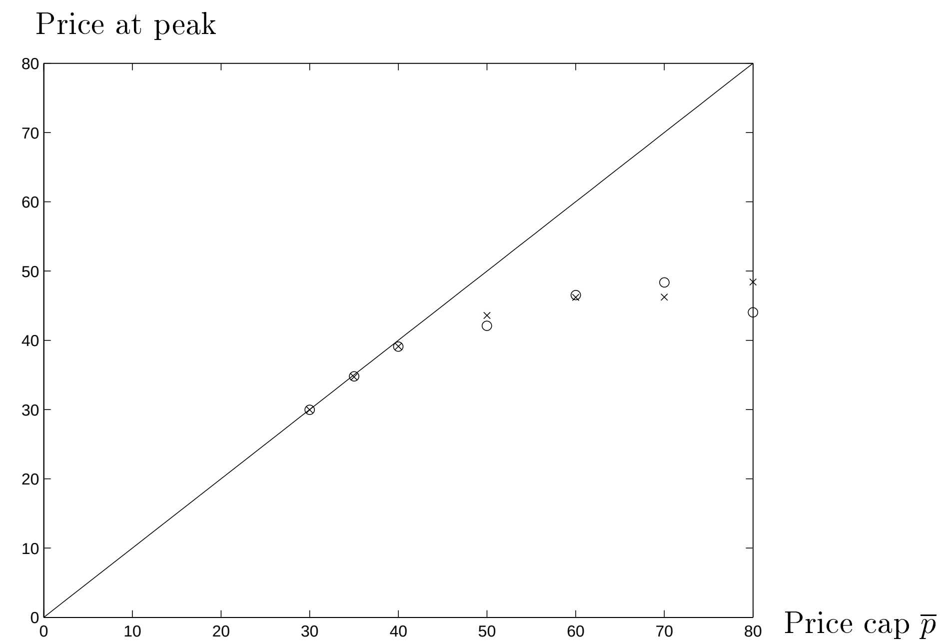

Figure 10: Price at peak versus price cap. For each value of price cap, the corresponding circle and cross shows the result at iteration from staring at two different starting functions.

then Cournot prices are a possible outcome in each pricing period. Cournot prices in each pricing period is a much less competitive outcome than even the least competitive supply function equilibrium.

To understand the significance of the number of pricing periods, we consider the extreme of a load-duration characteristic that is piece-wise constant and represents just four pricing periods, as illustrated in figure 11. This load-duration characteristic is not continuous and so violates that assumption in our formulation.

We label the pricing periods a, b, c, and d and assume that clearing prices p_a, p_b, p_c, and p_d, apply throughout periods a, b, c, and d, respectively. We consider supply functions that achieve the Cournot prices in each period. In particular, consider figure 12. The dashed line shows the relationship between prices and quantities under Cournot bidding, which was defined in section 3 and illustrated in figure 2. In each pricing period, the Cournot prices $p_a, p_b, p_c,\ p_d$, and corresponding quantities, $q_{ia}, q_{ib}, q_{ic},\ q_{id}$, are achieved by constructing a supply function that is constant independent of price in four price bands around p_a, p_b, p_c, and p_d. While we have not proved that this is actually an equilibrium in supply functions, the figure is suggestive that such a supply function could be an SFE for the piece-wise constant load-duration characteristic.

In contrast, when a firm is obliged to bid consistently over a time horizon having many pricing periods, it will not be possible to robustly achieve the Cournot outcome in each pricing period because the bands around each Cournot price will be much smaller. If there are a large number of pricing periods, then each firm must trade off profits from high prices at peak against sales at off-peak. That is, equilibrium prices will be lower than Cournot prices.

Green and Newbery showed that the obligation to bid consistently over a time horizon reduced the mark-up at off-peak to below that of Cournot prices [2]. However, the stability analysis in this paper shows that even at peak times the observed equilibrium prices will be lower than the Cournot price. Moreover, we have shown both theoretically under restrictive assumptions and numerically that there is a small range of stable SFEs. For example, consider symmetric firms with affine marginal costs as in the symmetric three firm example system in section 5.1. Figure 13 shows the wide range of SFEs between the most and least competitive SFEs. Figure 13 also shows the affine SFE, which is the only stable SFE and so it is the only SFE that is likely to be be observed in practice.

7 Policy implications and conclusion

The requirement to bid consistently over a time horizon with multiple pricing periods can help limit the exercise of market power, by depressing the prices that can be achieved in equilibrium. Such a requirement is compatible with and additional to other proposals for mitigation of market power, such as ([11]):

- long-term contracting,
- real-time pricing,
- price caps.

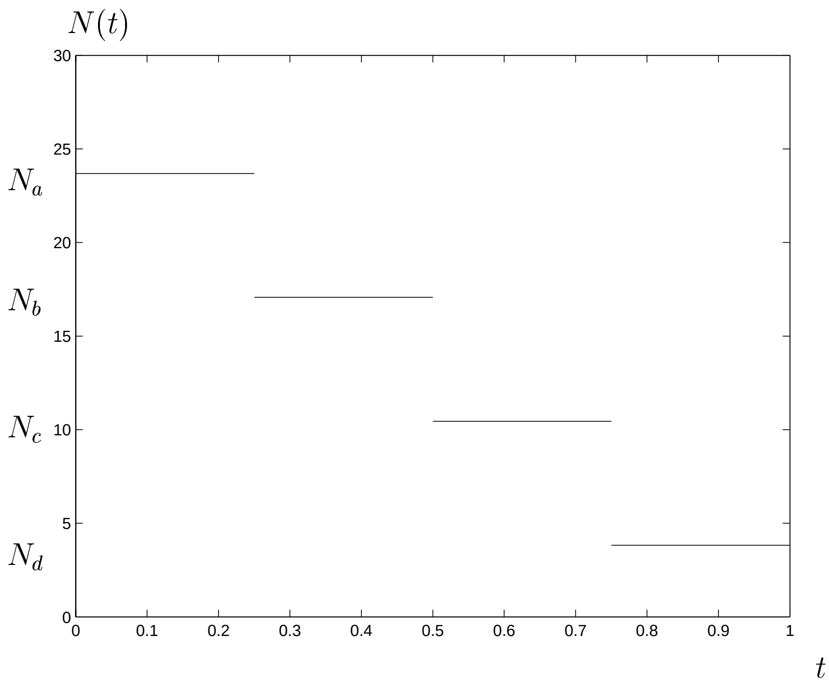

Figure 11: Piece-wise constant load-duration characteristic.

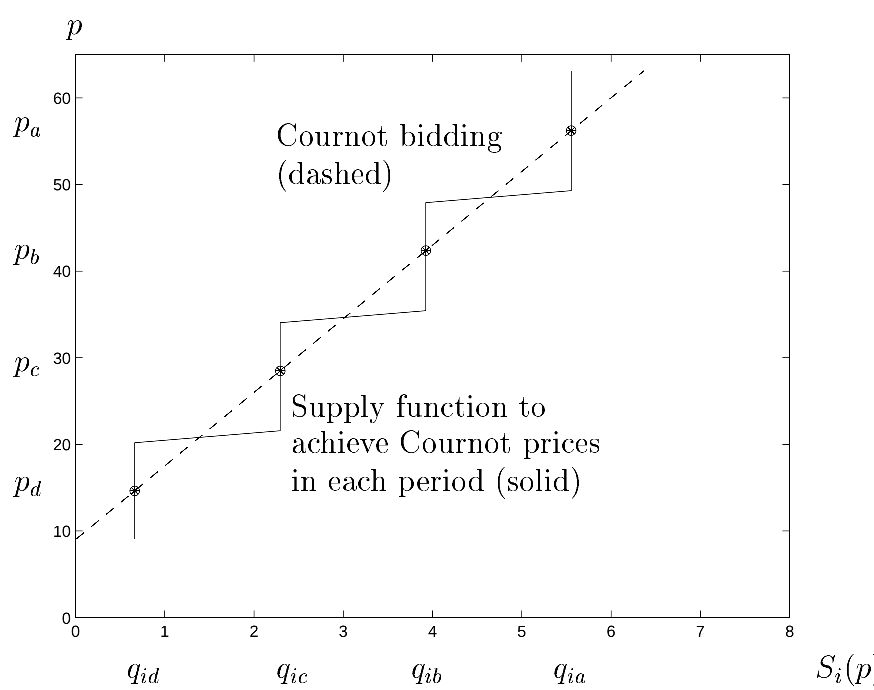

Figure 12: Bid supply function to achieve Cournot prices and quantities in a four period time horizon.

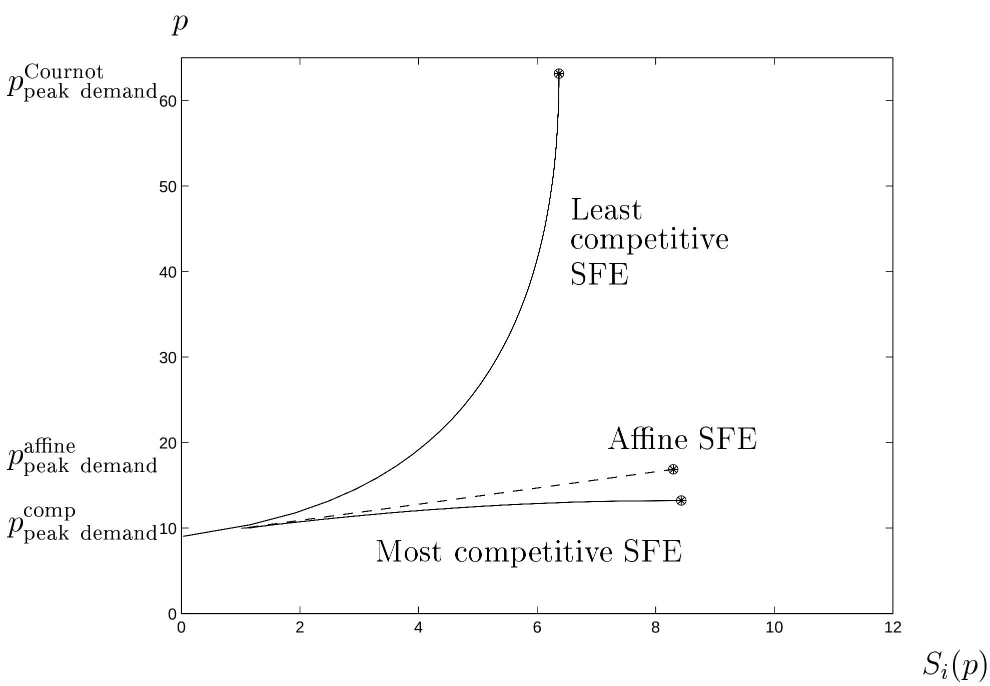

Figure 13: Illustration of significance of stability theorem.

In conclusion, we have analyzed stability of equilibria in a bid-based pool market and found that the requirement to bid consistently across a time horizon can limit the exercise of market power. Additionally, we find that there is typically only a small range of stable SFEs. Since some electricity markets have been set up with an obligation for consistent bids while others have not, it is important to observe that seemingly arcane differences in specification of market rules can have large effects on outcomes.

Acknowledgment

Much of this work was performed during visits by the first author to the John F. Kennedy School of Government, Harvard University in Fall 2000 and to the University of California Energy Institute in Summer 2001. The first author gratefully acknowledges the hospitality of the John F. Kennedy School of Government and the University of California Energy Institute. The first author was funded in part by the University of Texas Faculty Development Program, the National Science Foundation under grant ECS-0082506, and the University of California Energy Institute. The authors gratefully acknowledge the support of the Harvard Electricity Policy Group and the Harvard-Japan Project on Energy and Environment. The views expressed here are those of the authors and not necessarily those of any of these organizations. The authors would like to thank Severin Borenstein, Jim Bushnell, Ed Kahn, Shmuel Oren, Alex Rudkevich, and Frank Wolak for comments and suggestions.

References

[1] Paul D. Klemperer and Margaret A. Meyer. Supply function equilibria in oligopoly under uncertainty. *Econometrica*, 57(6):1243–1277, November 1989.

[2] Richard Green and David M. Newbery. Competition in the British electricity spot market. *Journal of Political Economy*, 100(5):929–953, October 1992.

[3] Richard Green. Increasing competition in the British electricity spot market. *The Journal of Industrial Economics*, XLIV(2):205–216, June 1996.

[4] Aleksandr Rudkevich. Supply function equilibrium in poolco type power markets: Learning all the way. Technical Report TCA Technical Paper Number 1299-1702, Tabors Caramanis and Associates, December 1999.

[5] Ross Baldick, Ryan Grant, and Edward P. Kahn. Linear supply function equilibrium: Generalizations, application, and limitations. University of California Energy Institute POWER Paper PWP-078, www.ucei.berkeley.edu/ucei/PDF/pwp078.pdf, August 2000.

[6] Ross Baldick and William Hogan. Capacity constrained supply function equilibrium models of electricity markets: Stability, non-decreasing constraints, and function space iterations. University of California Energy Institute POWER Paper PWP-089, www.ucei.berkeley.edu/ucei/PDF/pwp089.pdf, December 2001.

[7] Christopher J. Day and Derek W. Bunn. Divestiture of generation assets in the electricity pool of England and Wales: A computational approach to analyzing market power. *Journal of Regulatory Economics*, 19(2):123–141, 2001.

[8] Arthur R. Bergen and Vijay Vittal. *Power Systems Analysis*. Prentice-Hall, Upper Saddle River, NJ, second edition, 2000.

[9] Fernando L. Alvarado, Jianping Meng, Christopher L. DeMarco, and Wellington S. Mota. Stability analysis of interconnected power systems coupled with market dynamics. *IEEE Transactions on Power Systems*, 16(4):695–701, November 2001.

[10] Fernando Alvarado. That stability of power system markets. *IEEE Transactions on Power Systems*, 14(2):505–511, May 1999.

[11] Severin Borenstein. The trouble with electricity markets (and some solutions). University of California Energy Institute POWER Paper PWP-081, www.ucei.berkeley.edu/ucei/PDF/pwp081.pdf, January 2001.