Fourier Optics - Exam #1 Review

Ch. 2 2-D Linear Systems

A. Fourier Transforms, theorems. - handout --> your note sheet

B. Linear Systems

C. Applications of above - sampled data and the DFT (supplement to Goodman)

Ch. 3 Foundations of Scalar Diffraction

A. Greens Theorem. & Kirchhoff formulation, B.C.

B. Rayleigh-Sommerfeld formulation, B.C.

C. Angular Spectrum (equal to R-S integral & relate to Far Field) What you can and cannot easily calculate.

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Ch. 4 Fresnel and Fraunhofer Diffraction

A. Levels of approximation (Fresnel integral expression)

Convolution and Fourier transform expressions. Impulse response and transfer function; relate to angular spectrum.

B. Fraunhofer diffraction (scaling of the spatial and angular variables) - many examples of aperture functions and their diffraction patterns.

C. Fresnel diffraction: Fresnel zones (Fresnel number), focusing conditions and properties.

1 hr and 25 - 30 min. exam, one page of notes (note sheet) allowed, and calculator (no books or other notes)

Sample Exam Questions

1. (25 points) In the Rayleigh-Sommerfeld (R-S) treatment of scalar diffraction, a beneficial choice of Green's Functions is made. Describe (and sketch) the possible Green's Functions and show how (for only one case) that the choice leads to the relaxation of the Kirchhoff boundary condition. State the boundary condition as used in the R-S formulation (what it is and where applied), and state what is relaxed (as compared to Kirchhoff's original formulation).

Sample Exam Questions

2. (35 points) The objective of this problem is to compare the far-field diffraction angles of coherent light beams that come through square, circular, and Gaussian amplitude-profile holes.

a.) State the far-field diffraction angles from the z-axis to the first zero along the x_o-axis of the square and the circular holes (use the wavelength and the distance across the aperture, width of square and diameter of disk in your answers).

b.) Since the Gaussian has no zeros, we need a definition of the radius or diameter. If the Gaussian function is defined as:

$$u(x, y) = \exp\left[-\pi b^2 x^2\right] \exp\left[-\pi b^2 y^2\right]$$

I can define 2 possible choices that do not make the math too complicated.

(1) The radius is at x or y = 1/b where the amplitude becomes $e^{-\pi}$ (and the intensity becomes $e^{-2\pi}$). This is probably an unrealistically large of an estimate for the size.

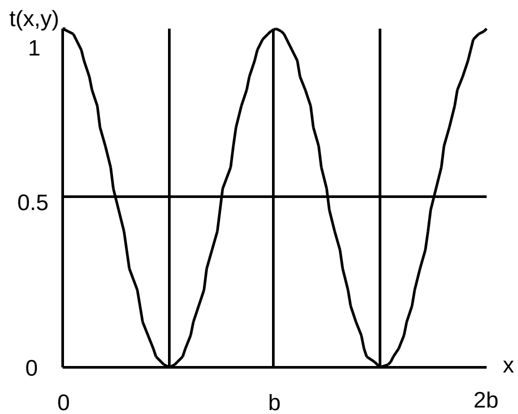
OR (2) x or y = $\dfrac{1}{\left(\sqrt{\pi}\right) b}$ where the amplitude becomes e^{-1} (and the intensity becomes e^{-2}). This is the conventional definition of the radius of a Gaussian laser beam.

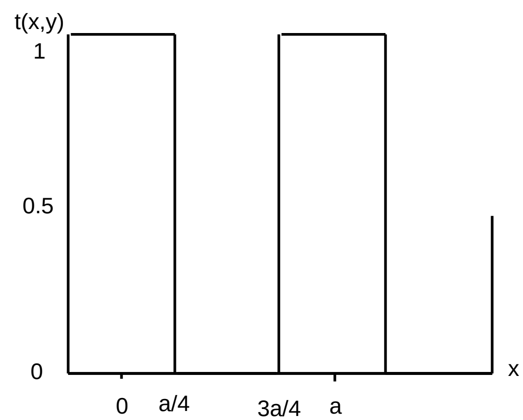
Find the far-field diffraction angle (from the z-axis to the point where the amplitude falls to either e^{-p} or $e^{-1)}$. For both these cases, don't forget that the radius is defined two ways, and this definition is applied both to the source aperture plane and to the far-field diffraction plane. Give the far field angle in terms of a number and lambda and a "DIAMETER" for the Gaussian aperture so that the angle may be compared to the two apertures in part a.).

Sample Exam Questions

3. (35 points) Compare the first-order far-field diffraction efficiency of a 1-D square-wave amplitude grating with a cosinusoidal amplitude grating.

$$t(x,y) = \frac{1}{2} + \frac{1}{2}\cos\left(2\pi\frac{x}{b}\right) \qquad t(x,y) = Comb\left(\frac{x}{a}\right) * \mathrm{Re}ct\left(\frac{x}{a/2}\right)$$





In both cases there is no y dependence and the gratings are approximately infinite in size.

Note that although the average amplitude transmissions are both 0.5, when the functions are squared to get intensity, the average power transmitted through the cosine is 3/8 and through the square-wave is 1/2. (This would be a trick question.)

Sample Exam Questions

QUESTION - What are the fractions of the incident intensity that is transmitted to the + (or-) 1 orders for the two cases? In this case the different constants that emerge from the Fourier transform are important.

Also, remember that the magnitude squared of the sum of delta functions contains only the sum of the squared terms and the cross terms become zero since $\delta(x)\ \delta(x-d)$ is zero unless d is zero.