

EE313 Linear Systems and Signals  
Analog Sinusoidal Modulation

Many ways exist to modulate a message signal  $m(t)$  to produce a modulated (transmitted) signal  $x(t)$ . For amplitude, frequency, and phase modulation, modulated signals can be expressed in the same form as

$$x(t) = A(t) \cos(2\pi f_c t + \Theta(t))$$

where  $A(t)$  is a real-valued amplitude function (a.k.a. the envelope),  $f_c$  is the carrier frequency, and  $\Theta(t)$  is the real-valued phase function. Using this framework, several common modulation schemes are described below. In the table below, the amplitude modulation methods are double sideband larger carrier (DSB-LC), DSB suppressed carrier (DSB-SC), DSB variable carrier (DSB-VC), and single sideband (SSB). The hybrid amplitude-frequency modulation is quadrature amplitude modulation (QAM). The angle modulation methods are phase and frequency modulation.

Modulation	$A(t)$	$\Theta(t)$	Carrier	Type	Use
DSB-LC	$A_c [1 + k_a m(t)]$	$\Theta_0$	Yes	Amplitude	AM radio
DSB-SC	$A_c m(t)$	$\Theta_0$	No	Amplitude	
DSB-VC	$A_c m(t) + \epsilon$	$\Theta_0$	Yes	Amplitude	
SSB	$A_c \sqrt{m^2(t) + [m(t) \star h(t)]^2}$	$\arctan\left(-\frac{m(t) \star h(t)}{m(t)}\right)$	No	Amplitude †	Marine radios
QAM	$A_c \sqrt{m_1^2(t) + m_2^2(t)}$	$\arctan\left(-\frac{m_2(t)}{m_1(t)}\right)$	No	Hybrid	Satellite
Phase	$A_c$	$\Theta_0 + k_p m(t)$	No	Angle	Underwater modems
Frequency	$A_c$	$2\pi k_f \int_0^t m(t) dt$	No	Angle	FM radio TV audio

†  $h(t)$  is the impulse response of a bandpass filter or phase shifter to effect a cancellation of one pair of redundant sidebands. For ideal filters and phase shifters, the modulation is amplitude modulation because the phase would not carry any information about  $m(t)$ .

Each analog TV channel is allocated a bandwidth of 6 MHz. The picture intensity and color information are transmitted using vestigial sideband modulation. Vestigial sideband modulation is a variant of amplitude modulation (not shown above) in which the upper sideband is kept and a fraction of the lower sideband is kept, or vice-versa. In an analog TV signal, the audio portion is frequency modulated.

The following quantity is known as the complex envelope

$$\tilde{x}(t) = A(t) e^{j\Theta(t)} = x_I(t) + j x_Q(t)$$

where  $x_I(t)$  is called the in-phase component and  $x_Q(t)$  is called the quadrature component. Both  $x_I(t)$  and  $x_Q(t)$  are lowpass signals, and hence, the complex envelope  $\tilde{x}(t)$  is a lowpass signal. An alternative representation for the modulated signal  $x(t)$  is

$$x(t) = \Re\{\tilde{x}(t) e^{j2\pi f_c t}\}$$