THE UNIVERSITY OF TEXAS AT AUSTIN
Dept. of Electrical and Computer Engineering

EE381K-14 Multidimensional Digital Signal Processing
Problem Set #1: Periodicity and Convolution

Date assigned: January 18, 2008
Date due: January 24, 2008

Reading: Dudgeon & Mersereau, Ch. 1

You may use any computer program to help you solve these problems, check answers, etc.

Homework is due on Thursday, January 24th, by 11:00 AM by hardcopy in lecture.

Regularly scheduled office hours for Prof. Evans are Wednesdays 10–11 AM, Thursdays 12:30–1:30 PM, and Fridays 9:00–10:00 AM in ENS 433B. Feel free to send questions by e-mail to bevans@ece.utexas.edu.

Be sure to submit your own independent homework solutions.

Problem 1.1 Periodicity Matrices

(a) A 2-D sequence $\tilde{x}[n_1, n_2]$ is known to be periodic with periodicity matrix

$$\boldsymbol{N} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}.$$

Show that $\tilde{x}[n_1, n_2]$ is also rectangularly periodic with horizontal and vertical periods $N_1 = N_2 = D = |N_{11}N_{22} - N_{12}N_{21}|$.

(b) How many "periods" of $\tilde{x}$ are contained in one of the rectangular periods?

Hints: The problem is asking you to visualize periodicity that is non-rectangular and periodicity that is rectangular. One thing to try is to choose small non-zero integers for N_{11}, N_{12}, N_{21} and N_{22}, and sketch the situation to see what happens.

The problem statement does not require that the two cases have the same number of samples.

Starting from the definition of a periodicity matrix:

$$x(\boldsymbol{n} + \boldsymbol{N}\boldsymbol{r}) = x(\boldsymbol{n})$$

for periodicity matrix $\boldsymbol{N}$ *and any integer vector* $\boldsymbol{r}$. *Let* $\boldsymbol{K}$ *be a non-singular square integer matrix, and let* $\boldsymbol{l}$ *be an integer vector. Substitute* $\boldsymbol{r} = \boldsymbol{K}\ \boldsymbol{l}$,

$$x(\boldsymbol{n} + \boldsymbol{N}\boldsymbol{K}\boldsymbol{l}) = x(\boldsymbol{n})$$

This means that $\boldsymbol{N}\boldsymbol{K}$ *is also a valid periodicity matrix.*

Problem 1.2 Convolution

Determine the 2-D convolution of the following two 2-D sequences.

$$\begin{aligned} x[n_1, n_2] &= u[n_1, n_2] \\ h[n_1, n_2] &= \delta[n_1 + n_2] \end{aligned}$$

Problem 1.3 More Convolution

Consider the sequence x defined by

$$x[n_1, n_2] = \begin{cases} 1, & 0 \leq n_1 \leq n_2 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the convolution of x with itself.

Problem 1.4 Filter Design

We wish to design a simple 3×3 lowpass filter that satisfies the following conditions:

$$H(\omega_1, \omega_2) = \begin{cases} 1, & (\omega_1, \omega_2) = (0, 0) \\ 0, & (\omega_1, \omega_2) = (\pi, 0), (\pi, \pi), (0, \pi), (-\pi, \pi), \\ & \qquad (-\pi, 0), (-\pi, -\pi), (0, -\pi), \text{ and } (\pi, -\pi) \end{cases}$$

Determine one possible choice for $h[n_1, n_2]$.

Hints: Use the definition of the 2-D frequency response. You should find that there are nine equations and nine unknowns.

Plot the magnitude and phase response of the filter you designed.

Problem 1.5 Inverse Fourier Transform

Compute the impulse response $h[n_1, n_2]$ of the ideal lowpass filter for which one period of the frequency response is shown in Figure 1. The response is one in the shaded regions and zero in the unshaded ones.

Hints: First, the figure shows the fundamental tile of the 2-D Fourier domain, which goes from $-\pi$ *to pi in each Fourier variable. This diamond-shaped lowpass frequency response is a linear shift-invariant approximation to the lowpass response of the human visual system. The filter design, therefore, can mimic to some extent what the eye actually outputs from the retina.*

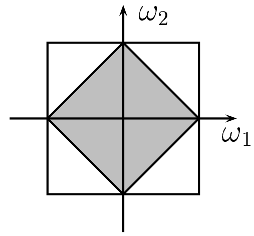

Figure 1: $H(\omega_1, \omega_2)$ for Problem 1.18.

This response is a rotated version of a separable square lowpass frequency response. The Fourier transform of a rotation in the frequency domain has an analogy in discrete the time domain. If we were in the continuous-space Fourier domain and we were inverse transforming to the continous-space domain, then a rotation in the continuous-space Fourier domain is the same rotation in the continuous-space domain. This view might simplify the calculations in that you could solve the problem first in the continous-space domain and then sample the solution in the discrete-space domain.

To look ahead, this frequency response is actually the response of the lowpass channel of a two-channel filter bank for a quincunx sampling grid. Each channel of the two-channel filter bank represents half of the Fourier domain, but in a non-separable way.