THE UNIVERSITY OF TEXAS AT AUSTIN
Dept. of Electrical and Computer Engineering

EE381K-14 Multidimensional Digital Signal Processing
Problem Set #2: Fourier Transforms and Sampling

Date assigned: January 26, 2008
Date due: January 31, 2008

Reading: Dudgeon & Mersereau, Ch. 1

You may use any computer program to help you solve these problems, check answers, etc.

Homework is due on Thursday, January 31st, by 11:00 AM by hardcopy in lecture.

Regularly scheduled office hours for Prof. Evans are Wednesdays 10–11 AM, Thursdays 12:30–1:30 PM, and Fridays 9:00–10:00 AM in ENS 433B. Feel free to send questions by e-mail to bevans@ece.utexas.edu.

Be sure to submit your own independent homework solutions.

Problem 2.1 Fourier Transforms

Find the Fourier transforms of the following sequences.

(a) $x_a[n_1, n_2] = a^{2n_1+n_2}u[n_1, n_2]$ where $|a| < 1$.

(b) $x_b[n_1, n_2, n_3] = a^{2n_1+n_2}u[n_1, n_2]$ where $|a| < 1$.

(c) $x_c[n_1, n_2] = a^{n_1}b^{n_2}\delta[n_1 - 4n_2]u[n_1, n_2]$. Assume that $|a| < 1$, $|b| < 1$.

Problem 2.2 Time-Varying Systems

Linear systems that are not time invariant can be described by their time varying impulse responses. Consider the *one-dimensional* case shown in Figure 1.

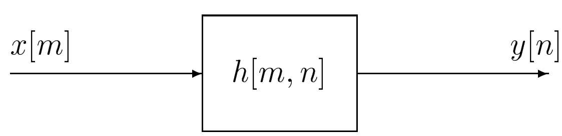

Figure 1: A one-dimensional time-varying system for Problem 2.2.

$h[m,n]$ is the response of the 1-D linear system at time n to an impulse applied at time m.

(a) Derive an expression for the output of such a system, $y[n]$ when the input is $x[m]$.

(b) Derive an expression for the impulse response of the cascade of two linear systems, if their individual impulse responses are $g[m,n]$ and $h[m,n]$.

(c) Can the order of the systems in (b) be interchanged without changing the behavior of the overall system? Explain.

(d) Generalize the result in (a) to a linear shift varying two-dimensional system for processing images.

Problem 2.3 Sampling a Lowpass Signal with an Elliptical Passband
A two-dimensional bandlimited signal has the elliptical support

$$H(\Omega_1, \Omega_2) = 0 \qquad \text{if } \frac{\Omega_1^2}{a^2} + \frac{\Omega_2^2}{b^2} \geq 1 \ .$$

(a) Determine a sampling matrix $\boldsymbol{V}$ that will allow $h(t_1, t_2)$ to be sampled at a minimum sampling rate without aliasing.

(b) What is the minimum sampling density (in samples per unit area)?

Problem 2.4 Sampling a Lowpass Signal with an Elliptical Passband
Two analog bandlimited waveforms have Fourier transforms with the regions of support indicated in Figure 2. For each determine the minimum sampling density (in samples per square meter) which will permit an exact reconstruction of the analog waveform. For each case sketch the optimal sampling raster.

Problem 2.5 Sampling a Lowpass Signal with an Elliptical Passband
A signal $x(t_1, t_2)$, with the bandlimited spectrum shown in Figure 3, is sampled at the sampling points sketched in Figure 4.

(a) Determine a sampling matrix, $\boldsymbol{V}$.

(b) Determine the corresponding aliasing matrix, $\boldsymbol{U}$.

(c) Sketch the spectrum of the sampled signal with respect to the continuous frequencies (Ω_1, Ω_2).

(d) Determine the largest value for the bandwidth, W, if there is to be no aliasing.

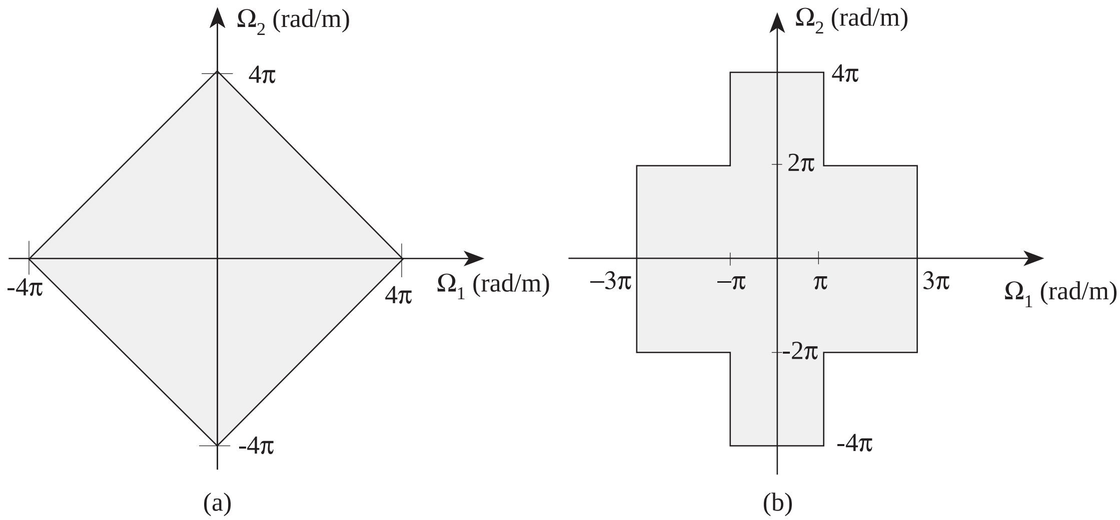

Figure 2:

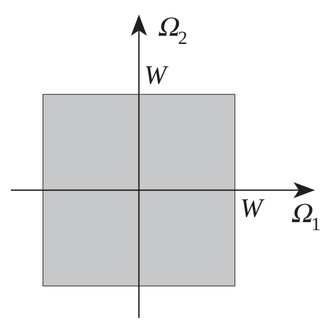

Figure 3:

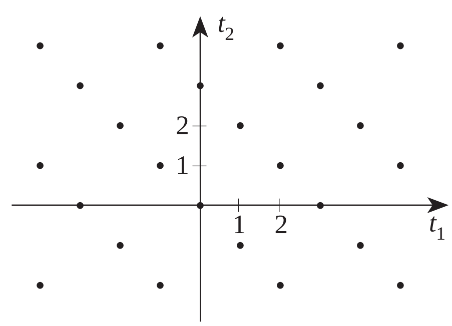

Figure 4: