You may use any computer program to help you solve these problems, check answers, etc.

Homework is due on Friday, February 25th, by 11:00 AM. You may turn in the homework to Prof. Evans’ office in ENS 433B, by fax to 471-6512, or by e-mail. If you submit the homework by fax, please let me know by e-mail.

Regularly scheduled office hours for Prof. Evans are Wednesdays 10–11 AM, Thursdays 12:30–1:30 PM, and Fridays 9:00–10:00 AM in ENS 433B. Feel free to send questions by e-mail to bevans@ece.utexas.edu.

Be sure to submit your own independent homework solutions.

Problem 4.1 Finite-Extent 2-D Convolution

Determine the two-dimensional convolution of the two sequences $x[n_1, n_2]$ and $h[n_1, n_2]$ sketched in Figure 1. (*Note: Both sequences have sample values equal to 1 at the locations indicated by large black dots and are zero elsewhere.)*

![Figure 1: The two sequences to be convolved in Problem 1.8.](image)

Problem 4.2 Non-Separable 2-D Discrete-Time Fourier Transform
\( H(\omega_1, \omega_2) \) is 1 in the shaded regions in Figure 2 and 0 in the unshaded regions. Determine \( h[n_1, n_2] \).

![Diagram](image)

**Figure 2: \( H(\omega_1, \omega_2) \) for Problem 4.2.**

**Problem 4.3** Elliptic Passband

The ideal circular lowpass filter has the impulse response

\[
h_c[n_1, n_2] = \frac{W J_1(W \sqrt{n_1^2 + n_2^2})}{2\pi \sqrt{n_1^2 + n_2^2}}
\]

and frequency response

\[
H_c(\omega_1, \omega_2) = \begin{cases} 
1, & \omega_1^2 + \omega_2^2 \leq W^2 < \pi^2 \\
0, & \text{otherwise.}
\end{cases}
\]

Determine the impulse response of the ideal elliptical lowpass filter that has the frequency response

\[
H_e(\omega_1, \omega_2) = \begin{cases} 
1, & \frac{\omega_1^2}{a^2} + \frac{\omega_2^2}{b^2} \leq W^2 < \pi^2 \\
0, & \text{otherwise.}
\end{cases}
\]

where \( 0 < a \leq b \leq \pi \). Plot the 2-D impulse response, \( h_e[n_1, n_2] \).

**Problem 4.4** Filtering Using the Discrete Fourier Transform

Consider an image \( x[n_1, n_2] \) of \( 512 \times 512 \) pixels to be filtered by a system with an impulse response \( h[n_1, n_2] \). The sequence \( h[n_1, n_2] \) is an \( 11 \times 11 \) sequence that is zero outside \( 0 \leq n_1 \leq 10 \) and \( 0 \leq n_2 \leq 10 \). Compare the computations

\[
y[n_1, n_2] = x[n_1, n_2] * h[n_1, n_2]
\]

and

\[
v[n_1, n_2] = DFT_{k_1,k_2}^{-1}(DFT_{n_1,n_2}\{x[n_1,n_2]\} \cdot DFT_{n_1,n_2}\{h[n_1,n_2]\})
\]
where the forward DFT and inverse DFT are $512 \times 512$ points in size. For what values of $(n_1, n_2)$ does $v[n_1, n_2]$ equal $y[n_1, n_2]$?

**Problem 4.5 1-D Discrete Sine Transform**

Develop an algorithm to compute the forward discrete sine transform using an appropriate discrete Fourier transform algorithm. The forward discrete sine transform is defined as

$$X_s[k] = \sum_{n=0}^{N-1} 2x[n] \sin \left(\frac{(2n + 1)k\pi}{2N}\right)$$

for $k = 0, 1, \ldots, N - 1$. 