THE UNIVERSITY OF TEXAS AT AUSTIN
Dept. of Electrical and Computer Engineering

EE381K-14 Multidimensional Digital Signal Processing
Problem Set #4: Practice Midterm #1 Based on the Fall 1996 Midterm #1

Date assigned:	February 18, 2008
Date due:	February 25, 2008

Reading: D&M, Chapters 1&2; Lectures 1–9

You may use any computer program to help you solve these problems, check answers, etc.

Homework is due on Friday, February 25th, by 11:00 AM. You may turn in the homework to Prof. Evans' office in ENS 433B, by fax to 471-6512, or by e-mail. If you submit the homework by fax, please let me know by e-mail.

Regularly scheduled office hours for Prof. Evans are Wednesdays 10–11 AM, Thursdays 12:30–1:30 PM, and Fridays 9:00–10:00 AM in ENS 433B. Feel free to send questions by e-mail to bevans@ece.utexas.edu.

Be sure to submit your own independent homework solutions.

Problem 4.1 Finite-Extent 2-D Convolution

Determine the two-dimensional convolution of the two sequences $x[n_1, n_2]$ and $h[n_1, n_2]$ sketched in Figure 1. (*Note:* Both sequences have sample values equal to 1 at the locations indicated by large black dots and are zero elsewhere.)

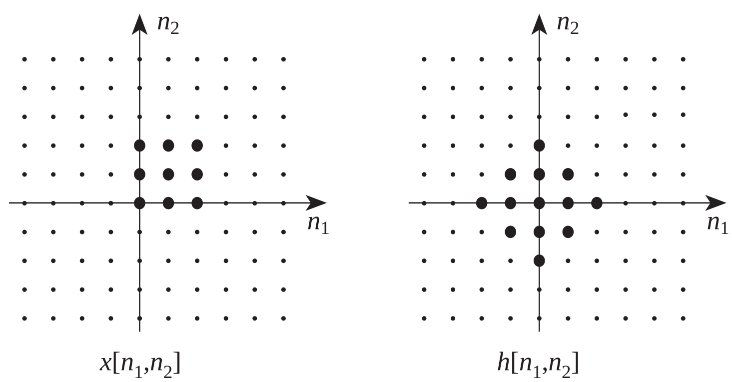
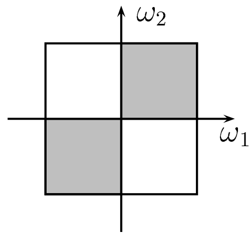


Figure 1: The two sequences to be convolved in Problem 1.8.

Problem 4.2 Non-Separable 2-D Discrete-Time Fourier Transform

$H(\omega_1, \omega_2)$ is 1 in the shaded regions in Figure 2 and 0 in the unshaded regions. Determine $h[n_1, n_2]$.



$H(\omega_1, \omega_2)$

Figure 2: $H(\omega_1, \omega_2)$ for Problem 4.2.

Problem 4.3 Elliptic Passband

The ideal circular lowpass filter has the impulse response

$$h_c[n_1, n_2] = \frac{W}{2\pi} \frac{J_1(W\sqrt{n_1^2 + n_2^2})}{\sqrt{n_1^2 + n_2^2}}$$

and frequency response

$$H_c(\omega_1, \omega_2) = \begin{cases} 1, & \omega_1^2 + \omega_2^2 \le W^2 < \pi^2 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the impulse response of the ideal elliptical lowpass filter that has the frequency response

$$H_e(\omega_1, \omega_2) = \begin{cases} 1, & \frac{\omega_1^2}{a^2} + \frac{\omega_2^2}{b^2} \le W^2 < \pi^2 \\ 0, & \text{otherwise.} \end{cases}$$

where $0 < a \le b \le \pi$. Plot the 2-D impulse response, $h_e[n_1, n_2]$.

Problem 4.4 Filtering Using the Discrete Fourier Transform

Consider an image $x[n_1, n_2]$ of 512×512 pixels to be filtered by a system with an impulse response $h[n_1, n_2]$. The sequence $h[n_1, n_2]$ is an 11×11 sequence that is zero outside $0 \le n_1 \le 10$ and $0 \le n_2 \le 10$. Compare the computations

$$y[n_1, n_2] = x[n_1, n_2] ** h[n_1, n_2]$$

and

$$v[n_1, n_2] = \mathcal{DFT}^{-1}_{k_1,k_2}\left(\mathcal{DFT}_{n_1,n_2}\{x[n_1, n_2]\} \cdot \mathcal{DFT}_{n_1,n_2}\{h[n_1, n_2]\}\right)$$

where the forward DFT and inverse DFT are 512×512 points in size. For what values of $(n1, n2)$ does $v[n_1, n_2]$ equal $y[n_1, n_2]$?

Problem 4.5 1-D Discrete Sine Transform

Develop an algorithm to compute the forward discrete sine transform using an appropriate discrete Fourier transform algorithm. The forward discrete sine transform is defined as

$$X_s[k] = \sum_{n=0}^{N-1} 2x[n] \sin \frac{(2n+1)k\pi}{2N}$$

for $k = 0, 1, \ldots, N-1$.