Computing Coset Vectors

FPD($\mathbf{S}$) = { $\mathbf{U}\,\mathbf{l}$ mod $\mathbf{S}$ | $\mathbf{l} \in$ FPD(Λ) }

1. FPD fundamental parallelepiped
2. S is an N x N resampling matrix (non-singular integer matrix) having Smith form $\mathbf{S} = \mathbf{U}\,\Lambda\,\mathbf{V}$
3. Vector modulo a matrix

$$\mathbf{x} \bmod \mathbf{M} = \mathbf{x} - \mathbf{M}\lfloor \mathbf{M}^{-1}\mathbf{x} \rfloor$$

4. $\mathbf{x} \bmod \mathbf{M} = \mathbf{x} - \mathbf{M}\,[\mathbf{M}^{-1}\,\mathbf{x}]$
5. FPD(Λ) is a rectangular prism that is of dimensions $\Lambda_{11}\times\Lambda_{22}\times\ldots\times\Lambda_{NN}$ where Λ_{ii} is the *i*th diagonal entry of Λ
6. $\mathbf{l}$ is a point in the rectangular prism of data FPD(Λ): $\{0, \Lambda_{11-1}]\times[0, \Lambda_{22}-1] \times \ldots \times[0, \Lambda_{NN}-1]$ assuming that $\Lambda_{11,}\,\Lambda_{22}, \ldots\, \Lambda_{NN}$ are positive.

Smith Forms for Upsampling and Downsampling

1. Smith form is not unique
2. $\mathbf{S} = \mathbf{U}\,\Lambda\,\mathbf{V}$
3. $\mathbf{S}^{-1} = \mathbf{V}^{-1}\,\Lambda^{-1}\,\mathbf{U}^{-1}$
4. Downsample [1] by $\mathbf{S}$: $x_d[\mathbf{n}] = x[\mathbf{S}\,\mathbf{n}] = x[\mathbf{U}\,\Lambda\mathbf{V}\,\mathbf{n}]$
5. Upsample [1] by $\mathbf{S}$:

$$x_u[\mathbf{n}] = \begin{cases} x[\mathbf{S}^{-1}\mathbf{n}] & \text{if } \mathbf{S}^{-1}\mathbf{n} \in I \\ 0 & \text{otherwise} \end{cases}$$

Upsampler and Downsampler in Cascade

1. In 1-D when can we swap the order of a down/upsampler cascade?
- L and M are relatively prime.
- M L = L M, which always true in 1-D.
2. The same conditions are true in m-D. Since matrix multiplication does not in general commute, the relative primeness can be with respect to the multiplication on the left or multiplication on the right. The product of L and M commutes when the rational matrix L^{-1} M has rational eigenvalues [3].

References

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