EE381K-14 Multidimensional DSP

Decimator Design

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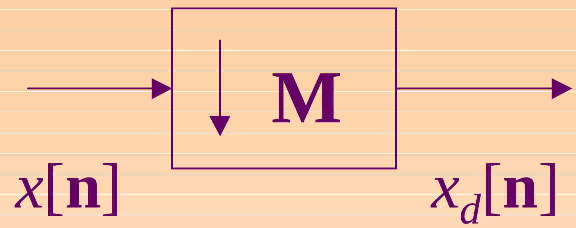
Multidimensional Downsampling

- **Downsample by M**
  - Input | det M | samples
  - Output first sample and discard others
- **Discards data**
- **May cause aliasing**



$$x_d[\mathbf{n}] = x[\mathbf{Mn}]$$

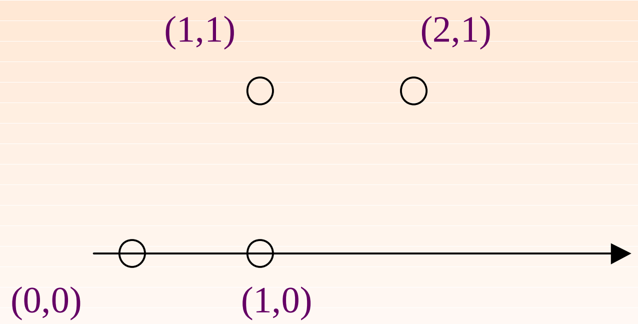
$\mathbf{k}_i$ is a coset vector

$$X_d(\boldsymbol{\omega}) = \frac{1}{|\det \mathbf{M}|} \sum_{i=0}^{|\det \mathbf{M}|-1} X(\mathbf{M}^{-t}(\boldsymbol{\omega} - 2\pi \mathbf{k}_i))$$

Coset Vectors

- Indices in fundamental tile of lattice(**M**)
  - |det **M**| coset vectors (origin always included)
  - Not unique for a given **M**
  - Compute using Smith form decomposition of **M**

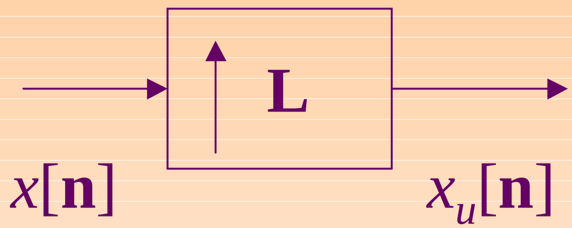
$$\mathbf{M} = \begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$$



Distinct coset vectors for **M**

Multidimensional Upsampling

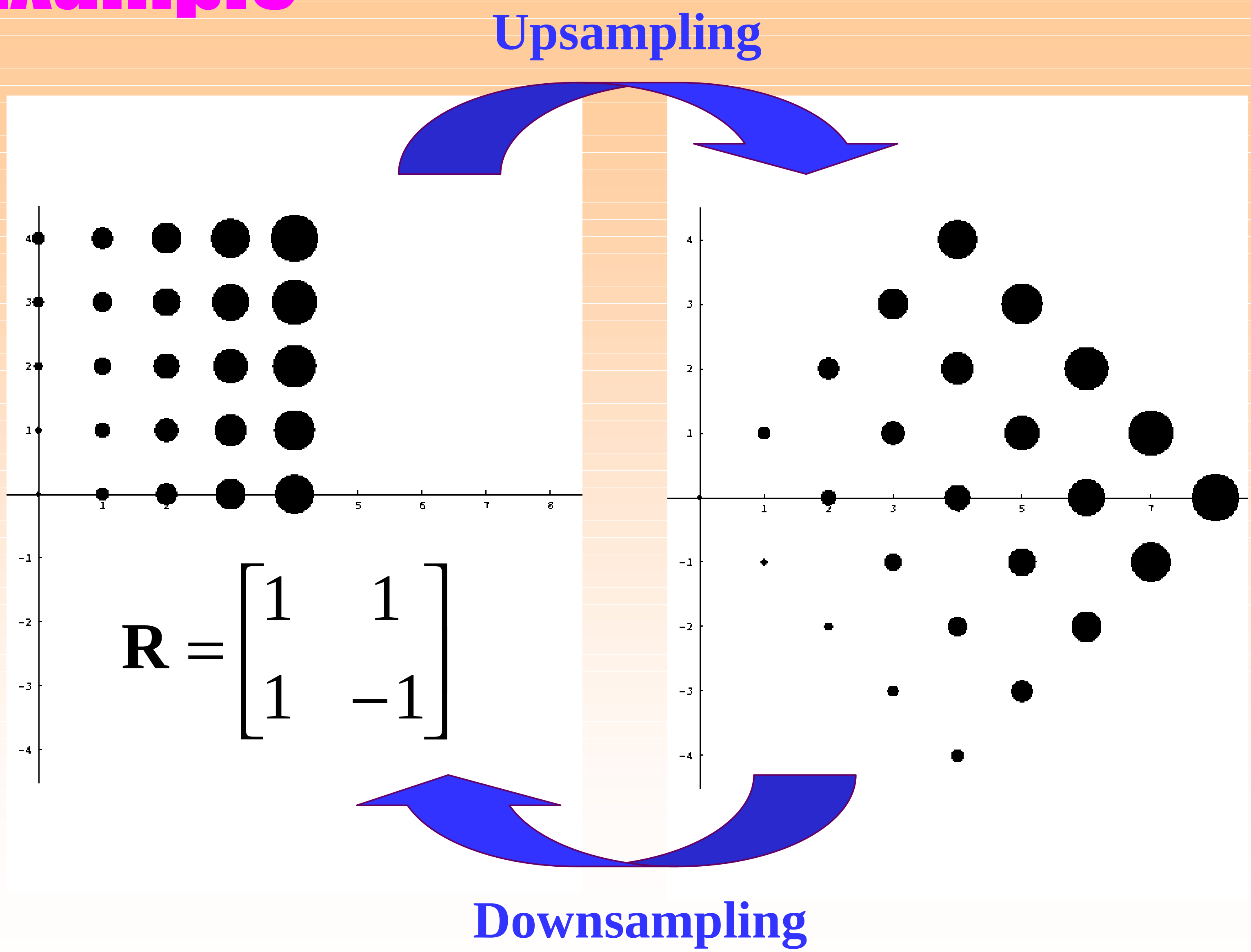
- Upsample by **L**
  - Input one sample
  - Output the sample and then |det **L**| - 1 zeros
- Adds data
- May cause imaging



$$X_u(\boldsymbol{\omega}) = X(\mathbf{L}\,\boldsymbol{\omega})$$

$$x_u[\mathbf{n}] = \begin{cases} x[\mathbf{L}^{-1}\mathbf{n}] & \text{if } \mathbf{L}^{-1}\mathbf{n} \in R_{\mathfrak{I}} \\ 0 & \text{otherwise} \end{cases}$$

Example



$$\mathbf{R} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Rational Rate Change

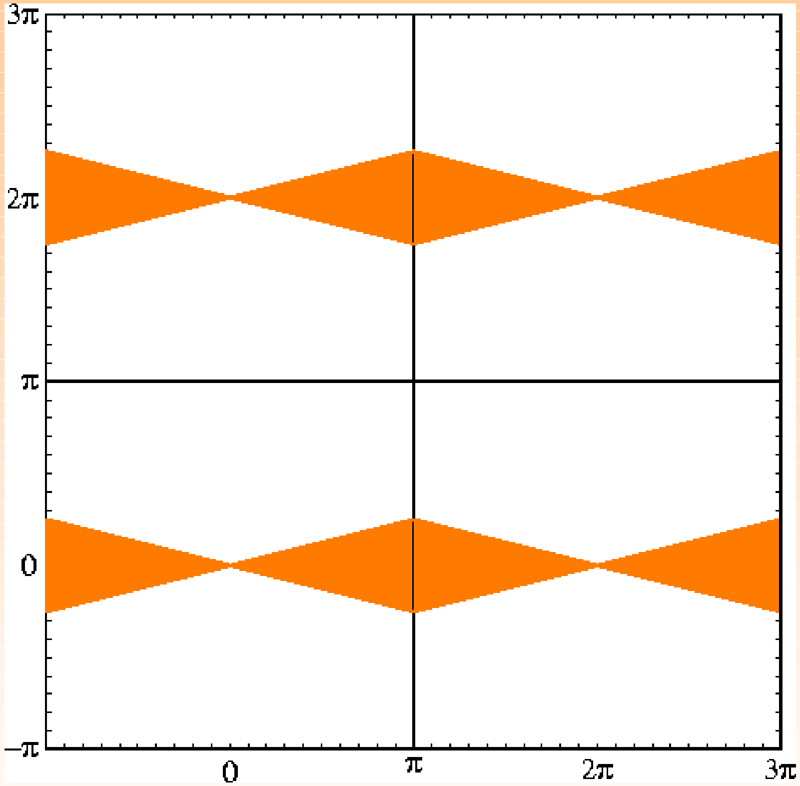
- Rational rate change
  - In one-dimension: $H = L^{-1} M$
  - In multiple dimensions: $\mathbf{H} = \mathbf{L}^{-1} \mathbf{M}$
- Interpolation generalizes
  - Interpolation filter: columns of π $\mathbf{L}^{-1}$ define two adjacent sides of parallelogram passband
- Decimation generalizes
  - Decimation filter: columns of π $\mathbf{M}^{-1}$ define two adjacent sides of parallelogram passband

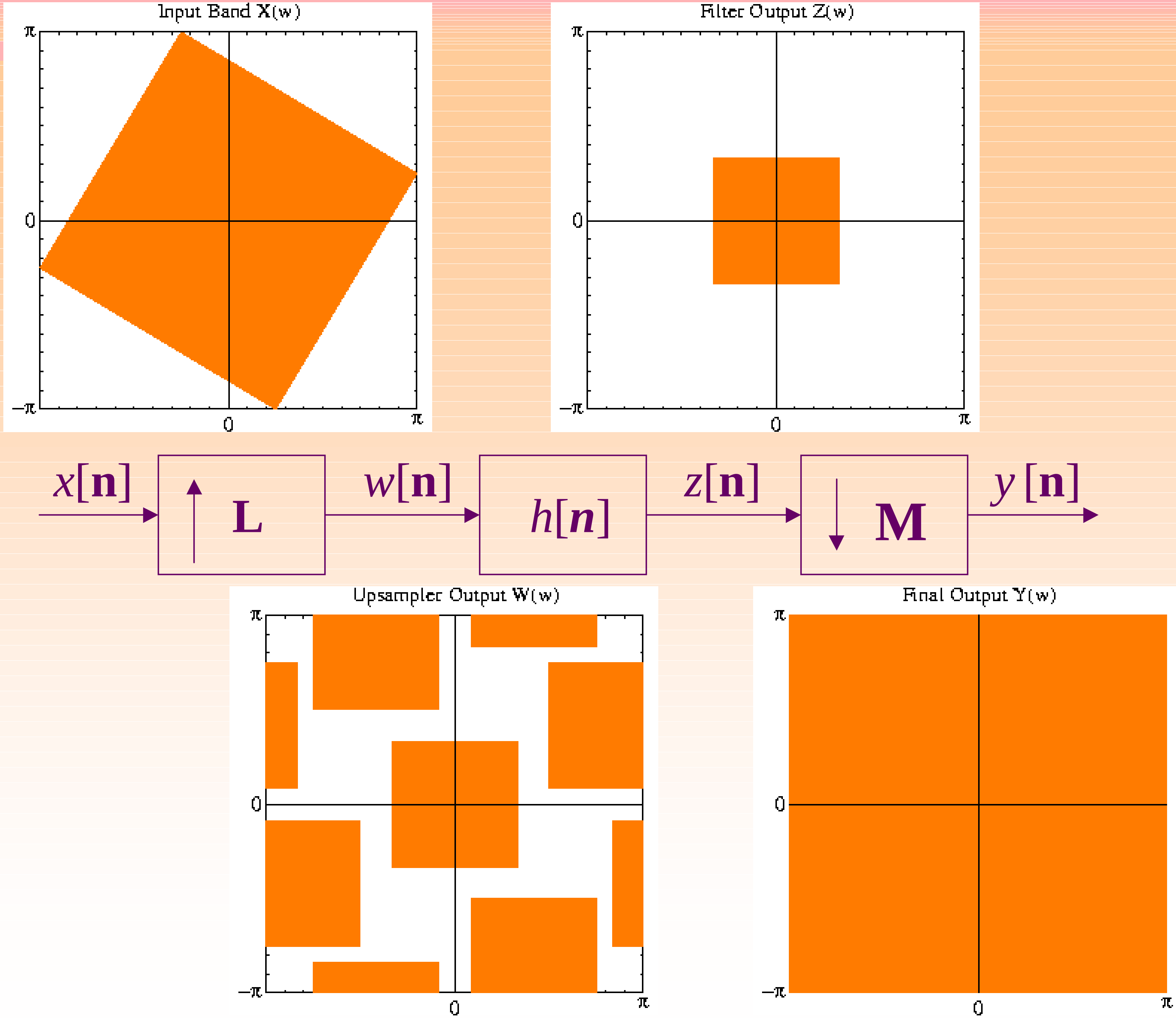
2-D Narrowband Signals

- Image Processing
  - Digitized pictures are often oversampled
- Video Processing
  - Quincunx decimation on NTSC video has little perceptual effect
- Seismic Processing
  - Fan filtering for seismic migration

2-D Narrowband Signals

- Example: Fan Filters
  - Velocity filters for position-time data
  - Periodic extension reveals passbands are either hexagons or parallelograms
- Can be resampled at the Nyquist rate




Input Band X(w)
π
0
−π
0
π
Filter Output Z(w)
π
0
−π
0
π
x[n]
L
w[n]
h[n]
z[n]
M
y [n]
Upsampler Output W(w)
π
0
−π
0
π
Final Output Y(w)
π
0
−π
0
π

2-D Decimation Systems

- Pick vertices of parallelogram to be rational multiples of π [Chen and Vaidyanathan]
- Compute rational matrix **H** from vertices
  - H maps parallelogram onto square fundamental frequency tile

$$-\pi < \omega_1 < \pi \text{ and } -\pi < \omega_2 < \pi$$

  - Using two adjacent parallelogram vertices

$$\mathbf{H}\begin{bmatrix}\mathbf{v}_1 & \mathbf{v}_2\end{bmatrix} = \begin{bmatrix}\pi & \pi \\ \pi & -\pi\end{bmatrix}$$

Factoring Rational Matrix H

- Factor H = L^{-1} M by Smith-McMillan Form of H

$$\mathbf{H} = \mathbf{U}\,\Lambda\,\mathbf{V} = \mathbf{U}\,\Lambda_{\mathbf{L}}^{-1}\,\Lambda_{\mathbf{M}}\,\mathbf{V}$$

$$\mathbf{H} = \left(\Lambda_{\mathbf{L}}\,\mathbf{U}^{-1}\right)^{-1}\left(\Lambda_{\mathbf{M}}\,\mathbf{V}\right) = \mathbf{L}^{-1}\,\mathbf{M}$$

- Enhancements
  - Allow user to sketch a region and circumscribe it with a parallelogram of minimum area [Evans, Teich, Schwarz, 1994]
  - Add a modulator at input to shift center of parallelogram to the origin