UNIVERSITY OF TEXAS AT AUSTIN
Dept. of Electrical and Computer Engineering

Mid-Term #1

Date: October 2, 1996 Course: EE 381K

Name: ____
Last, First

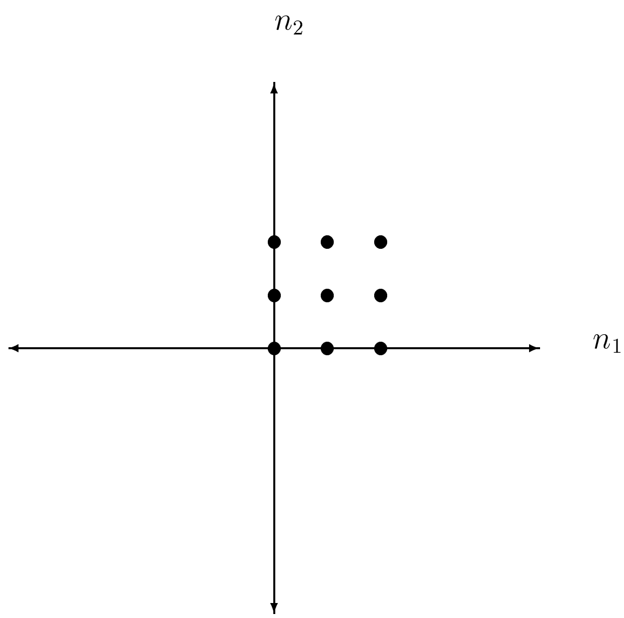
Alias: ____

- The exam will last 90 minutes.
- You may use any books during the exam
- The only notes you may use are those taken for this class, which includes course handouts.
- Calculators are allowed.
- You may use any standalone computer system, i.e., one that is not connected to a network.
- All work should be performed on the midterm itself. If more space is needed, then use the backs of the pages.

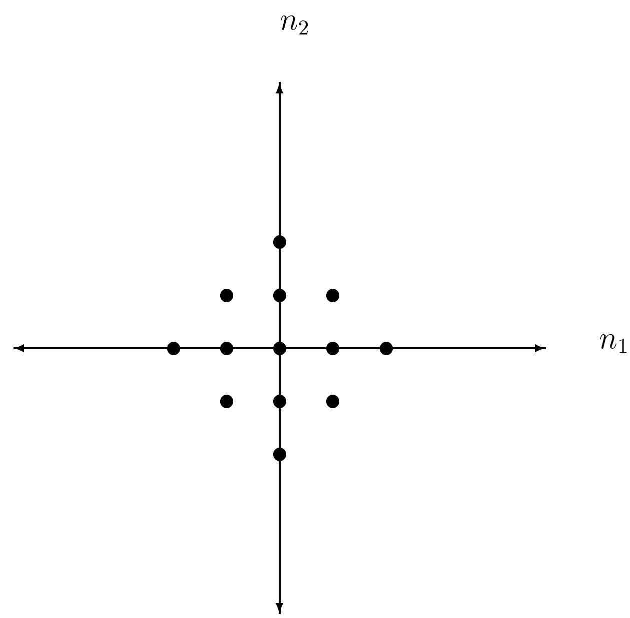
Problem	Point Value	Your Score	Topic
1	15		2-D Convolution
2	15		2-D DTFT
3	20		2-D Sampling
4	20		2-D DFT
5	20		1-D DST
6	10		1-D Spectrum and 2-D DCT
Total	100		

Problem 1.1 Finite-Extent 2-D Convolution. 15 points.

Determine the two-dimensional convolution of the two sequences x[n1, n2] and h[n1, n2] sketched below. Both sequences have sample values equal to 1 at the locations indicated by the black dots.



$x[n_1, n_2]$

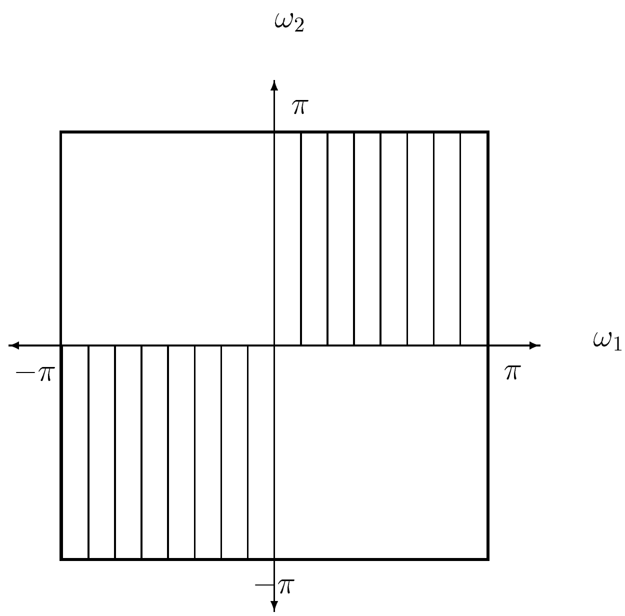


$h[n_1, n_2]$

Problem 1.2 Non-Separable 2-D Discrete-Time Fourier Transform. 15 points.

$H(\omega_1, \omega_2)$ is 1 in the shaded regions in the figure below and 0 in the unshaded regions. Determine $h[n_1, n_2]$.

Hint: You can think of this non-separable frequency response as a modulated version of a separable frequency response.



$H(\omega_1, \omega_2)$

Problem 1.3 Elliptic Passband. 20 points.

A two-dimensional bandlimited signal has the elliptic support

$$H(\omega_1, \omega_2) = \begin{cases} 1 & \text{if } \frac{\Omega_1^2}{a^2} + \frac{\Omega_2^2}{b^2} < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine a sampling matrix V that will allow $h(t_1, t_2)$ to be sampled at a minimum sampling rate without aliasing.

(b) What is the minimum sampling density (in samples per unit area)?

Problem 1.4 Filtering Using the Discrete Fourier Transform. 20 points.

Consider an image $x[n_1, n_2]$ of 512×512 pixels to be filtered by a system with an impulse response $h[n_1, n_2]$. The sequence $h[n_1, n_2]$ is an 11×11 sequence that is zero outside $0 \leq n_1 \leq 10$ and $0 \leq n_2 \leq 10$. Compare the computations

$$y[n_1, n_2] = x[n_1, n_2] ** h[n_1, n_2]$$

$$v[n_1, n_2] = \mathcal{DFT}^{-1}_{k_1,k_2} \left(\mathcal{DFT}_{n_1,n_2}\{x[n_1, n_2]\} \cdot \mathcal{DFT}_{n_1,n_2}\{h[n_1, n_2]\} \right)$$

where the forward DFT and inverse DFT are 512×512 points in size. For what values of (n_1, n_2) does $v[n_1, n_2]$ equal $y[n_1, n_2]$?

Problem 1.5 1-D Discrete Sine Transform. 20 points.

Develop an algorithm to compute the forward discrete sine transform using an appropriate discrete Fourier transform algorithm. The forward discrete sine transform is defined as

$$X_s[k] = \sum_{n=0}^{N-1} 2\,x[n]\,\sin\frac{(2n+1)k\pi}{2N}$$

Problem 1.6 Potpourri.

(a) Computing the power spectrum of a discrete signal by computer. 5 points.

We want to plot the power spectrum of a discrete signal $x[n]$ that is defined for $n = -2, -1, 0, 1, 2,$ The power spectrum is the square of the absolute value of the discrete Fourier transform. How would you compute the power spectrum given that your favorite computer language has a built-in fast Fourier transform (FFT)?

Which DFT coefficient holds the zero frequency (DC) value?

(b) Fast implementations of the discrete cosine transform. 5 points.

1. How did Hyesook's implementation avoid the need of a transform on the input data?

2. To what DFT algorithm is Hyesook's implementation similar?