UNIVERSITY OF TEXAS AT AUSTIN
Dept. of Electrical and Computer Engineering

Mid-Term #2

Date: October 30, 1996 Course: EE 381K

Name: ____
Last, First

Alias: ____

- The exam will last 90 minutes.
- You may use any books during the exam
- The only notes you may use are those taken for this class, which includes course handouts and homework solutions.
- Calculators are allowed.
- You may use any standalone computer system, i.e., one that is not connected to a network.
- All work should be performed on the midterm itself. If more space is needed, then use the backs of the pages.

Problem	Point Value	Your Score	Topic
1	15		2-D IIR Filtering
2	15		3-D IIR Filter Stability
3	15		2-D LSI System
4	15		Constrained 2-D FIR Filter Design
5	20		McClellan Transformation
6	20		Potpouri
Total	100		

Problem 2.1 Two-Dimensional IIR Filtering

A 2-D system is described by the difference equation

$$y[n_1, n_2] - 0.9\,y[n_1, n_2 - 1] + 0.5\,y[n_1 - 1, n_2 - 1] = x[n]$$

Find the impulse response of $y[n_1, n_2]$ for $0 \le n_1 \le 3$ and $0 \le n_2 \le 3$ under the assumption that $y[n_1, n_2] = 0$ for $n_1 < 0$ or $n_2 < 0$. (You can give the answer as numbers plotted on a grid of (n_1, n_2) points as we do for 2-D numeric convolution problems.)

Problem 2.2 Stability of a 3-D IIR Filter. 20 points.

Consider a first-octant 3-D IIR filter with system function

$$H_Z(z_1, z_2, z_3) = \frac{1}{1 - \frac{2}{5}z_1^{-1} - \frac{3}{10}z_2^{-1} + \frac{2}{5}z_3^{-1}}$$

Note that the support of a first-octant 3-D filter is for $n_1 \geq 0$, $n_2 \geq 0$, and $n_3 \geq 0$. Determine whether or not the filter is stable and justify your answer.

Problem 2.3 2-D LSI System. 20 points.

Consider the 2-D linear shift-invariant system described by the two-dimensional difference equation

$$y[n_1, n_2] = a\, y[n_1 + 1, n_2 - 1] + b\, y[n_1 + 1, n_2] + c\, y[n_1, n_2 + 1] + x[n_1, n_2]$$

(a) Since the system is LSI, determine the support of the impulse response $h[n_1, n_2]$.

(b) If $x[n_1, n_2] = u[-n1, -n2]$. determine an appropriate set of initial conditions so that $y[n1, n2]$ can be recursively computed.

Problem 2.4 Constrained 2-D FIR Filter Design. 20 points.

We want to design a 5×5 FIR filter to approximate the ideal frequency response

$$I(\omega_1, \omega_2) = \begin{cases} 1 & \left(\frac{\omega_1}{0.5\pi}\right)^2 + \left(\frac{\omega_2}{0.25\pi}\right)^2 = 1 \\ 0 & \text{otherwise} \end{cases}$$

The support of the filter to be designed is

$$-2 \leq n_1 \leq 2 \quad \text{and} \quad -2 \leq n_2 \leq 2$$

We saw in class that the frequency response can be written in the form

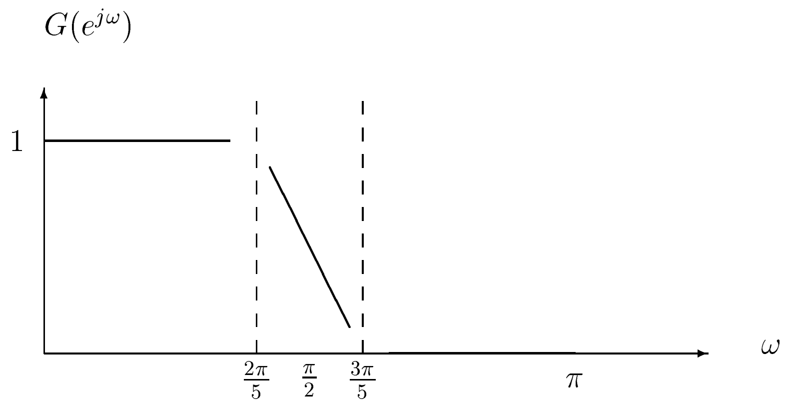
$$H(\omega_1, \omega_2) = \sum_{i=1}^{F} a_i \, \phi_i(\omega_1, \omega_2)$$

(a) If $h[n_1, n_2]$ is to have the same symmetries as $i[n_1, n_2]$, what is F, the number of independent coefficients?

(b) Assuming that $a_i = h[n_1, n_2]$ for some n_1 and n_2, determine the set of basis functions $\{\phi_i(\omega_1, \omega_2)\}$.

Problem 2.5 McClellan Transformation

A McClellan transformation can be used to design zero-phase FIR filters of any dimensionality from a 1-D zero-phase FIR prototype. In particular, it can design one-dimensional filters. Let $G(e^{j\omega})$ be a $(2N+1)$-point zero-phase FIR filter with the frequency response



A 1-D filter with frequency response $H(e^{j\Omega})$ is designed using the transformation

$$\cos\omega \longrightarrow F(\Omega) = \cos 2\Omega = 2\cos^2\Omega - 1$$

(a) Draw and label a curve of Ω vs. ω for this transformation. (This is the one-dimensional equivalent of a topographic map or contour plot that we used in transforming one-dimensional filters to multiple dimensions.)

(b) Draw and label $H(e^{j\Omega})$ for $0 \leq \Omega \leq 0$.

(c) Determine the length of $h[n]$.

Problem 2.6 Potpourri.

(a) 10 points. Back by popular demand is the Discrete Sine Transform $X_s[k]$ of the discrete-time signal $x[n]$:

$$X_s[k] = \sum_{n=0}^{N-1} 2\, x[n] \, \sin\left(\frac{\pi(2n+1)k}{2N}\right)$$

On the last exam, you developed an algorithm for computing the forward Discrete Sine Transform. Give an explicit formula (algorithm) for the inverse Discrete Sine Transform.

(b) 5 points. Dr. Harold Stone's talk was entitled "Fourier-Wavelet Techniques in Image Searching". Give two reasons why his template matching was not robust.

(c) 5 points. Dr. Alan Bovik's talk was entitled "FOVEA: Foveated Vergent Active Stereo System for Dynamic Three-Dimensional Scene Recovery."

i. In stereo vision, two images are taken by cameras placed side-by-side and depth of objects in the scene is computed. Computation of depth is an ill-defined problem in that the depth computation does not have a unique solution. How did the FOVEA System overcome this?

ii. How did the FOVEA System reduce the amount of computation it performed to compute each surface in the scene?