Effects of Steering Delay Quantization in Beamforming

James Fung

Applied Research Laboratories

The University of Texas at Austin

Literature Survey

EE 381K Multi-Dimensional Signal Processing

Abstract

The effect of steering delay quantization on beamformer frequency and azimuth response is not well understood. This project will attempt to analyze the relationship between steering delay quantization, with its corresponding impact on computational complexity, and beamformer response error. Results found here should benefit both conventional beamforming and modern adaptive beamforming applications.

I. Introduction

Through temporal and spatial sampling, it is possible to determine the frequency and spatial characteristics of a signal despite interference. Acoustic applications include the detection, localization, and characterization of objects or acoustic sources, and this branch of signal processing in both the time and space domains is commonly referred to as beamforming.

The most intuitive example of a beamformer is the delay-sum beamformer (Fig. 1). For simplicity, this paper will restrict itself to linear, uniformly spaced arrays of omni-directional sensors. Suppose that we are interested in a signal emanating from a point source in a particular direction with propagation speed c. If the source is distant enough, it is customary to model the signal as a plane wave. The name delay-sum beamformer is derived from the fact that the sensor outputs are delayed so that a wavefront arrives at the summation in phase. The signal then adds coherently while noise and interference signals add incoherently.

$$b(t,\theta) = \sum_{k=1}^{N} x_k(t - \tau_k(\theta)) \tag{1}$$

$$\tau_k(\theta) = k\frac{d\sin\theta}{c} \tag{2}$$

where $b(t,\theta)$ is the beamformer output, $x_k(t)$ are the sensor outputs, and $\tau_k(\theta)$ are the necessary delays to steer the beamformer toward direction of arrival θ.

An azimuthal and frequency beamformer response is shown in Fig. 2. Note that the response has quantifiable mainlobe width and sidelobe level characteristics that can vary considerably with respect to frequency [1]. Much like traditional temporal filters, there is a fundamental trade-off between mainlobe width and sidelobe levels. Weighting coefficients, also called shading coefficients, similar to time-domain windowing coefficients are almost always applied to the sensors to shape the response to specifications. The discussion of shading coefficients lies outside the scope of this paper.

II. Background

To implement each necessary delay using analog circuitry would be cost prohibitive. However, discrete time systems have the problem that simple delays must be integer multiples of the sampling period. With a uniformly spaced linear array, there exists a set of beams, called synchronous beams, which can be exactly formed using these delays. However, if the A/D converters only sample at the Nyquist rate of the signal of interest, these beams are quite sparse and offer little angular resolution.

One solution is to simply choose the time sample that is closest to the desired delay. However, this alters the array pattern in a way that has no simple closed-form solution except in special cases. For example, Fig. 3(a) is the quantized counterpart of the ideal response in Fig. 3(b), where $\tau_k = -kT_s/2$, T_s is the sampling period, and the discrete time delays have been rounded down to the nearest integer delay [2]. Note that even in this simple case, the effect can be quite large. Little has been published on the consequences of time quantization on beamformer response, and this is the problem my project will focus on.

Most beamforming research today has moved away from data-independent beamforming to time-varying algorithms that alter the beamformer according to information extracted from the sensor and beamformer outputs. One large sub-category of this is adaptive beamforming, which varies the weights on the different sensors so that the beamformer converges to some statistical optimum. Such adaptive algorithms lie outside the scope of this project, but they should benefit from the results found here since many of their algorithms rely on well-known data-independent beamforming methods. For example, adaptive beamformers can become unstable, so [3] combines the robust qualities of a filter-and-sum beamformer with adaptability through the control of one parameter, which can be tuned to optimize the filter toward a given criterion. For two relatively modern comparisons of different data-dependent beamforming methods, see [4, 5].

III. Comparison of Conventional Low-Pass Beamformers

Mucci [6] gives a comparison of the major data-independent beamformer algorithms. My project will focus on low-pass applications, so this section will summarize the performance of three low-pass beamformers: delay-sum, interpolation, and DFT. Throughout this section, the variables are defined as follows: N = number of sensors, N_B = number of beams.

Delay-Sum Beamformer. The primary disadvantage of the delay-sum beamformer is that, to avoid serious beamformer response degradation due to steering delay quantization, the A/D converters must sample considerably faster than the Nyquist rate. Not only does this significantly increase the cost of the A/D converters, but it also requires a large amount of memory to store all of these samples. This algorithm requires $N_B N$ MACs per output sampling period. Almost always, the delay-sum beamformer is an inferior choice for a design. However, given its intuitive implementation, it is often used as a benchmark for other algorithms, especially since it is computationally efficient.

Interpolation Beamformer. A detailed description of interpolation beamformers (Fig. 4) is given in Section IV. The sampling frequency of an interpolation filter is often close to the Nyquist rate, much lower than the equivalent delay-sum beamformer, with corresponding savings in the memory required. The algorithm in Fig. 4 requires $N_B N_C$ MACs per output sampling period, where N_C is the length of the FIR interpolation filter. Since beamforming and interpolation are both linear, shift-invariant operations, they can be exchanged. The resulting pre-beamforming interpolation algorithm would require $N_C N$ MACs per output sampling period.

DFT Beamformer. Taking the Fourier transform of (1) and approximating $X_k(\omega)$ with a K-point DFT:

$$B(m,\theta) = \sum_{k=0}^{N-1} X_k[m] e^{-j2\pi m F_i \tau_k(\theta)/K} \tag{3}$$

which can implemented as shown in Fig. 5 using the efficient FFT algorithm. F_i is the sampling frequency of the A/D converters. One advantage of this method is that steering delay quantization is completely avoided, and so F_i can be set to the Nyquist rate. For a linear, uniformly-spaced array, it is possible to take advantage of the linear dependence of (2) with respect to k, and we can form multiple beams using another FFT:

$$B[m, \theta_n] = \sum_{k=1}^{N} X_k[m] e^{-j2\pi kmn/N} \quad (4)$$

$$\theta_n = \sin^{-1}\left(\tfrac{nKc}{F_i N d}\right) \quad (5)$$

The best scenario for computational complexity is approximately $N^2\log(N)$ MACs per output sampling period, which is more efficient the higher N is. However, note that this would require N^2 words of memory to store input data, which is considerable. Also, the θ_n are not uniformly spaced, as may be required, and their positions are determined by other beamformer parameters.

IV. Interpolation Beamformers

The fundamental paper on interpolation beamformers remains [7]. Instead of sampling at the higher frequency necessary for the delay-sum beamformer, the interpolation beamformer's A/D converters can sample close to the Nyquist rate of the signal. Samples to be used in the beamformer are digitally interpolated. This decrease in sampling frequency has a corresponding reduction in memory. The trade-off for decreased demand on the analog components is the added computational burden of the digital interpolators. However, since the zero-padding means the data is sparse, the additional computational complexity is fairly low.

One way to decrease beamformer response errors due to steering delay quantization is to minimize the maximum possible delay quantization error by increasing the interpolation factor and creating a finer partition of the sampling period. However, this proportionally increases the interpolation filter length and computational complexity. This trade-off between beamformer accuracy and computational complexity will be key topic in my investigation.

As stated in Section II, to avoid distortions in beamformer response due to steering delay quantization, a beamformer could be constrained to only use synchronous beams. Increasing angular resolution requires a larger interpolation factor, M, with a directly proportional increase to the length of the interpolation filter. Another version of the interpolation beamformer, using a polyphase decomposition, is shown Fig. 6 [8]. Here, $T_s' = T_s/M$, a single post-upsampling discrete-time increment. H_k' are the DTFTs of $h_k'[n]$, where $h_k'[n] = h[nM + k]$ and $h[n]$ is the FIR interpolation filter. The memory requirement of this algorithm is equal to that of the interpolation beamformer described above, and the computational complexity is $N_B N_C N/M$ MACs per output sampling period. Thus, this implementation outperforms the post-beamforming interpolation and pre-beamforming interpolation structures when $M > N_E$ and $M > N_B$, respectively. In other words, since there is an affine relationship between M and angular resolution, the polyphase decomposition is more efficient if the number of synchronous beams is large compared to the number of sensors.

Another recent variation on the interpolation beamformer is given in [9]. Here, researchers experimented with implementing short delays with analog circuitry and long delays digitally. Initial results are promising, showing improved control of zeros in the beamformer response without increased sampling rates.

V. Digital Interpolation

The general structure for digital interpolation is shown in Fig. 7. To recover the frequency content of the signal after zero-padding, $H(f)$ should be an ideal low-pass filter with digital frequency cutoff at π/M, M being the interpolation factor. Design of such filters is a classic time-domain filtering problem. For data-independent, non-time-varying applications, the interpolation filter is optimized beforehand for a certain performance characteristic, such as filter-length to minimize computational complexity using the Parks-McClellan algorithm.

[10] argues against following the Parks-McClellan algorithm blindly. For short filter lengths, Parks-McClellan filters can have unexpected amplitude and phase errors at the high end of the signal band where the beamforming process is more sensitive to such errors. The paper goes on to describe procedures to minimize beamformer errors at the high end of the spectrum and to minimize interpolation error at a specific frequency.

It is also possible to implement an interpolation beamformer with IIR filters [11]. By using a bank of all-pass fractional delay filters in polyphase configuration to exploit the sparsity of offered by zero-padding, this setup offers computational savings over those using FIR interpolation filters. All-pass filters are used to combat the problem IIR filters have with quantization errors. However, one of the key disadvantages of IIR filters is still present: such a beamformer output cannot have linear phase, and is thus unsuitable for certain applications.

Much of modern research into interpolation deals with the design of fractional delay filters [12, 13]. While these can greatly improve beamforming accuracy, as shown in [9], they suffer from the fact that they are designed for a very specific delay and thus a filter must be stored and processed for each sensor in each beam, greatly increasing computation complexity.

VI. Conclusion

A brief introduction to beamforming and a preliminary investigation of steering delay quantization effects on beamformer response were presented. This project will attempt to analyze the effect of steering delay quantization on the frequency-azimuth response of a general interpolation beamformer. As a yardstick for beamformer accuracy and computational complexity, the delay-sum and DFT beamformers were described. As an important component in the interpolation filter, a survey of research on digital interpolators was also given.

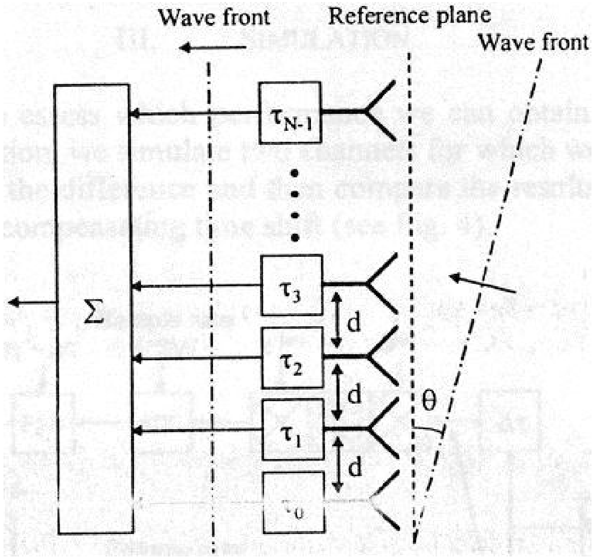


Fig. 1. Delay-sum beamformer. (Fig. 1, [9])

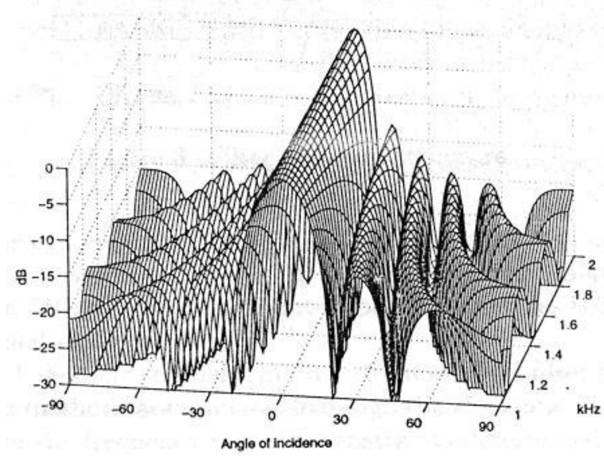
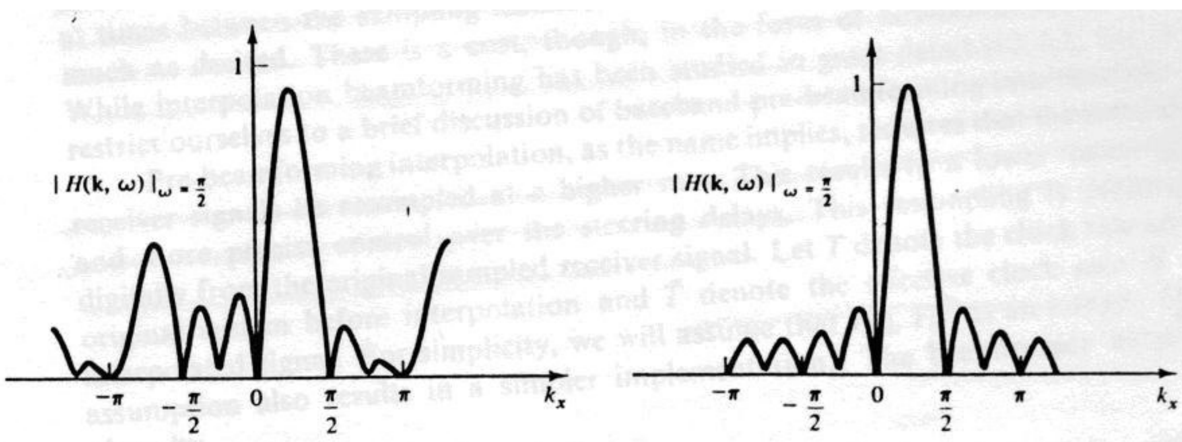


Fig. 2. Frequency and azimuthal response of a simple beamformer (Fig. 1, [1])



(a) (b)

Fig. 3. Example of the effect of steering delay quantization on the wavenumber-frequency of a beamformer. (a) Quantized beamformer. (b) Ideal beamformer. (Fig. 6.9, [2])

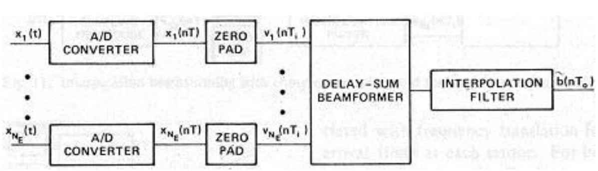


Fig. 4. Interpolation beamformer with post-beamforming interpolation. (Fig. 9, [6])

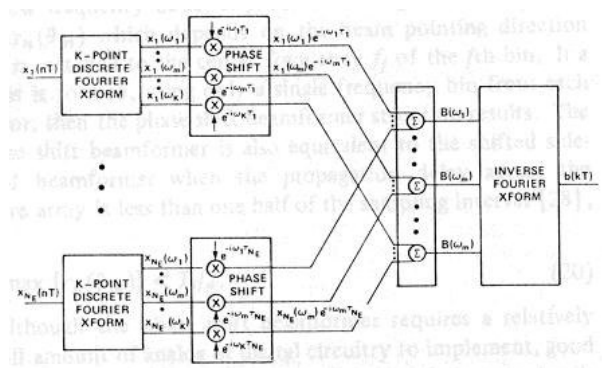


Fig. 5. DFT beamformer. (Fig. 15, [6])

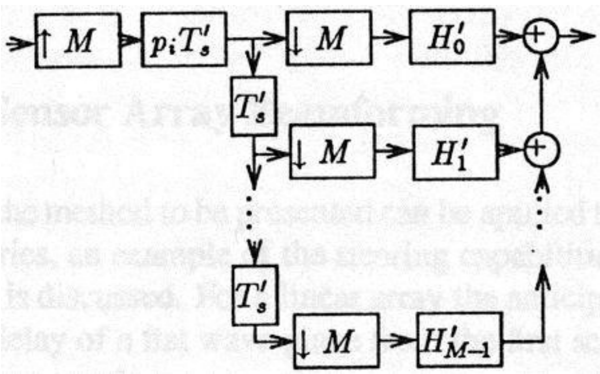


Fig. 6. Polyphase interpolation filter along one sensor. (Fig. 5(c), [8])

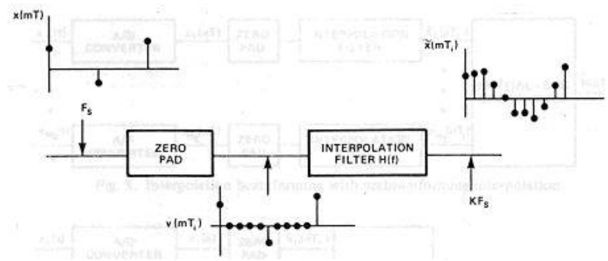


Fig. 7. Digital interpolation. (Fig. 6, [6])

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