Tone Dependent Color Error Diffusion

Literature Survey

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Abstract

Image halftoning converts a high-resolution image to a low-resolution image, e.g. an 8-bit grayscale image to a binary image, for printing and display. Conventional error diffusion halftoning produces worms and other objectionable artifacts. Tone Dependent error diffusion (Li and Allebach) helps reduce these artifacts by controlling diffusion of quantization errors based on the input graylevel value. Allebach *et al.* design error filters weights and thresholds for each (input) graylevel optimized based on a Human Visual System (HVS) model. Color HVS models would significantly impact the design of optimum error filters for (tone-dependent) color error diffusion. A survey of tone dependent grayscale halftoning methods and the linear color vision model used by Evans *et al.* is presented.

I. Introduction

Digital Halftoning is the process of transforming a continuous tone image (grayscale or color) to an image with reduced number of levels so that it can be displayed (or printed) on devices with limited reproduction palettes. Halftoning is more complicated than simply truncating each multi-bit intensity to the lower resolution. Simple truncation would give poor image quality because the quantization error would be spread equally over all spatial frequencies.

Halftoning methods in current use may be categorized as classical screening, search based methods and error diffusion. Screening applies a periodic array of thresholds to each graylevel of the multi-bit image. Pixels are converted to black if they are below the threshold or white otherwise. Classical screening is limited by the fundamental tradeoff between spatial resolution and rendered graylevels. The dither array cell should be as small as possible for increased spatial resolution but as large as possible to reproduce more graylevels. Direct binary search (DBS) [1] produces high quality halftones by iteratively searching for the best binary pattern to match a given grayscale image by minimizing a distortion criterion. The distortion criterion incorporates a linear spatially-invariant model of the human visual system as a weighting function [2]. Due to its implementation complexity however, it is impractical for use as a direct halftoning method in desktop printers. DBS is still implicity used for the purpose of screen design. Importantly, DBS serves as a practical upper bound on achievable halftone quality for other algorithms. Error diffusion [3] generates high quality halftones at an implementation cost greater than that of screening but significantly less than DBS. Computationally, screening amounts to pixel-parallel thresholding, whereas error diffusion requires a neighborhood operation and thresholding. The neighborhood operation distributes the quantization error due to thresholding to the unhalftoned neighbors of the current pixel. The term "error diffusion" refers to the process of diffusing the quantization error along the path of the image scan. Recently, tone dependent error diffusion halftoning algorithms have been developed [5], [6] for grayscale error diffusion. These methods include using error filters with different values for different graylevels in the input image. The quantizer threshold is also modulated based on the input graylevel [5]. In this project, I propose to formulate the design of tone dependent color error diffusion halftoning systems. The aim is to perform an extension of the tone dependent algorithm for grayscale halftoning in [5] to color while incorporating a color HVS model in the error filter design.

Section II summarizes the key ideas in grayscale and color error diffusion halftoning. Section III describes tone dependent error diffusion halftoning for grayscale images. Section IV explains a pattern-color separable HVS model employed in [7],[8] for color error diffusion halftoning. Section V discusses design and implementation issues in tone dependent color error diffusion. Example halftones generated by the methods discussed in the report are presented in section VI. Section VII concludes the report.

II. Background

We use $\mathbf{m}$ to denote a 2-D spatial index (m_1, m_2).

In grayscale halftoning by error diffusion, each grayscale pixel is thresholded to white or black, and the quantization error is fed back, filtered, and added to the neighboring grayscale pixels [3]. The system block diagram shown in Fig. 1, is also known as a noise-shaping feedback coder. In Fig. 1, $x(\mathbf{m})$ denotes the graylevel of the input image at pixel location $\mathbf{m}$, such that $x(\mathbf{m}) \in [-1, 1]$. The output halftone pixel is $b(\mathbf{m})$, where $b(\mathbf{m}) \in \{-1, 1\}$. Here, 1 is interpreted as the absence of a printer dot and -1 is interpreted as the presence of a printer dot. $Q(\cdot)$ denotes the standard thresholding quantizer function given by

$$Q(x) = \begin{cases} +1 & x \geq 0 \\ -1 & x < 0 \end{cases} \tag{1}$$

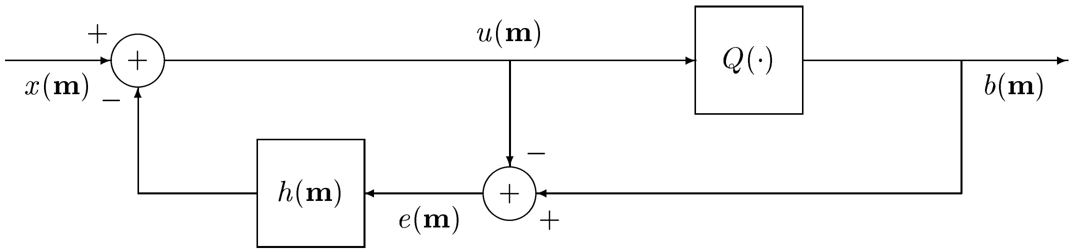

Fig. 1. System block diagram for grayscale error diffusion halftoning where $\mathbf{m}$ represents a two-dimensional spatial index (m_1, m_2) and $h(\mathbf{m})$ is the impulse response of a fixed 2-D nonseparable FIR error filter having scalar-valued coefficients.

The error filter $h(\mathbf{m})$ filters the previous quantization errors $e(\mathbf{m}) \in [-1, 1]$:

$$h(\mathbf{m}) * e(\mathbf{m}) = \sum_{\mathbf{k} \in \mathcal{S}} h(\mathbf{k})\, e(\mathbf{m} - \mathbf{k}) \tag{2}$$

Here, $*$ means linear convolution, and the set $\mathcal{S}$ defines the extent of the error filter coefficient mask. The error filter output is fed back and added to the input. Note that $(0,0) \notin \mathcal{S}$. The mask is causal with respect to the image scan. To ensure that all of the quantization error is diffused, $h(\mathbf{m})$ must satisfy the constraint

$$\sum_{\mathbf{k} \in \mathcal{S}} h(\mathbf{m}) = 1 \tag{3}$$

The quantizer input $u(\mathbf{m})$ and output $b(\mathbf{m})$ are given by

$$u(\mathbf{m}) = x(\mathbf{m}) - h(\mathbf{m}) * e(\mathbf{m}) \tag{4}$$

$$b(\mathbf{m}) = Q(u(\mathbf{m})) \tag{5}$$

Although an error filter is typically lowpass, the feedback arrangement causes the quantization error to be highpass filtered, i.e. pushed into high frequencies where the human eye is least sensitive. The feedback arrangement sharpens the original image by passing low frequencies and amplifying high frequencies. Traditional grayscale error diffused halftones appear sharper than the original and contain highpass noise [10].

Vector Error Diffusion (VED) applies error diffusion in three-dimensional color space. This is generally done in two ways. The first method employs separable filtering in each plane in each color plane while cleverly modifying the quantization process (Vector Quantization) to render the nearest attainable color at each pixel [11]. An alternative framework uses matrix-valued filters (multifilters) [7] to take into account the correlation amongst color planes. A matrix gain model predicts the sharpening and noise shaping characteristics of the color error diffusion scheme.

III. Grayscale Tone Dependent Error Diffusion

Tone-dependent error diffusion methods involve using error diffusion filters with different sizes and values for different graylevels [5], [6]. Optimal error weighting matrix design for selected graylevels based on "blue-noise" spectra was introduced in [12]. The tone dependent error diffusion (TDED) algorithm in [5] is based on the idea that by searching for weights and thresholds to minimize a visual cost function for each graylevel, the algorithm can be made to produce halftone quality similar to that of DBS [1].

For the TDED algorithm in [5], the error filter and threshold matrix, denoted by $h(\mathbf{m})$ and $t[\mathbf{m}; a]$ respectively, are functions of input pixel value a. The binary output $b(\mathbf{m})$, is determined by

$$b(\mathbf{m}) = \begin{cases} +1, & \textbf{if } u(\mathbf{m}) \geq t[\mathbf{m}; x(\mathbf{m})] \\ -1, & \textbf{otherwise.} \end{cases} \tag{6}$$

The quantization error $e(\mathbf{m})$ and the quantizer input $u(\mathbf{m})$ are then computed as in conventional grayscale error diffusion. The threshold matrix used by Li and Allebach [5] is based on a binary DBS pattern for the input graylevel 0.5:

$$t[\mathbf{m}; a] = \begin{cases} t_u(a) & \text{if } p[\mathbf{m}, 0.5] = 0 \\ t_l(a) & \text{otherwise.} \end{cases} \tag{7}$$

where $t_u(a)$ and $t_l(a)$ are tone dependent parameters satisfying $t_u(a) \geq t_l(a)$. The function $p[\mathbf{m}, \mathbf{0.5}]$ is a halftone pattern generated by DBS that represents a constant patch with graylevel 0.5. By substituting (7) into (6), the thresholding process can be represented by

$$b(\mathbf{m}) = \begin{cases} +1 & \text{if } u(\mathbf{m}) \geq t_u(x(\mathbf{m})) \\ -1 & \text{if } u(\mathbf{m}) < t_l(x(\mathbf{m})) \\ p[\mathbf{m}, 0.5] & \text{otherwise.} \end{cases} \tag{8}$$

For the error filter design, the authors choose the magnitude of the DFT of the DBS pattern as an objective spectrum for the halftone pattern for input graylevel values in the midtones (21-235). For the highlight and shadow regions (graylevel values in 0-20 and 234-255) the objective spectrum is the DFT of the graylevel patch. Let $B^{DBS}(k,l)$ and $B^{TDED}(k,l)$ denote the DFT of the DBS and the tone dependent error diffusion patterns, respectively. The goal is then to search for the tone dependent parameter vector $v = (t_u(a), t_l(a), h(\mathbf{m}; a))$ that minimizes

$$J = \sum_k \sum_l (|B^{TDED}(k,l)|^2 - |B^{DBS}(k,l)|^2) \tag{9}$$

subject to the constraints

$$t_u(a) + t_l(a) = 1 \tag{10}$$

$$t_u(a) \geq t_l(a) \tag{11}$$

$$\sum_{\mathbf{k} \in \mathcal{S}} h(\mathbf{k}; a) = 1 \tag{12}$$

$$h(\mathbf{k}; a) \geq 0 \quad \forall \quad \mathbf{k} \in \mathcal{S} \tag{13}$$

A serpentine scan is used in the design. Note that $B^{DBS}(k,l)$ would be replaced by $X(k,l)$ (the DFT of the input graylevel patch) for the highlight and shadow regions. The algorithm to search for the optimum tone dependent parameter vector v_{opt} is described in [5].

Since each graylevel requires an error filter, TDED entails an increase in memory. The number of distinct error filters is reduced to half though based on symmetry conditions i.e. $h(\mathbf{k}; a) = h(\mathbf{k}; 1-a)$. The use of a serpentine scan in TDED also limits parallelism, since it can only be implemented as a serial process. In [5] the authors use a *2-row serpentine* scan, where they process two consecutive rows in one direction and the next two rows in the opposite direction. The 2-row serpentine scan overcomes diagonal worms seen with the raster scan and is more parallelizable.

IV. Linear Color Vision Model

Damera-Venkata and Evans [7] employ a linear color model based on the pattern color separable model by Wandell *et al.* [14], [15]. The linear color model consists of (1) a linear transformation $\tilde{\mathbf{T}}$, and (2) separable spatial filtering on each channel. Each channel uses a different spatial filter. The filtering in the z-domain is a matrix multiplication by a diagonal matrix $\mathbf{D}(\mathbf{z})$. In the spatial domain, the linear HVS model $\tilde{\mathbf{v}}(\mathbf{m})$ is computed as

$$\tilde{\mathbf{v}}(\mathbf{m}) = \tilde{\mathbf{d}}(\mathbf{m})\tilde{\mathbf{T}} \tag{14}$$

A complete HVS model is uniquely determined by the color space transformation and associated spatial filters. Monga, Geisler and Evans [8] evaluate popular color spaces in halftoning applications to find the best HVS model for color error diffusion. Using both objective as well as subjective measures, they identify the transformation to the Linearized CIELab color space [17] as perceptually most accurate. The linearized CIELab color space is obtained by linearizing the CIELab space about the D65 white point [17] in the following manner:

$$Y_y = 116\,\frac{Y}{Y_n} - 16 \tag{15}$$

$$C_x = 500\left[\frac{X}{X_n} - \frac{Y}{Y_n}\right] \tag{16}$$

$$C_z = 200\left[\frac{Y}{Y_n} - \frac{Z}{Z_n}\right] \tag{17}$$

Hence $\tilde{\mathbf{T}}$ (based on a sRGB monitor) is sRGB $\longrightarrow$ CIEXYZ $\longrightarrow$ Linearized CIELab. The $\mathbf{Y}_y$ component is similar to the luminance and the $\mathbf{C}_x$ and $\mathbf{C}_z$ components are similar to the R-G and B-Y opponent color components. The original transformation to the CIELab from CIEXYZ is a non-linear one

$$L^* = 116f\left(\frac{Y}{Y_n}\right) - 16 \tag{18}$$

$$a^* = 500\left[f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right] \tag{19}$$

$$b^* = 200\left[f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right] \tag{20}$$

where

$$f(x) = \begin{cases} 7.787x + \frac{16}{116} & \text{if } 0 \leq x \leq 0.008856 \\ x^{1/3} & \text{if } 0.008856 < x \leq 1 \end{cases}$$

The values for X_n, Y_n and Z_n are as per the D65 white point [18].

The nonlinearity in the transformation from CIELab distorts the spatially averaged tone of the images, which yields halftones that have incorrect average values [17]. The linearized color space overcomes this, and has the added benefit that it decouples the effect of incremental changes in (Y_y, C_x, C_z) at the white point on (L, a, b) values:

$$\nabla_{(Y_y, C_x, C_z)}(L^*, a^*, b^*)|_{D65} = \frac{1}{3}\mathbf{I} \tag{21}$$

The spatial filters operate more aggresively on the luminance channel and are based on the luminance frequency response by Nasanen and Sullivan [19] and the chrominance frequency response by Kolpatzik and Bouman in [20].

V. Tone Dependent Color Error Diffusion

The extension of TDED to color is non-trivial. There are two alternatives: 1) using separable filters for each color plane or 2.) use matrix valued filters for capturing the correlation amongst color planes as in [7].

In either case the key issue is the design of the error filters for each RGB triplet in the input image. The color HVS model significantly impacts the design. In my approach, I intend to use the color vision model based on the transformation to the Linearized CIELab space [8] as discussed in Section IV with the key difference that the visual filters would be applied in the FFT (frequency) domain after taking the FFT of the transformed image. I propose a separable design for each color plane by considering RGB triplets on the diagonal line through the color cube i.e. $(R, G, B) = (0,0,0), (1,1,1)...(255,255,255)$. The error filters would be trained so as to minimize the visually weighted squared error between the magnitude spectra of the halftone image and the spectra of the constant color patch for a given RGB triplet.

VI. Simulation Results

The design of the error filter is the key to high quality error diffusion halftoning methods. The Floyd-Steinberg error filter was designed by trial-and-error to give four dyadic taps. Jarvis [21] and Stucki [22] proposed dyadic 12-tap error filters to reduce worms. The resulting halftones are however significantly sharper. Fig. 2 shows an example of Floyd-Steinberg and Tone Dependent error diffused halftones of the *barbara* image. For comparison purposes, a direct binary search halftone is also shown in Fig. 2. Visual inspection confirms a better dot distribution for the tone dependent error diffused halftone. Worms are broken by diffusing the quantization errors to a wider area at extreme gray levels [5]. The halftone in Fig. 2(c) is however not sharpened too much as is the case when using the Jarvis or Stucki error filters.

VII. Conclusion

Tone dependent error diffusion methods for grayscale halftoning are shown to produce halftone quality comparable to DBS by threshold modulation and by using error filters dependent on the input graylevel value. Future work involves the design and analysis of tone dependent color error diffusion halftoning systems. Tone dependent color quantization could also be considered. The design of optimum (in visual quality) separable tone dependent error filters for each color plane, is the focus of this project.

References

[1] M. Analoui and J. Allebach, "Model based halftoning using direct binary search," *Proc. SPIE Human Vision, Visual Processing, and Digital Display III*, vol. 1666, pp. 109–121, Feb. 1992.

[2] S. H. Kim and J. P. Allebach, "Impact of human visual system models on model based halftoning," *IEEE Trans. on Image Processing*, vol. 11, no. 3, pp. 258–269, Mar. 2002.

[3] R. Floyd and L. Steinberg, "An adaptive algorithm for spatial grayscale," *Proc. Soc. Image Display*, vol. 17, no. 2, pp. 75–77, 1976.

[4] D. Anastassiou, "Error diffusion coding for A/D conversion," *IEEE Trans. on Circuits and Systems*, vol. 36, no. 9, pp. 1175–1186, Sept. 1989.

[5] P. Li and J. P. Allebach, "Tone dependent error diffusion," *SPIE Color Imaging: Device Independent Color, Color Hardcopy, and Applications VII*, vol. 4663, pp. 310–321, Jan. 2002.

[6] R. Eschbach, "Reduction of artifacts in error diffusion by means of input-dependent weights," *J. Electronic Imaging*, vol. 2, no. 4, pp. 352–358, Oct. 1993.

[7] N. Damera-Venkata and B. L. Evans, "Design and analysis of vector color error diffusion halftoning systems," *IEEE Trans. on Image Processing*, vol. 10, no. 10, pp. 1552–1565, Oct. 2001.

[8] V. Monga, W. S. Geisler, and B. L. Evans, "Linear, color separable, human visual system models for vector error diffusion halftoning," *IEEE Signal Processing Letters*, vol. accepted for publication, 2003.

[9] J. Mannos and D. Sakrison, "The effects of a visual fidelity criterion on the encoding of images," *IEEE Trans. on Info. Theory*, vol. 20, no. 4, pp. 525–536, July 1974.

[10] R. Ulichney, *Digital Halftoning*, MIT Press, Cambridge, MA, 1987.

[11] H. Haneishi, H. Yaguchi, and Y. Miyake, "A new method of color reproduction in digital halftone image," in *Proc. IS&T Conf. 47th Annual Conf.*, Cambridge, MA, May 1993.

[12] V. Ostromoukhov, "A simple and efficient error-diffusion algorithm," *Proc. Int. Conf. on Computer Graphics and Interative Tech.*, Aug. 2001.

[13] H. R. Kang, *Digital Color Halftoning*, SPIE Optical Engineering Press, 1999.

[14] A. B. Poirson and B. A. Wandell, "Appearance of colored patterns: Pattern-color separability," *J. Opt. Soc. Am. A*, vol. 10, no. 12, pp. 2458–2470, Dec. 1993.

[15] X. Zhang and B. A. Wandell, "A spatial extension of CIELAB for digital color image reproduction," *SID Digest of Technical Papers*, pp. 731–734, 1996.

[16] M. D. Fairchild, *Color Appearance Models*, Addison-Wesley, 1998.

[17] T. J. Flohr, B. W. Kolpatzik, R. Balasubramanian, D. A. Carrara, C. A. Bouman, and J. P. Allebach, "Model based color image quantization," *Proc. SPIE Human Vision, Visual Proc. and Digital Display IV*, vol. 1913, pp. 270–281, 1993.

[18] "What is sRGB? - Introduction to the standard default RGB color space developed by Hewlett-Packard and Microsoft," http://www.srgb.com/aboutsrgb.html, 1999.

[19] J. Sullivan, L. Ray, and R. Miller, "Design of minimum visual modulation halftone patterns," *IEEE Trans. on Systems, Man, and Cybernetics*, vol. 21, no. 1, pp. 33–38, Jan. 1991.

[20] B. Kolpatzik and C. Bouman, "Optimized error diffusion for high quality image display," *Journal of Electronic Imaging*, vol. 1, pp. 277–292, Jan. 1992.

[21] J. Jarvis, C. Judice, and W. Ninke, "A survey of techniques for the display of continuous tone pictures on bilevel displays," *Computer Graphics and Image Processing*, vol. 5, pp. 13–40, May 1976.

[22] P. Stucki, "MECCA—a multiple-error correcting computation algorithm for bilevel hardcopy reproduction," Research Report RZ1060, IBM Research Laboratory, Zurich, Switzerland, 1981.

(a) Original Barbara image (b) Floyd-Steinberg error diffusion

(c) Tone Dependent Error Diffusion (d) Direct binary search

Fig. 2. Comparison of classical Floyd-Steinberg and Tone Dependent error diffusion methods with the iterative direct binary search method.