Broadband Beamforming

Project Report

Multidimensional Digital Signal Processing

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Abstract

Broadband wireless channels, where high data rates are transmitted, are extremely dispersive in nature. A fundamental challenge in the design of equalizers for the broadband case lies in reducing complexity. Broadband finite impulse response (FIR) beamformers employ a space-time antenna array which reduces the multipath delay spread to narrowband levels. The beamformers should additionally preserve the whiteness of the channel noise at the beamformer output to allow for the application of trellis based equalizers. The power complementarity property has been used to address this issue in the literature. Techniques to design FIR filters that preserve the whiteness of the channel noise when the received signal is oversampled are studied in this project.

1 Introduction

Over the last decade, we have witnessed an explosive growth in cellular communications and the Internet. These trends indicate a strong potential in the future for mobile broadband wireless data communications. A key obstacle to reliable wireless communications is frequency-selective multipath channels. For narrowband channels, trellis based decoding represents an effective method to combat intersymbol interference (ISI) in frequency-selective multipath channels [1]. However, in the broadband case, multipath dispersion is quite severe and results in the channel memory increasing linearly with the data rate. Since the size of the trellis grows exponentially with the channel memory, the direct application of trellis based decoding algorithms becomes unfeasible due to their high complexity. Techniques to overcome this effect include channel shortening equalizers and other equalization techniques that are not trellis based. Co-channel interference (CCI) from adjacent users is a serious issue in cellular systems and interference cancellation is another important factor in equalizer design.

2 Background

Several approaches have been adopted to reduce equalizer complexity without sacrificing too much in performance in terms of ISI mitigation. Multiple Input Multiple Output (MIMO) systems use space-time antenna arrays at both the transmit and receive ends to enhance diversity. A known drawback of symbol-spaced equalizers is that they are highly sensitive to the phase of the sampling at the receiver [1, 2]. *Fractionally spaced equalizers*, where the equalizer taps are placed closer together in time than the symbol interval are used to overcome this effect. Fractionally-spaced equalizers have been shown to be effective in equalizing MIMO channels [3] and can be designed using the theory of biorthogonal partners [4]. Design of equalizers for MIMO channels is discussed in detail in [3, 5].

Adaptive *frequency-domain* equalizers for broadband wireless communications have been proposed in [6]. Frequency-domain equalizers exhibit linear complexity growth with increase in channel memory and are well-suited for broadband channels. A feasible alternative is hence to use an adaptive equalizer that operates in the spatial-frequency domain and uses either least mean square (LMS) or recursive least squares (RLS) adaptive processing [6]. Reduced-complexity techniques for broadband wireless channels have also been investigated [7]. Methods to allow the receiver to find burst and symbol timing and a modified decision-feedback equalizer structure are proposed.

Another approach that has been considered is to employ a broadband *beamformer* followed by a finite impulse response (FIR) filterbank as the front end of a communications receiver followed by a *maximum a posteriori* (MAP) sequence detector as part of the back end [8]. Trellis based decoders are based on the maximum likelihood sequence estimation (MLSE) criterion and are optimum from a probability of error viewpoint [1]. However, the application of MLSE algorithms becomes unfeasible in the broadband case due to their high complexity. The ISI can be reduced to narrowband levels by using a broadband beamformer where the antenna array observations are processed by an FIR filterbank [9]. Optimal MAP equalization is then performed at the receiver output. The FIR filter coefficients are chosen to minimize interference [10]. However, the noise at the output of such a receiver is colored and hence, the resultant signal cannot be applied to a trellis-based equalizer.

Space-time receivers can be designed to preserve the whiteness of the channel noise while reducing ISI [8]. To ensure that the noise at the beamformer output remains white, the filterbank is required to have the *power complementarity* property [11]. An N-channel FIR filterbank $\{W_1(z), W_2(z)\ldots, W_N(z)\}$ is said to be power complementary if

$$\sum_{i=1}^{N} W_i(z)\tilde{W}_i(z) = 1 \tag{1}$$

The tilde on transfer functions stands for complex conjugation followed by reciprocation of functional argument, i.e., $\tilde{W}(z) = W^*(z^{-1})$. Design procedures for beamformers with the power complementarity constraint have been proposed [8]. This design assumes that the noise at the input of the beamformer filterbank is white. However, oversampling at the pulse shaping receive filter colors the noise and this coloring has to be incorporated into the power complementarity constraint. In this paper, filters are designed taking into account this coloring of the noise.

This paper is organized as follows. Section 3 describes the signal model and the beamforming optimization problem. Section 4 presents simulation results and compares this design to previous design methods. Finally, in section 5, conclusions and future work are presented.

3 Problem Formulation

We consider a digital communication system where a symbol sequence is transmitted using a pulse shaping waveform $f(t)$. The modulated signal has the complex baseband representation given by

$$s(t) = \sum_{m} f(t - mT)x_m \tag{2}$$

where T is the symbol period. This signal is passed through a frequency-selective wireless channel that is modeled by an L-ray complex impulse response given by

$$g(t) = \sum_{l=1}^{L} a_l \delta(t - \tau_l) \tag{3}$$

where a_l denotes the complex reflection coefficient specifying the amplitude and phase of the lth ray and τ_l represents the associated time delay. We assume that the channel is an Additive White Gaussian Noise (AWGN) channel so that the signal at the input of the antenna element is given by

$$u(t) = \sum_{m} \sum_{l=1}^{L} a_l f(t - mT - \tau_l)x_m + \nu_i(t)$$

where the additive noises $\nu_i(t)$ are independent with 0 mean.

The corresponding multipath signal is then received by an N-element evenly spaced linear antenna array, where the first element is used as the reference point for all observations. We assume that the spacing between antenna elements is d and that the lth multipath signal impinges on the antenna array at an angle θ_l measured with respect to the normal to the array. Assuming that a receive filter with impulse response $r(t)$ is used in each antenna element and the resulting waveform is sampled with period T_s, the sampled noisy observation at the output of the ith antenna element can be expressed as

$$z_i(nT_s) = \sum_{i=1}^{L} h_i(nT_s - mT)x_m + v_i(nT_s) \tag{4}$$

for $1 \leq i \leq N$. If $p(t) = r(t) * f(t)$ denotes the pulse obtained by convolving the transmit and receive filter impulse responses, then

$$h_i(nT_s) = \sum_{i=1}^{L} a_l e^{-j(i-1)\phi_l} p(nT_s - \tau_l)$$

represents the discrete-time channel impulse response seen by the ith antenna element, where

$$\phi_l = 2\pi \frac{dsin(\theta_l)}{\lambda}$$

is the inter-antenna phase factor for the lth multipath component and λ denotes the carrier wavelength.

The noise term is given by

$$v_i(nT_s) = \int_{-\infty}^{\infty} \nu_i(t) r(nT_s - t)dt \tag{5}$$

The beamformers have channel shortening as their main goal and this can best be accomplished by sampling the impulse responses h_i of the antenna elements above the Nyquist rate. Selecting the sampling period T_s as an integer fraction of T allows the application of a fractionally spaced equalizer to the beamformer output. Although beamforming and sampling at the baud rate T is possible, it is more susceptible to timing phase errors. The structure of the beamformer is shown in Fig.1.

For each i, $1 \leq i \leq N$, the beamformer applies an FIR filter $W_i(z)$ to the sequence $z_i(nT_s)$ and then combines the resulting output to generate the observation sequence $y(nT_s)$. We assume that the order of the filters $W_{i)}(z)$ is M. Denoting the coefficients of $W_i(z)$ as $w_i^*(n)$, the beamformer output is given

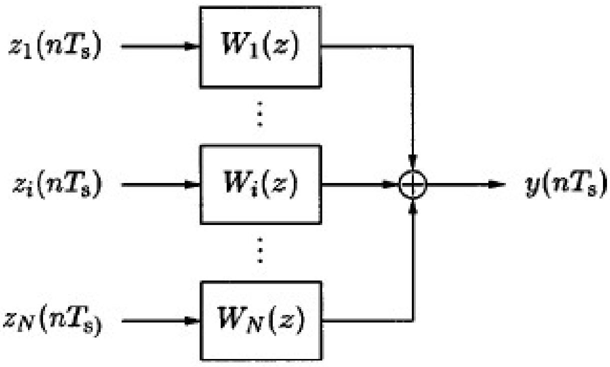


Figure 1: Broadband Beamformer [1].

by

$$\begin{aligned} y(nT_s) &= \sum_{i=1}^{n} w_i^*(n) * z_i(nT_s) = \sum_{i=1}^{N}\sum_{p=0}^{M} w_i^*(p) z_i((n-p)T_s) \\ &= \sum_{m} h(nT_s - mT)x_m + v(nT_s) \end{aligned}$$

where

$$h(nT_s) = \sum_{i=1}^{N}\sum_{p=o}^{M} w_i^*(p) h_i((n-p)T_s), \quad v(nT_s) = \sum_{i=1}^{N}\sum_{p=o}^{M} w_i^*(p) v_i((n-p)T_s)$$

represent the composite channel impulse response and measurement noise generated by the space-time beamformer.

The filters $W_i(z)$ must be selected such that the composite noise $v(nT_s)$ remains white so that the output can be processed by a trellis based decoder. Let us now look at the statistical properties of the noise $v_i(nT_s)$. The autocorrelation of the noise sequence is given by

$$\begin{aligned} C_v(n,m) &= E(v_i(nT_s)v_i^*(mT_s)) \\ &= \int_{-\infty}^{\infty} \nu_i(t) r(nT_s - t)dt \int_{-\infty}^{\infty} \nu_i^*(t') r^*(mT_s - t')dt' \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \nu_i(t)\nu_i^*(t') r(nT_s - t) r^*(mT_s - t') dt dt' \\ &= \sigma^2 \int_{-\infty}^{\infty} r(t) r^*(t - (n-m)T_s) dt \end{aligned}$$

where σ^2 is the variance of the noise sequence obtained by projecting $\nu(t)$ onto a set of complete orthonormal basis functions used to represent the received signal. At symbol spacing, the autocorrelation function of the receive filter is $\delta(n-m)$ and the noise sequences $v_i(nT_s)$ are also white. However, when we sample at a rate greater than the symbol spacing, $v_i(nT_s)$ is colored. Hence, for the noise at the output of the beamformer to be white, we require the filters to satisfy an altered power complementarity property of the form

$$R(z)\sum_{i=1}^{N} W_i(z)\tilde{W}_i(z) = 1 \tag{6}$$

where $R(z)$ is the z-transform of $C_v((n-m)T_s)$. Under this constraint, the coefficients of the filter can be chosen in a number of ways to shorten the effective channel impulse response. The mean squared error between the transmitted signal and the beamformer output is used as the objective function here. If

$$\mathbf{w} = \left[\begin{array}{c} w_1(0)\ldots w_1(M)\ldots w_N(0)\ldots w_N(M) \end{array}\right]',$$

$$\mathbf{z(n)} = \left[\begin{array}{c} z_1(nT_s)\ldots z_1((n-M)T_s)\ldots z_N(nT_s)\ldots z_N((n-M)T_s) \end{array}\right]'$$

denote the vector formed by the complex conjugates of the beamformer coefficients and the vector of observations employed by the beamformer at time n respectively, then

$$y(nT_s) = \mathbf{w}^H\mathbf{z}(n)$$

The beamformer error is given by

$$\tilde{s}(nT_s) = s(nT_s) - \mathbf{w}^H\mathbf{z}(n)$$

If the joint second order statistics of $s(nT_s)$ and $\mathbf{z}(n)$ are denoted as

$$E\left[\left[\begin{array}{c} s(nT_s) \\ (z(n) \end{array}\right]\left[\begin{array}{cc} s^*(nT_s) & \mathbf{z}^H(n) \end{array}\right]\right] = \left[\begin{array}{cc} r_s & \mathbf{r}_{zs}^H \\ \mathbf{r}_{zs} & \mathbf{R}_z \end{array}\right]$$

the beamformer error can be expressed as

$$J(\mathbf{w}) = E\left[|\tilde{s}(nT_s)|^2\right] = ((\mathbf{w}-\mathbf{a})^H\mathbf{R}_z(\mathbf{w}-\mathbf{a}) + b \tag{7}$$

where $\mathbf{a} = \mathbf{R}_z^{-1}\mathbf{r}_{zs}$ and $b = r_s - \mathbf{r}_{zs}^H\mathbf{R}_z^{-1}\mathbf{r}_{zs}$. The power complementarity constraint (12) can be expressed in the Discrete Fourier Transform (DFT) domain by taking the $2M+1$ point DFT to implement the linear convolution as a cyclic convolution. Hence,

$$R(k)\sum_{i=1}^{N} W_i(k)\tilde{W}_k(z) = 1, \quad 0 \leq k \leq 2M \tag{8}$$

where by symmetry, only the first $M+1$ values have to be considered. This constraint can be expressed in vector form as

$$c_k(\mathbf{w}) = \mathbf{R}(\mathbf{k})(\mathbf{w}^H \mathbf{C}_k \mathbf{w}) - 1 = 0, \quad 0 \leq k \leq M$$

where

$$\mathbf{C}_k = \mathbf{I}_N \otimes \Omega_k$$

and $\otimes$ denotes the Kronecker product of two matrices. Ω_k is a Toeplitz matrix with entries

$$\Omega_k(l,m) = e^{-j2\pi k(l-m)/(2M+1)}$$

By construction, the matrices $\mathbf{C}_k$ are non-negative definite for all values of k so that the beamforming problem reduces to the minimization of a positive definite quadratic objective function under non-negative definite quadratic constraints, whose domain is not convex. The Lagrangian associated with the minimization of (15) under (17) can be expressed as

$$L(\mathbf{w},\lambda) = J(\mathbf{w}) + \lambda^T \mathbf{c}(w) \tag{9}$$

where $\lambda = [\lambda_0 \quad \lambda_1 \ldots \lambda_M]^T$ represents the vector of Lagrange multipliers and $c(\mathbf{w}) = [c_0(\mathbf{w}) \quad c_1(\mathbf{w}) \ldots c_M(\mathbf{w})]^T$. $L(\mathbf{w},\lambda)$ with lambda fixed is minimized when $\bigtriangledown_{\mathbf{w}}(\mathbf{w},\lambda) = 0$ which gives

$$\mathbf{w}_{opt}(\lambda) = \left(\mathbf{R} + \sum_{k=0}^{M} \lambda_k \mathbf{C}_k\right)^{-1} \mathbf{Ra}$$

The dual function is hence given by

$$\begin{aligned} G(\lambda) &= L(\mathbf{w}_{opt}(\lambda),\lambda) \\ &= -\mathbf{a}^H\mathbf{R}\left(\mathbf{R} + \sum_{k=0}^{M} \lambda_k \mathbf{C}_k\right)^{-1} \mathbf{Ra} - \sum_{k=0}^{M} \frac{\lambda_k}{\mathbf{R}(K)} + d \end{aligned}$$

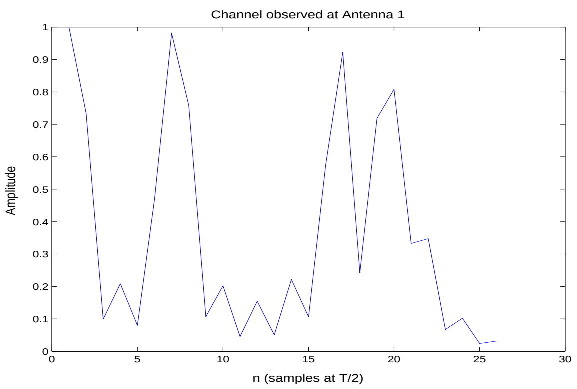
where d represents the term obtained by regrouping the constants. Its domain is given by

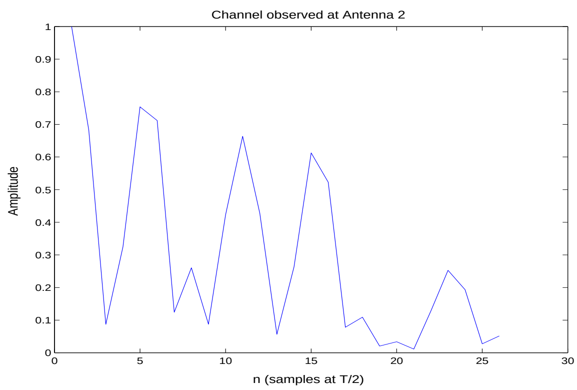
$$D = \left(\lambda \in \mathbb{R}^{M+1} : \mathbf{R} + \sum_{k=0}^{M} \lambda_k \mathbf{C}_k > 0 \right)$$

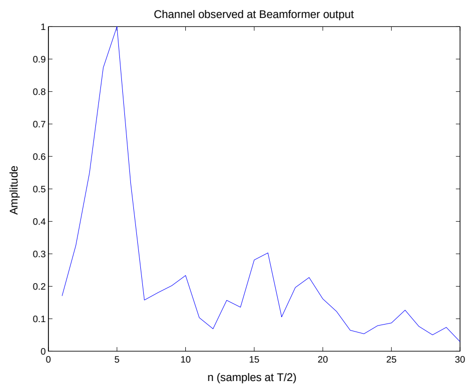
Thus, the unconstrained minimization of $-G(\lambda)$ over D, which is convex, gives the optimal solution to the dual problem. Since the dual problem is unconstrained, the minimum is unique and can be determined by standard Newton or gradient methods.

4 Simulation Results

Transmit and receive filters whose combined response has a raised cosine spectrum with roll-off factor of 0.2 were used. The stationary broadband channel was simulated according to (6). The maximum multipath was chosen to be $12T$ and $T_s = T/2$. Results for a 2 branch beamformer are shown in Fig.2.







The channel is seen to be effectively shortened to about 3 baud intervals. This design was seen to reduce the variance of the noise at the beamformer output to 0.092, while the design proposed in [8] had an output noise variance of 0.7959.

5 Conclusion

The design of power complementary broadband beamformers with oversampling at the receiver was examined in this paper. Beamformers of this type seek to shorten the effective channel impulse response while preserving the whiteness of the additive channel noise. A Lagrangian approach is used to obtain an approximate solution to the problem. This paper assumes that the signal is received in the presence of additive noise. Future work might include extending the design to the case where interfering signals may also be present.

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