Broadband Beamforming

Literature Survey

Multidimensional Digital Signal Processing

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Spring 2003

Abstract

Broadband wireless channels, where high data rates are transmitted, are extremely dispersive in nature. A fundamental challenge in the design of equalizers for the broadband case lies in reducing complexity. Broadband finite impulse response (FIR) beamformers employ a space-time antenna array which reduces the multipath delay spread to narrowband levels. Adaptive frequency-domain equalizers that exhibit relatively low complexity growth with increase in channel memory have also been considered to address this issue. This survey studies practical equalizer structures that have been proposed in the literature to combat multipath dispersion in broadband channels.

1 Introduction

Over the last decade, we have witnessed an explosive growth in cellular communications and the Internet. These trends indicate a strong potential in the future for mobile broadband wireless data communications. Trellis based decoding represents an effective method to combat intersymbol interference (ISI) in frequency-selective multipath channels [1]. However, in the broadband case, multipath dispersion is quite severe and results in the channel memory increasing linearly with the data rate. Since the size of the trellis grows exponentially with the channel memory, direct application of trellis based decoding algorithms becomes unfeasible due to their high complexity. Techniques to overcome this effect include channel shortening equalizers and other equalization techniques that are not trellis based. Co-channel interference (CCI) from adjacent users is a serious issue in cellular systems and interference cancellation is another important factor in equalizer design.

2 Background

Several approaches have been adopted to reduce equalizer complexity without sacrificing too much in performance in terms of ISI mitigation. One approach that has been considered is to employ a broadband *beamformer* followed by an FIR filterbank as the front end of a communications receiver followed by a *maximum a posteriori* (MAP) sequence detector as part of the back end [2]. Section 3 discusses the design of these FIR beamformers. Multiple Input Multiple Output (MIMO) systems use space-time antenna arrays at both the transmit and receive ends to enhance diversity. *Fractionally-spaced* equalizers have been shown to be effective in equalizing MIMO channels [3]. Section 4 provides a detailed description of such equalizers. Section 5 gives an overview of adaptive *frequency-domain* equalizers for broadband wireless communications proposed in [4]. Frequency-domain equalizers exhibit relatively low complexity growth with increase in channel memory and are well-suited for broadband channels. In Section 6, results of simulations performed by the authors using each of these methods are compared. Finally, in Section 7, conclusions and future research plans for the project are presented.

3 Broadband Beamformers with Power Complementary filters

Trellis based decoders are based on the maximum likelihood sequence estimation (MLSE) criterion and are optimum from a probability of error viewpoint [1]. Adaptive linear prefilters can be used to force the overall impulse response of narrowband channels to approximate a desired truncated impulse response of acceptable duration [5]. However, the application of MLSE algorithms becomes unfeasible in the broadband case due to their high complexity. The ISI can be reduced to narrowband levels by using a broadband beamformer where the antenna array observations are processed by an FIR filterbank [6]. The structure of the beamformer is shown in Fig. 1. Optimal MAP equalization is then performed at the receiver output. The FIR filter coefficients are chosen to reduce the effects of secondary paths [7]. However, the noise at the output of such a receiver is colored and hence, the resultant signal cannot be applied to a trellis-based equalizer.

Space-time receivers can be designed to preserve the whiteness of the channel noise while reducing ISI [2]. To ensure that the noise at the beamformer output remains white, the filterbank is required to have the *power complementarity* property [8]. An N-channel FIR filterbank $\{W_1(z), W_2(z) \ldots, W_N(z)\}$ is said to be power complementary if

$$\sum_{i=1}^{N} W_i(z)\tilde{W}_i(z) = 1 \tag{1}$$

The tilde on transfer functions stands for complex conjugation followed by reciprocation of functional argument, for example, $\tilde{W}(z) = W^*(z^{-1})$.

Under this constraint, the filter coefficients can be chosen in a variety of ways to shorten the channel impulse response. When a mean square error (MSE) criterion is employed, the filter coefficients are selected to minimize the mean square error between the beamformer output and the transmitted signal. Alternatively, in the zero-forcing approach, the goal is to force the impulse response to be zero outside a certain range. Both formulations require minimizing a positive definite quadratic equation under non-negative definite quadratic constraints.

Lagrangian relaxation can be used to solve this problem where the power complementarity property is expressed as a set of non-negative definite quadratic constraints [2]. Then the dual mathematical problem reduces to the minimization of a convex function over a convex domain, which is easily solvable by standard optimization techniques. The resulting solution obeys the constraints only approximately but typically provides a good approximation. However, as the order of the FIR filters increases, the number of constraints becomes large which makes it difficult to ensure that the Lagrangian vector stays in the convex domain. An alternative design technique is proposed for this case based on the lattice parametrization of FIR filterbanks [9]. The lattice structure achieves the power complementarity property structurally, without the need to impose explicit filter constraints. The beamformer is then constructed incrementally, one lattice stage at a time, to minimize the MSE. The resulting solution is only locally optimal but the MSE decreases monotonically as more lattice stages are added.

Broadband transmission also suffers from CCI due to the presence of adjacent users. The effect of CCI can be interpreted along with the white channel noise as colored noise at the receiver to design broadband beamformers which whiten this colored noise, in addition to shortening the desired channel impulse response [10].

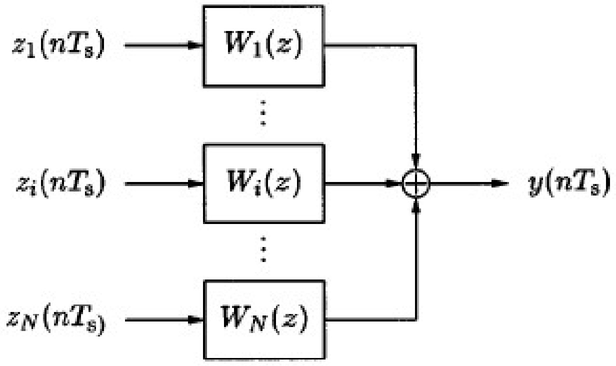

Figure 1: Broadband Beamformer.[1].

4 MIMO Biorthogonal Partners

In typical linear equalizer structures, the equalizer taps are spaced at the reciprocal of the symbol rate. A known drawback of these symbol-spaced equalizers is that they are highly sensitive to the phase of the sampling at the receiver [1, 11]. Fractionally spaced equalizers where the equalizer taps are placed closer together in time than the symbol interval are used to overcome this effect. The additional redundancy in allowing the equalizer to operate at a higher rate makes the equalizer design more flexible and allows for FIR solutions. Fig. 2 shows the structure of a fractionally spaced equalizer.

Fractionally spaced equalizers can be designed using the theory of biorthogonal partners [12]. Design of equalizers for MIMO channels is discussed in detail in [3, 13]. A MIMO transfer functions $\mathbf{H}(z)$ is said to be a left biorthogonal partner of $\mathbf{F}(z)$ with respect to an integer M if

$$[\mathbf{H}(z)\mathbf{F}(z)]_{\downarrow M} = \mathbf{I} \tag{2}$$

where $[x(n)]_{\downarrow M}$ denotes decimation by an integer M. The zero-forcing fractionally spaced equalizer is hence simply the left biorthogonal partner of the channel matrix. Conditions for the existence of stable FIR biorthogonal partners of a MIMO transfer function are derived. The FIR biorthogonal partner, if it exists, is not unique and this property is exploited to minimize the noise power at the receiver. Design of a polynomial matrix to minimize the noise component of $y(n)$ reduces to a linear estimation problem for a vector process. Fractional biorthogonal partners where the amount of oversampling at the receiver is not an integer, but a rational number, have also been explored [14].

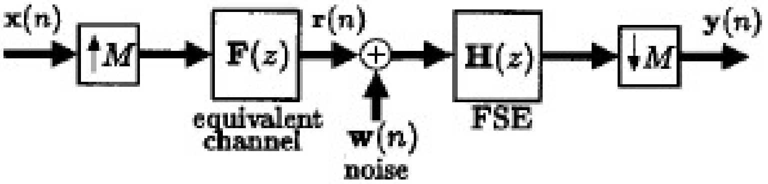

Figure 2: Digital Communication System with a Fractionally-Spaced Equalizer.[3].

5 Frequency-Domain Equalization

The complexity of conventional equalization approaches generally grows linearly or even exponentially with the number of symbols of dispersion. Broadband channels may hence call for low complexity solutions. Reduced-complexity techniques for broadband wireless channels have been investigated [15]. Methods to allow the receiver to find burst and symbol timing and a modified decision-feedback equalizer structure are proposed.

Frequency-domain equalization (FDE) is a technique that exhibits relatively low complexity growth with increasing channel memory. A feasible alternative is hence to use an adaptive equalizer that operates in the spatial-frequency domain and uses either least mean square (LMS) or recursive least squares (RLS) adaptive processing [4]. A prefix of length greater than the channel memory is inserted before each block of data transmitted to ensure that the block appears to have cyclic properties at the receiver. Block diagrams of the transmitted block and the receiver are shown in Fig. 3. The received signals are sampled such that aliasing distortion is avoided and transformed using a fast fourier transform (FFT) algorithm. The discrete frequency components of each diversity branch are then weighted suitably and combined. Computational complexity of this technique is considerably lesser than time-domain equalization.

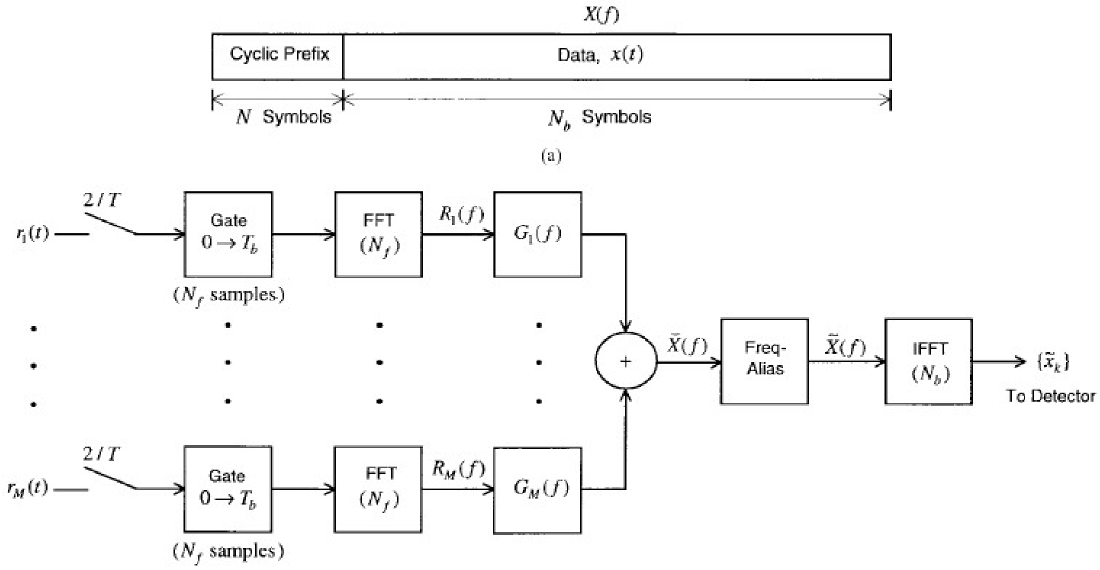

Figure 3: Transmitted block and Receiver of a Frequency-Domain Equalizer.[4].

6 Simulation Results

This section presents simulation results to illustrate the performance of the broadband equalization algorithms discussed so far. Broadband beamformers were simulated using an 8-PSK constellation format and a stationary broadband transmission channel described by a five-path fading model [2]. At the receiver end, Lagrangian relaxation and lattice filters are used to synthesize fifth order beamformers for antenna arrays with 2, 3 and 4 elements. The left part of Fig. 4 shows the channel impulse response observed at the four-branch beamformer output at 10 dB SNR. The right part of Fig. 4 shows the performance of a MAP equalizer applied to the beamformer output in terms of the symbol error rate. It is seen that broadband beamformers are effective in reducing the length of the multipath channel impulse response to six coefficients for the assumed channel model and beamformer order. Furthermore, for a five-ray channel model, the addition of more antenna elements provides only a marginal improvement in diversity gain and channel shortening and there is no need to use more than 4 antenna elements. The Lagrangian optimization problem can be solved easily using standard gradient or Newton techniques as long as the length of the FIR filters is not too large. This structure has the advantage that the MAP equalizer produces soft outputs that can be included in a turbo equalization iteration.

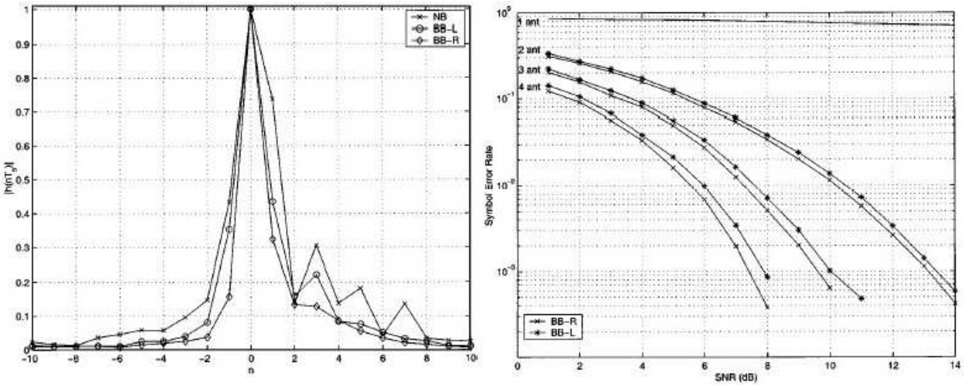

Figure 4: (Left) Channel impulse response seen at the four-branch beamformer output at 10 dB SNR. NB: Narrowband Beamformer BB-L: Broadband Beamformer - Lattice BB-R: Broadband Beamformer - Lagrangian. (Right) Performance of a MAP equalizer applied to the beamformer output.[2].

Simulations were performed to equalize two MIMO channels, a square 3×3 channel and a rectangular 2×3 channel, using a fractionally-spaced equalizer [3]. In both cases, a downsampling factor of 2 was chosen and the constellation was chosen to be 16-QAM. Fig. 5 shows the probability of error as a function of the estimator order for both these channels with SNR = 18 dB. The first measurement for an estimator order of 0 corresponds to the case where there is no optimization of the equalizer. It is seen that the probability of error can be reduced by several orders of magnitude by exploiting the redundancy of the biorthogonal partner. Complexity of design is dictated by the order of the estimator as design involves inverting polynomial matrices derived from the estimator. With increasing order of the estimator, it is seen that the performance of the equalizer does not improve after a certain point.

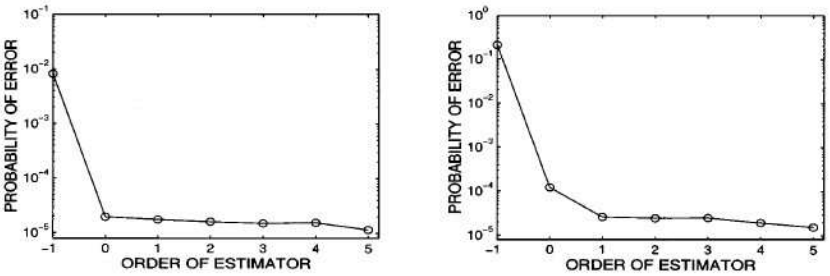

Figure 5: Probability of error as a function of the estimator order. (Left) Square 3 × 3 channel. (Right) 2 × 3 channel.[3].

Simulation results in terms of the bit error rate performance of the frequency-domain equalizer are also shown [4]. The complexity of FDE is shown to grow only a little faster than linearly with the bit rate in [4]. QPSK modulation is assumed with a data rate of 8 Mbps, 64 symbols of dispersion and 256 point FFTs. Both the LMS and RLS algorithms were seen to converge very close to the minimum MSE solution. Fig. 6 shows the bit error rate performance of a quasi-static channel [4]. The probability of error in this case is seen to be very close to results achieved in [2].

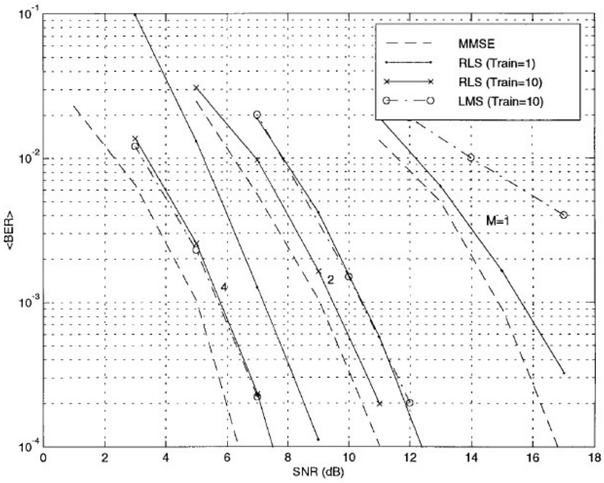

Figure 6: Bit Error Rate Performance of FDE.[4].

7 Conclusion

This survey examined in detail equalization techniques for broadband wireless channels. The design of broadband beamformers with the power complementarity property as proposed in [2] was discussed in detail. However, sampling above the Nyquist rate at the receiver input induces coloring of the noise. This factor has not been taken into account in the model proposed in [2]. This will alter the power complementarity constraint (1). It is proposed to study this effect by incorporating this coloring in the signal model and the constrained beamforming problem.

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