Extract Object Boundaries in Noisy Images using Level Set

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Literature Survey

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Abstract

Finding object contours in noisy images is a challenging task because of the amorphous nature of the object and the lack of sharp boundaries. Classical edge-based segmentation methods have the drawback of not connecting edge segments to form a distinct and meaningful boundary. Many level set approaches, which can deal with changes of topology and the presence of corners, have been developed to extract object boundaries. Previous researchers have used image gradient, edge strength, area minimization and region intensity to define the speed function. However, no paper mentions the edge/gradient direction. Our approach will incorporate direction and magnitude in the speed function.

Section 1: Introduction

Many of the computer vision applications involve the decomposition of image into region with some homogeneous properties, which are related to the nature of the applications. The boundary of an object is an important feature for the object detection, classification and tracking. Edge based approaches are not suitable for boundary extraction in noisy images [1]. They will detect edges that are not part of an object's boundary or miss parts of a boundary when the intensity contrast is weak. In general, additional effort is needed to connect the incomplete edges into a distinct and meaningful object boundary.

Several approaches have been proposed to extract object boundaries in images using closed curves. Roughly speaking, there are two types of boundary search approaches. One uses a closed contour represented by a parameterized curve. The problem of finding the desirable contour is posed as an energy minimization problem. The classical Euler-Lagrange formulation of the active contour is called 'snake' [2]. This kind of method relies on an initial guess of the boundary, image features and parameters. Moreover, its performance suffers from the change of topology and the presence of corners.

To overcome these problems, the level set approach has been proposed [3]. The guiding principle of level set methods is to describe a closed curve γ in R^2 as the zero level set of a higher dimension function $\Phi(x, y, t)$ in R^3. Instead of propagating the curve γ directly, we consider the evolution of function $\Phi(x, y, t)$ with a speed function F and extract the zero level set of points to obtain the boundary curve. Since level set methods represent the curve in an implicit form, they greatly simplify the management of the contour evolution, especially for handling topological changes. Most of the challenges in level set methods result from the need to construct an adequate model for the speed function. This review only focuses on the differential convex function. An algorithm for minimizing a non-differentiable convex function over a convex feasibility has been proposed in [4].

Section 2: Background

We consider the generation of a family of contours. Let an initial curve r_0 undergo deformation in a Euclidean plane. Let $r(x,y,t)$ denote the family of curves generated by the propagation of r_0 in the outward normal direction $\vec{N}$ with the speed *F*. We ignore the tangential velocity because it does not influence the geometry of the deformation, but only its parameterization [5]. The curve velocity $r_t(x,y,t)$ is denoted by

$$r_t(x,y,t) = F\vec{N}, \qquad (1)$$

where *F* is a scalar function and $\vec{N}$ is a unit normal vector.

According to the level set method, we can express the closed curve $r(t)$ in an implicit form as

$$r(x,y,t) = \{(x,y) \mid \Phi(x,y,t) = 0\}, \text{ or } \Phi(r(t),t)) = 0. \qquad (2)$$

By the chain rule,

$$\Phi_t + \Phi_r \cdot r_t = \Phi_t + \nabla\Phi \cdot F\vec{N} = \Phi_t + \nabla\Phi \cdot F\frac{\nabla\Phi}{|\nabla\Phi|} = 0,$$

yielding the movement equation of curves,

$$\Phi_t + F|\nabla\Phi| = 0, \text{ with } \Phi(x,y,t=0) = r_0. \quad (3)$$

The above motion equation (3) is a partial differential equation in one higher dimension than the original problem. Given the initial value, it can be solved by means of difference operators in a fixed grid via

$$\Phi_{i,j}^{n+1} = \Phi_{i,j}^{n} - \Delta t \cdot h \cdot (\max(F_{i,j},0)\nabla^+ + \min(F_{i,j},0)\nabla^-), \quad (4)$$

where *n* is the iterative time, *h* is the grid step, Δt is the time step, $F_{i,j}$ is the speed value of pixel (i, *j*), $\Phi_{i,j}^n$ is the level value of pixel (i, *j*) at time *n* and

$$\nabla^+ = (\max(D^{-x},0)^2 + \min(D^{+x},0)^2 + \max(D^{-y},0)^2 + \min(D^{+y},0)^2)^{0.5}$$
$$\nabla^- = (\max(D^{+x},0)^2 + \min(D^{-x},0)^2 + \max(D^{+y},0)^2 + \min(D^{-y},0)^2)^{0.5}$$

$$D^{-x} = \Phi_{i,j} - \Phi_{i-1,j},\ D^{+x} = \Phi_{i+1,j} - \Phi_{i,j},\ D^{-y} = \Phi_{i,j} - \Phi_{i,j-1},\ D^{+y} = \Phi_{i,j+1} - \Phi_{i,j}.$$

This implementation allows the function Φ to automatically follow topological changes and corners during evolution. The speed function F plays a key role in the level set method.

Section 3: Prior Work

The speed function is essentially a decreasing function of some features. These features should have very high values at the final shape boundary. In general, speed function models can be classified as edge-based, region-based and Motion-based.

3.1 Speed Function Due to Image Gradient

Caselle et al. [6] proposed the geometric active contour followed by Malladi et al. [7]. The active contour was extended to extract the surfaces of 3-d objects [8].

The model proposed by Caselles and Malladi was based on the following speed function:

$$F = (a + \varepsilon k) / |1 + \nabla G_\sigma * I|, \qquad (5)$$

where k is the curvature of the curve, a, ε and p are constants and $|1 + \nabla G_\sigma * I|$ is the edge gradient using a Gaussian filter G_σ with a known standard deviation σ. Since the stop criterion is the magnitude of the gradient, the speed slows down at strong edges. The drawback of this model is that it only detects objects with edges defined by strong gradients. F is never small enough to stop the curve evolution in a noisy image and the curve may extend beyond the boundary. Moreover the pulling back force is not strong hence it may not be able to pull back the expanding contour if it were to propagate and cross the desired boundary. In paper [9], a curvature profile acts as a boundary regularization term specific to the shape being extracted. This method needed a prior model about the final shape.

Yezzi et al. [10] tried to solve the above problems by introducing an extra pull back term. This can be expressed as

$$F = (a + \varepsilon k) / |1 + \nabla G_\sigma * I| - \nabla |1 + \nabla G_\sigma * I| \cdot \nabla\Phi / |\nabla\Phi| \qquad (6)$$

where the second term, $\nabla |1 + \nabla G_\sigma * I| \cdot \nabla\Phi / |\nabla\Phi|$ denotes the projection of an attractive force vector on the normal to the surface. The force is the gradient of a potential filed, which is given by the edge magnitude. The weakness of this technique is that it still suffers from the boundary leaking for complex structures.

Bottigi et al. [11] successfully applied an exponential speed function, $F = \exp(-a(G_\sigma * I - I_0))$, in mammography.

3.2 Speed Function Due to Region Intensity

Zhu et al. [12] proposed a statistical and variation approach which combines the geometrical features of snakes/balloons and region growing. Ho et al. [13] used the region competition to segment 3-D brain tumor.

Chan et al. [14] proposed an active model based on Mumford-Shan segmentation technique and the level set method. Their model can extract objects whose boundaries are not necessarily defined by gradient or with very smooth boundaries. They introduced the energy function $F(c_1, c_2, C)$, defined by

$$F(c_1, c_2, C) = u \cdot Length(C) + v \cdot Area(inside(C)) + \lambda_1 \int_{inside(C)} |u_0(x,y) - c_1|^2 \, dxdy + \lambda_2 \int_{outside(C)} |u_0(x,y) - c_2|^2 \, dxdy \qquad (7)$$

where $u \geq 0, v \geq 0, \lambda_1, \lambda_2 > 0$ are fixed parameters, $u_0(x,y)$ is the intensity of pixel (x,y), C is the curve, while the constants c_1 and c_2 depending on C, are the average of u_0 inside or outside the curve C. Finding the object boundary turns out to be the minimization of the energy $F(c_1, c_2, C)$. For the level set formulation of the variation active contour model, they deduced the associate Euler-Lagrange equation for Φ as

$$\frac{\partial\Phi}{\partial t} = \delta_\varepsilon(\Phi)[u \cdot div(\frac{\nabla\Phi}{|\nabla\Phi|}) - v - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2] = 0 \;\; in\,(0,\infty) \times \Omega,$$

$$\Phi(0,x,y)=\Phi_0(x,y)\, in\, \Omega\,,\quad \frac{\delta_\varepsilon(\Phi)}{|\nabla\Phi|}\frac{\partial\Phi}{\partial\vec{n}}=0\, on\, \partial\Omega \tag{8}$$

where $\vec{n}$ denotes the exterior normal to the boundary $\partial\Omega$, $\partial\Phi/\partial\vec{n}$ denotes the normal derivation of Φ at the boundary, and $\delta_\varepsilon(\Phi)$ is the regulation function.

The problem with this method is that we have to estimate the intensity distribution of the region; however, the distribution model may degrade in a noisy image.

3.3 Speed Function Due to Motion

Extracting object boundary in image sequences can use motion information [15, 16]. Level set based approach can successfully deal with the challenging problem of tracking non-rigid objects that can not be easily parameterized and allow the changes of topology of the object being tracked. Therefore, the inter-frame difference density function has been considered in the speed function [17, 18, and 19].

Paragios and Deriche [17] proposed a geodesic active contour to extract and track moving objects in image sequences. The boundary was determined by a probabilistic edge detector that was based on the analysis of the inter-frame difference using a mixture model as well as the input image. The speed function was given as

$$F=r(g(I_D,\sigma_D)K+\nabla g(I_D,\sigma_D)\cdot\frac{\nabla\Phi}{|\nabla\Phi|})+(1-r)(g(|\nabla I|,\sigma_T)K+\nabla g(|\nabla I|,\sigma_T)\cdot\frac{\nabla\Phi}{|\nabla\Phi|}) \tag{9}$$

where r is the constant, K is the curvature, $g(I_D,\sigma_D)$ is a Gaussian function estimating the inter-frame difference I_D with the standard deviation σ_D, and $g(|\nabla I|,\sigma_T)$ is a Gaussian function estimating the gradient of input image I with the standard deviation σ_T. In their approach, the region to be tracked is assumed to have strong intensity boundaries and the background is assumed to be smooth.

In [18], the authors presented a variation method for extracting a moving object against a still background over a sequence of image frames. They introduced a geometrical constraint into the derivation of the Euler-Lagrange equations, such that the segmentation of each individual frame can be interpreted as a closed boundary of an object while integrating information over the entire sequence.

In contrast to [17], A.Mansouri [19] presented a generic tracking problem as a Bayesian estimation problem without assuming motion models. The Bayesian estimate problem is ultimately formulated as an energy minimization problem leading to the solution via Euler-Lagrange descent equations. These Euler-Lagrange equations are formulated as level set partial differential equations. He assumed that the region's luminance/chrominance statistic vary litter from frame to frame in the sequence.

Section 4: Conclusions

The advantages of level set were summarized in [20]. We will emphasize the importance of the gradient direction in the speed function. The introduction of the gradient direction in our project will overcome the disadvantages in the general level set methods that are summarized as follows.

a). The speed function may not turn out to be zero in multiple objects segmentation.

Fig. 1 shows a thermal image of a rat and four circular calibration emitters with very different intensities. The speed function of the brighter emitters can easily be reduced to zero while the dark emitters with low contrast from the background are likely to be missed by the evolving curve under the same model parameters. The active contour meets another problem in segmenting the rat and emitters, i.e. different shapes. Additional care must to be taken to set different model parameters for the rat and emitters. In contrast, our proposed method segments all five objects using the same parameters.

Fig. (1) A rat with four circular calibration emitters.

b). Gaps in Boundaries.

Gaps in boundaries are not a problem in the active contour model because the smoothing restriction and internal iterate values make the contour complete. However, they are a drawback of the level set method when applied to noisy images. The contour in the level set method is in an implicit form, which may simply leak through gaps.

c). Embedding the object.

If one object has one or more objects located inside, the general level set method and the active contour will not capture all objects of interest.

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