Measurements of Acoustic Pressure and Velocity Vector in Source Localization using Acoustic Intensity Sensors

Khalid Miah

Literature Survey

EE 381K Multi Dimensional Digital Signal Processing, Spring, 2005

Abstract – **Identifying acoustic sources in terms of their relative location is an important factor in active noise control. Traditional source localization measurements uses scalar pressure sensors which proved to be less accurate. Nehorai *et al.* proposed a different approach for source localization using vector natured acoustic intensity sensors. Four-dimensional (4-D) intensity based algorithm and three-dimensional (3-D) velocity covariance method were used for direction-of-arrival (DOA) estimation in free-space scenario. Hawkes *et al.* extended the free-space intensity based algorithm to account boundary reflections for DOA estimation. Performance analysis of these algorithms for both free-space and reflection boundary cases have been investigated in this report.**

I. INTRODUCTION

For active noise control, it is important to identify acoustic sources in terms of their relative location and power output. Localization of acoustic sources using vector sensor models will be analyzed in this report. Traditionally, localization of acoustic sources in fluid (air/water) fields is done with arrays of distributed sensors in which output of each sensor is a scalar quantity corresponding to the acoustic pressure [1], [2]. The time difference of arrivals of the acoustic waves between the sensors are then used for source localization [3]. Nehorai *et al.* considered a different approach for solving this problem by using and array of vector sensors whose output is a vector corresponding to acoustic pressure and acoustic particle velocity. The main advantage of these vector sensors over traditional scalar sensors is that they make use of more available acoustic information. Thus vector sensors outperform scalar sensor arrays in accuracy of source localization [1].

Source localization is one of the main applications of the vector natured intensity and energy density measurements. Acoustic or sound intensity is considered one of the fundamental quantities of acoustic fields [5]. It can be described as the rate of energy flow at a point in space through a unit area . Energy density describes the energy in a unit volume of space [4]. While acoustic intensity and energy density are different quantities, they can both be measured using the same vector sensor [6]. In this report, measurements of acoustic intensity will be referred to as measurements of acoustic pressure and particle velocity vectors.

The need of statistical distributions of acoustic quantities becomes obvious in encountering the difficulty of applying the modal theory [7] of acoustics at high frequencies in a large and irregular shaped space. In contrast to the modal theory of room acoustics, statistical properties of room acoustics is independent of the shape of the enclosed space [8]. In late 1960s, Waterhouse and Lubman laid the foundation of statistical properties of reverberant sound fields [9], [10]. Budhianto *et al.* derived the acoustic pressure and particle velocity related distributions for reverberation room environment with some constraints.

In this report, direction-of-arrival (DOA) of sound sources will be estimated using vector sensors utilizing statistical properties of the acoustical fields. A useful quality measure for direction estimation in 3-D space is the normalized asymptotic mean-square angular error (MSAE) between unit vector at the sensor pointing toward the source and its estimate. Algorithms for estimating DOA using 3-D covariance and 4-D intensity methods were developed [1] for free-space scenario. Intensity based algorithm has been extended to account boundary reflection for DOA estimation where vector sensors are free floating in the water column, located on the seabed, or on the ground [11]. As for this project, I am planning to analyze performance of different methods for both free-space and reflection boundary cases. These methods of source localization could be used in passive acoustic surveillance system to monitor sound sources in industry settings and even in battlefield or ocean without giving away the location of the probe itself.

Section II explains background and measurement models of the two algorithms. Section III summarizes the intensity based algorithm and velocity covariance method for free-space case. Section IV summarizes the intensity based algorithm to account for boundary reflection. Section V includes a brief analysis of the different methods. Section VI includes an implementation and conclusion of the report.

II. BACKGROUND

A quality measure for direction-of-arrival (DOA) estimation in 3-D space is the normalized asymptotic mean-square angular error (MSAE) between ***u*** and its estimate $\hat{\boldsymbol{u}}$. In this report, ***u*** is a unit vector at the sensor pointing towards the source, that is

$$\boldsymbol{u} = \begin{bmatrix} \cos\theta_1 \cos\theta_2 \\ \sin\theta_1 \cos\theta_2 \\ \sin\theta_2 \end{bmatrix} \tag{1}$$

where θ_1 and θ_2 are the azimuth and elevation angles of ***u***, respectively (see Fig. 1). Thus, $\theta_1 \in [0, 2\pi]$ and $|\theta_2| \le \pi/2$. The acoustic particle velocity and pressure at position **r** and time *t* can be denoted as

$\boldsymbol{v}(\mathbf{r}, t)$ and $p(\mathbf{r}, t)$. Under the plane wave at the sensor assumption, it can be shown [12], [13] that

$$\boldsymbol{v}(\mathbf{r}, t) = -\boldsymbol{u} \cdot \frac{p(\mathbf{r}, t)}{\rho_0 c} \tag{2}$$

where ρ_0 is the ambient density and c is the sound speed in the medium.

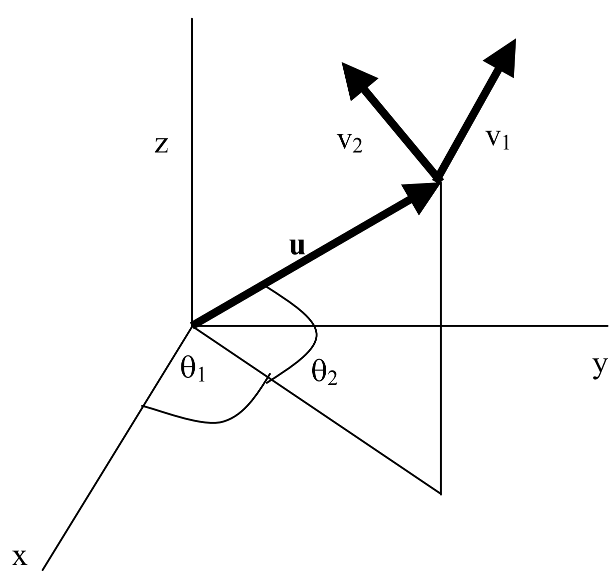

Fig. 1: The Orthonormal Vector Triad ($\boldsymbol{u}$, $\mathbf{v}_1$, $\mathbf{v}_2$) [1].

Phasor representation of acoustic pressure and acoustic particle velocity vectors will be used in the measurement model. Then, the pressure part of the model can be derived to

$$y_p(t) = P(t) + e_p(t) \tag{3}$$

Similarly, the velocity part of the measurement can be derived to

$$\boldsymbol{y_v}(t) = P(t).\boldsymbol{u} + \boldsymbol{e_v}(t) \tag{4}$$

Now, combining (3) and (4), we have

$$\begin{bmatrix} y_p(t) \\ \boldsymbol{y_v}(t) \end{bmatrix} = \begin{bmatrix} 1 \\ \boldsymbol{u} \end{bmatrix} P(t) + \begin{bmatrix} e_p(t) \\ \boldsymbol{e_v}(t) \end{bmatrix} \tag{5}$$

where $P(t)$ is the phasor representation of the acoustic pressure, $e_p(t)$ and $\boldsymbol{e_v}(t)$ are the noise components of pressure and particle velocity vectors, respectively. Now, let δ be the angular error between $\boldsymbol{u}$ and its estimate $\hat{\boldsymbol{u}}$, then $\delta = 2\sin^{-1}(\|\hat{\boldsymbol{u}} - \boldsymbol{u}\|/2)$. Then MSAE can be defined as $\lim_{N\to\infty}\{N\,E(\delta^2)\}$. For a regular model [14], the MSAE of any regular direction estimator is bounded from below by

$$\mathrm{MSAE}_{\mathrm{CR}} = \mathrm{N}.\{\cos^2(\theta_2).\mathrm{CRB}(\theta_1) + \mathrm{CRB}(\theta_2)\} \tag{6}$$

where $\mathrm{CRB}(\theta_1)$ and $\mathrm{CRB}(\theta_2)$ are, respectively, the CRB (Cramer-Rao bound) variances of the azimuth and elevation angles of the source.

Now, for a single-source single-vector sensor case, the CRB can be shown as

$$\mathrm{CRB}(\theta) = \frac{1+\rho}{2N\rho_v\rho}\begin{bmatrix} 1/\cos^2\theta_2 & 0 \\ 0 & 1 \end{bmatrix} \tag{7}$$

It can be observed from (7) that ρ_v is the signal-to-noise ratio (SNR) of the velocity measurement in each sensor component, while ρ is an equivalent SNR of both the pressure and velocity-vector measurements. Combining both (6) and (7), a compact expression for the lower bound of the MSAE of a single-source single-vector sensor measurements can be derived [1] as

$$\mathrm{MSAE}_{\mathrm{CR}} = \frac{1+\rho}{\rho_v\rho}. \tag{8}$$

III. FREE-SPACE MEASUREMENTS

In this section, two simple algorithms for estimating DOA of a single acoustic source using the measurements of a single vector sensor in free space will be analyzed. The following assumptions are made for the free-space measurements:

- Wave is traveling in a quiescent, homogeneous, and isotropic fluid.
- Plane wave at the sensor.
- Band-limited spectrum signal.
- The source signal sequence is identically distributed Gaussian process with zero-mean.
- The noise $\boldsymbol{e}(\mathrm{t})$ is complex Gaussian with zero mean.

A. 4-D Intensity Based Algorithm

This algorithm stems from the fact that the $\boldsymbol{u}$ is the unit vector in the opposite direction of the sound intensity vector. This algorithm computes

$$\hat{\boldsymbol{s}} = \frac{1}{N}\sum_{t=1}^{N} \mathrm{Re}\{y_p(t)\bar{\boldsymbol{y}}_{\mathbf{v}}(t)\} \tag{9}$$

$$\hat{\boldsymbol{u}} = \hat{\boldsymbol{s}} / \|\hat{\boldsymbol{s}}\| \tag{10}$$

The statistical performance of this estimator is analyzed and the results are summarized in the following:

- If $|P(t)|^2, |e_p(t)|, |\mathbf{e_v}(t)|$ have finite first-order moments, then $\hat{\boldsymbol{u}} \to \boldsymbol{u}$ almost surely [1].
- If $|P(t)|^2, |e_p(t)|, |\mathbf{e_v}(t)|$ have finite fourth-order moments, then the MSAE (with Gaussian assumption omitted) of $\hat{\boldsymbol{u}}$ is

$$\mathrm{MSAE} = \frac{1+\rho_v}{\rho_v \rho_p}. \tag{11}$$

For the Gaussian case, the ratio between the MSAE of this estimator and the $\mathrm{MSAE_{CR}}$ is

$$\frac{\mathrm{MSAE}}{\mathrm{MSAE_{CR}}} = \frac{\rho(1+\rho_p)}{\rho_p(1+\rho)} = 1 + \frac{\sigma_p^2/\sigma_v^2}{1+\rho}. \tag{12}$$

Thus, it can be shown that this estimator becomes efficient if $\sigma_v >> \sigma_p$, implies that $\rho \approx \rho_p$ [1].

B. Velocity Covariance Based Algorithm

In this algorithm, only acoustic particle-velocity vector measurements and its covariance matrix structure is used to estimate $\boldsymbol{u}$. Using (4), the data covariance in this case is

$$\mathbf{R} = \sigma_s^2 \; . \; \boldsymbol{u}\boldsymbol{u}^T + \boldsymbol{\sigma}_\mathbf{v}^2 I \tag{13}$$

where matrix **R** has an eigenvector ***u*** (or –***u***) associated with its largest eigenvalue, and *I* is the identity matrix. Now for some given unit vector $\boldsymbol{u}'$, this algorithm computes $\hat{\mathbf{R}}$,

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{t=1}^{N} \mathrm{Re}\left\{\mathbf{y}_\mathbf{v}(t)\mathbf{y}_v^*(t)\right\} \tag{14}$$

The statistical performance of this estimator is analyzed and the results [1] are summarized in the following:

- If $|P(t)|, |\boldsymbol{e}_\mathbf{v}(t)|$ have finite second-order moments, then $\hat{\boldsymbol{u}} \to \boldsymbol{u}$ almost surely.
- $|P(t)|, |\boldsymbol{e}_\mathbf{v}(t)|$ have finite fourth-order moments, then $\sqrt{\boldsymbol{N}}(\hat{\boldsymbol{u}} - \boldsymbol{u})$ is asymptotically normally distributed.
- If $\boldsymbol{e}_\mathbf{v}(t)$ is Gaussian and $|P(t)|$ has finite eighth-order moment, then the estimator has an optimal MSAE and is given by

$$\mathrm{MSAE} = \mathrm{MSAE}_{\mathrm{CR}} = \rho^{-1} + \rho^{-2}; \quad \rho = \rho_v \tag{15}$$

where $\rho_v \triangleq \sigma_s^2 / \sigma_v^2$ [1].

IV. REFLECTION BOUNDARY MEASUREMENTS

Acoustic Vector Sensor (AVS) located on the ground or seabed needs to account for boundary characteristics (reflection coefficient). Underwater and airborne acoustic sources can be localized in 3-D space using these freely drifting, moored, or ground based AVS [11]. Some of the fundamental assumptions for reflection boundary measurements are as follows:

- A point source radiating spherically symmetric waves and has a simple point image with (complex) amplitude R, relative to the source.
- The three velocity components are aligned with the coordinates of the AVS or that the orientation of the AVS is known.

Although the intensity based algorithm is used to estimate DOA, the 3-D intensity vector is not parallel to **u**. Therefore, the exact same method used for free space case cannot be used in this case to find the elevation angle of the source. The measurement of a single AVS can be written as

$$\boldsymbol{y}(\mathrm{t}) = \boldsymbol{h}p(\mathrm{t}) + \boldsymbol{e}(\mathrm{t}) \qquad \text{for t} = 1, 2, \ldots\ldots, \mathrm{N} \tag{16}$$

where ***h*** is the sensor's steering vector and is defined [11] as

$$\boldsymbol{h} = \begin{bmatrix} 1+R \\ (1+R)\cos\phi\cos\psi) \\ (1+R)\sin\phi\cos\psi \\ (1-R)sin\psi \end{bmatrix}. \tag{17}$$

where ϕ and ψ are the azimuth and elevation angle of the source, respectively. Therefore, the reflection coefficient can derived [11] as

$$R = \frac{Z_{in} - \rho_0 c/\sin\psi}{Z_{in} + \rho_0 c/\sin\psi} \tag{18}$$

where $Z_{in} = -\frac{(1+R)\rho_0 c}{\sin\psi(1-R)}$.

Based on the horizontal component of acoustic intensity, the azimuth of the source can be estimated from

$$\hat{\boldsymbol{u}}_h \cong \begin{bmatrix} \cos\hat{\phi} \\ \sin\hat{\phi} \end{bmatrix} = \frac{\hat{\boldsymbol{s}}}{\|\hat{\boldsymbol{s}}\|} \to \boldsymbol{u}_h \cong \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}. \tag{19}$$

Since the magnitude of the horizontal component of acoustic intensity depends on the elevation angle ψ, so will the accuracy of $\hat{\boldsymbol{u}}_h$. With proper modification, the analysis of the azimuthal estimator can be used to show that the asymptotic

$$\text{MSAE} = \lim_{N\to\infty} N\, E(\hat{\phi} - \phi)^2 = \frac{1 + |1+R|^2 \rho}{2\rho^2 |1+R|^4 \cos^2\psi} \tag{20}$$

where $\rho = \sigma_s^2 / \sigma^2$ is the signal-to noise ratio (SNR). Using, the vertical component of the acoustic intensity, the elevation angle for the ground and seabed case can be estimated as $\hat{\psi} = \mathrm{Re}\left\{\cos^{-1}\left|\frac{\rho_0 c}{\hat{\chi}\bar{Z}_{in}}\right|\right\}$ and $\hat{\psi} = Re\left\{cos^{-1}\left|\frac{n}{\sqrt{\hat{\chi}^2\eta^2 + 1}}\right|\right\}$, respectively. In this estimation, $\hat{\chi} = \frac{1-\bar{R}(\hat{\psi})}{1+\bar{R}(\hat{\psi})}\tan\hat{\psi}$ where $\bar{R}$ is the functional form of R [11].

There is no known expression for the MSAE_∞ of both azimuth and elevation angle estimators in the ground and seabed scenarios. However, under the assumption of Gaussian signals and noise, it can be shown [11] that

$$\text{MSAE}_\text{b} = \frac{1}{2\rho}\left(1 + \frac{1}{\rho|\boldsymbol{h}|^2}\right)\cdot\left(\frac{\cos^2\phi}{|\partial\boldsymbol{h}/\partial\phi|^2} + \frac{1}{|\partial\boldsymbol{h}/\partial\psi|^2 - \left|(\partial\boldsymbol{h}^{\boldsymbol{H}}/\partial\psi)\boldsymbol{h}\right|^2 / |\boldsymbol{h}|^2}\right) \tag{21}$$

V. ANALYSIS OF DIFFERENT METHODS

For the free-space case, algorithms discussed in this report are suitable for real-time applications and can be developed in the time domain. They can give a direction estimate instantly, i.e. with one time sample and simple to implement. These two algorithms are equally applicable to sources of various types, including wide-band and non-Gaussian. The MSAE of the intensity based algorithm is nearly optimal and that of the covariance method is optimal in the Gaussian noise case. In general, computation of MSAE in reflection boundary case is more difficult than that of the free-space case because of the inclusion of the boundary characteristics . The intensity based algorithm used to estimate DOA requires that both azimuth and elevation angle be calculated separately. The lower bound of MSAE is a function of the SNR ρ and the elevation angle ψ while in the free-space case $MSAE_b$ is just a function of the SNR [11]. In both free-space and boundary reflection cases, these algorithms do not depend on time delays and therefore do not require data synchronization and localization calibration between different sensor components [3].

VI. IMPLEMENTATION AND CONCLUSIONS

In this project, I am planning to simulate numerical examples of the intensity based algorithm and velocity covariance method for the free-space case. Intensity based algorithm for reflection boundary (ground) case will also be simulated and then be compared with the free-space case. I will contact authors of the relevant papers ([11], [2], [3]) to acquire experimental data for the simulation. The MATLAB Signal Processing toolbox would be used to perform necessary simulations. If time permits, use of velocity vector covariance method in DOA estimation for reflection boundary case will also be investigated. It can be noted that DOA estimation in reflection boundary case gets complicated if acoustic waves hits the sensors after reflecting from different boundaries which may not have same characteristics. This could be an area of interest for researchers involved in acoustic source localization.

REFERENCES

[1] E. Paldi and A. Nehorai, "Acoustic Vector-Sensor Array Processing," *IEEE Trans. Sig. Proc.*, vol. 42, no. 9, pp. 2481-2491, September 1994.

[2] M.Hawkes and A.Nehorai, "Surface-mounted acoustic vector-sensor array processing," *IEEE Proc. Int. Conf. Acoust., Speech Signal Process,* Atlanta, GA, pp. 3170-3173, May 1996.

[3] E. Paldi and A. Nehorai, "Performance Analysis of two Direction Estimation Algorithms using an Acoustic Vector Sensor ," *IEEE Proc. Int. Conf. Acoust., Speech Signal Process,* Minneapolis, MN, pp. 360-363, April 1993.

[4] D. Blackstock, *Fundamentals of Physical Acoustics*, New York: John Wiley and Sons, 2000, pp. 4 -55

[5] Lord Rayleigh [Strutt, J. W.], *The theory of sound*, London: McMillan and Co., 1937, pp. 1-18

[6] T. J. Poterek, "*Energy Density Analysis of Acoustic Intensity*", MS Thesis, The University of Texas at Austin, Austin, Texas, May 2001.

[7] M. Möser, *Engineering Acoustic- An introduction to noise control*, Berlin: Springer-Verlag, 2004, pp. 156 – 170.

[8] M. H. W. Budhiantho and E. L. Hixson, "Acoustic Velocity related Distributions," *J. Acoust. Soc. Am.*, vol. 100, no. 4, pt.2, pp. 2838-2845, November 1996.

[9] R.V. Waterhouse, "Statistical Properties of Reverberant Sound Fields," *J. Acoust. Soc. Am.,* vol. 43, no. 6, pp. 1436-1444, June 1968.

[10] D. Lubman, "Fluctuation of Sound with Position in a Reverberation Room," *J. Acoust. Soc. Am.*, vol. 44, no. 6, pp. 1491-1502, December 1968.

[11] M. Hawkes and A. Nehorai, "Wideband Source Localization Using a Distributed Acoustic Vector-Sensor Array," *IEEE Trans. Sig. Proc.*, vol. 51, no. 6, pp. 1479-1491, June 2003.

[12] P. M. Morse and K. U. Ingrad, *Theoretical Acoustics*, New York: McGraw-Hill, 1968.

[13] A. D. Pierce, *Acoustics – An Introduction to Its Physical Principles and Applications*, New York: McGraw-Hill, 1981.

[14] A. Nehorai and E. Paldi, "Vector-Sensor Array Processing for Electromagnetic Source Localization," *IEEE Trans. Sig. Proc.*, vol. 42, no. 2, pp. 376-398, February 1994.