Channel Estimation for Wired MIMO Communication Systems

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Abstract

This report addresses training-based channel modeling and estimation for a wired multiuser multicarrier communications system. The special case of a multiple-input multiple-output (MIMO) channel is considered where the different users transmit at the same time and over the same bandwidth. In the report, I will introduce the data transmission with the multicarrier modulation and training sequence designs based on the wired MIMO communication systems. The report also will present a MIMO channel model. Then, an optimal channel estimation method by training sequences is presented and analyzed.

I. Introduction

Communication systems that use multiple transmitters and receivers are often called multiple-input multiple-output (MIMO) systems. The bonded Asymmetric Digital Subscriber Line (ADSL) is a wired MIMO communication system [11]. The current challenges for MIMO systems are still the transmission power, bandwidth, and computational complexity and connection speeds [1]. To estimate an unknown channel is a very important and necessary work before transmitting the real signals since the channel is commonly time-varying. The channel estimation can be performed by sending a known training sequence, by transmitting pilot signals, and by using cyclic statistics of the received signal [2] [3]. The wired communication channel is slowly time-varying, so a preamble training sequence is often used to estimate the channel.

In this report, Section II introduces some key techniques and describes the multicarrier data transmission and reception for MIMO systems. Section III describes Training-Based MIMO channel models. Section IV presents and analyzes training sequence design methods. Section V presents and discusses an optimal channel estimation method by training sequences. Section VI concludes and summarizes this report.

II. Background

All transmission channels are fundamentally analog and thus may exhibit a wide variety of transmission effects. Modulation converts a stream of input bits into equivalent analog signals that are suitable for the transmission line. A primary impairment in communications is inter-symbol interference (ISI) which is caused by the memory in the channel. To combat ISI, a receiver usually uses an equalizer to compensate for the spreading in time and distortion in frequency caused by the channel [12]. Since the equalizer is designed in terms of the channel, it is very important to estimate the channel at the receiver for the equalizer. One technique to avoid ISI, without sacrificing the transmission rate, is Multicarrier Modulation (MCM). MCM divides a broadband channel into narrowband subchannels that have their own center carrier frequencies. There is no ISI in subchannels if each subchannel is ideally sampled and has constant gain. Because of MCM's robustness to multipath, and the ease of implementing MCM using the fast Fourier transform (FFT), the MCM concept is growing rapidly in practical importance. It has been implemented in several wireline and wireless high-speed data communications standards. The discrete multitone modulation (DMT) is a MCM application in the wired communication system (e.g. ADSL and VDSL system) [4] [11].

A Multicarrier Modulation transmitter is shown below [5]:

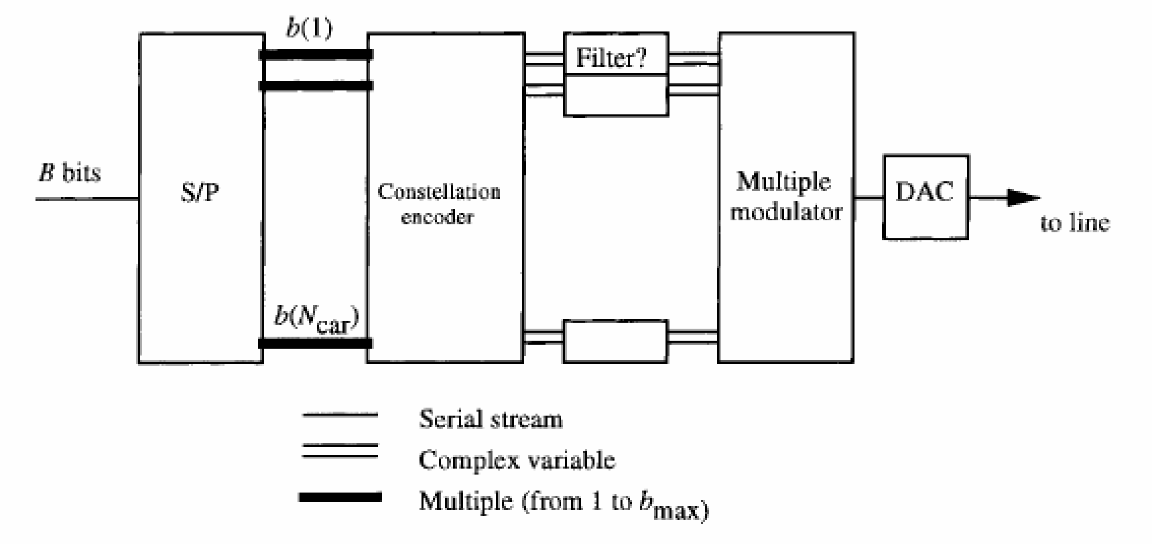

Figure 1 Multicarrier Modulation Transmission

The receiver is the dual of the transmitter assuming that the receiver has adjusted to the channel impairments and transmitter imperfections. The input of the S/P converter is a sequence of symbols of B bits each; the output for each symbol is N_{car} groups of $b(n)$ bit each. That is B= $\sum_{n\le N_{car}} b(n)$. The groups of $b(n)$ are then constellation-encoded, perhaps filtered, and then modulated onto N_{car} subcarriers.

III. MIMO Channel Modeling [4]

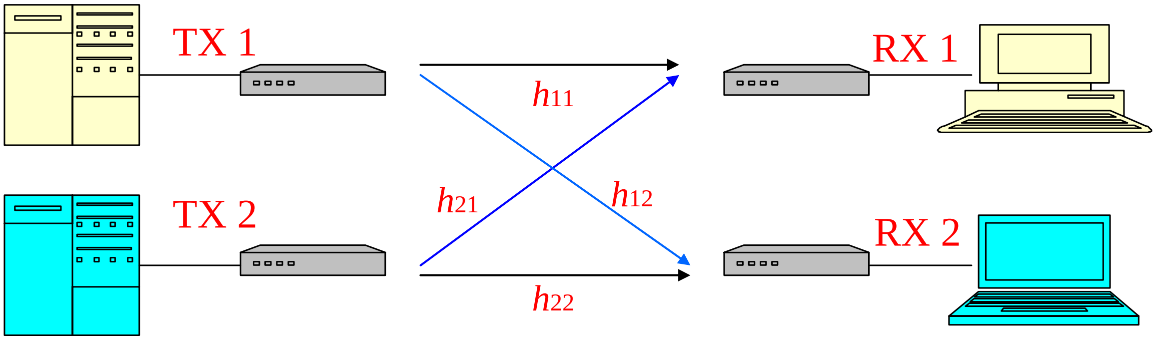

Figure 2 MIMO with two transmitters and two receivers

Consider a system that employs two-transmit and two-receive antennas simply. Two training signals and are transmitted over four wired channels

$$h_{ij}(L) = \left[h_{ij}(0) \cdots h_{ij}(L-1)\right]^T, \; i \text{ or } j = 1,2 \qquad (1)$$

where $(\bullet)^T$ denotes the transpose operation. h_{11} and h_{22} are main direct channel impulse responses between the same transmitter and receiver. h_{12} and h_{21} are cross channel impulse responses between the different transmitter and receiver. Each channel is modeled as a finite-impulse response (FIR) filter with L taps. The input training sequences $\mathbf{s}_1$ and $\mathbf{s}_2$ belong to a finite-signal constellation and are transmitted in data blocks where each block consists of N_{i} information symbols and N_t training symbols. For two transmitters, the receiver uses the $2N_t$ known training symbols to estimate the unknown $2L$ channel coefficients.

The observed training sequence output that does not have interference from information or preamble symbols can be expressed as [6]

$$\mathrm{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathrm{Sh} + \mathrm{z} = [S_1(L, N_t) \;\; S_2(L, N_t)] \begin{bmatrix} h_{11}(L) & h_{12}(L) \\ h_{21}(L) & h_{22}(L) \end{bmatrix} + \mathrm{z} \qquad (2)$$

where y and z are of dimension $2(N_t - L + 1) \times 1$, z is assumed to be additive white Gaussian noise(AWGN). S_1 and S_2 are Toeplitz matrices of dimension $(N_t - L + 1) \times L$, and

$$S_i(L, N_t) = \begin{bmatrix} s_i(L-1) & \cdots & s_i(0) \\ s_i(L) & \cdots & s_i(1) \\ \vdots & \ddots & \vdots \\ s_i(N_t - 1) & \cdots & s_i(N_t - L) \end{bmatrix}, \; i = 1,2$$

IV. Training Sequences Design

A known training sequence is transmitted to estimate the channel impulse response before data transmission in digital communication systems. Training sequences are periodic or aperiodic [7]. In either case, the power spectrum of the training sequence is approximately at over the transmission bandwidth. The suggested training sequence for channel estimation in a DMT system is a pseudo-random binary sequence with N samples [4]. The training sequence is made periodic by repeating N samples or adding a cyclic prefix. Obviously, the training sequence design can be based on a time domain or frequency domain. A time-domain optimization method is introduced in [8]. A disadvantage of the time-domain method is that an exhaustive search for the optimal training sequence of length N requires 2^N possible sequences. A frequency-domain method is proposed to reduce the computational cost in [9]. However, the frequency-domain method cannot always find the optimal periodic training sequence in terms of the mean-squared channel estimation error [8]. A summary and comparison for these design methods is shown below. A training sequence is said to be perfect or optimal if it have impulse-like autocorrelation and zero crosscorrelation [6] and L is the channel taps:

Domain	**Method**	**Minimum MSE**	**Searching Complexity**	**Optimal Sequence**
Time	Periodic (LS)[8]	Yes	High(2^N)	Yes
	Aperiodic [7]	No	Medium(N^2)	Yes
	L-Perfect (MIMO) [6]	Almost	Low($N\log_2 N$)	Sometimes
Frequency	Periodic [9]	No	Low($N\log_2 N$)	Sometimes

Table1 Comparison of Training Sequence Design methods

So, the design is a kind of tradeoff between the searching complexity and sequence performances.

V. MIMO Channel Estimation

The objective was to determine a training sequence which optimized the mean-squared estimation error for a least-squares type channel estimator. According to the model shown as above, a simple and intuitive method to estimate the MIMO channel impulse responses, i.e. h_{11}, h_{22}, h_{12} and h_{21}, is to send the training sequence **s** at only one transmitter by turning off another transmitter during one training time slot. This method is very low complexity and even doesn't need to design a good training sequence. However, it is very time consuming. It needs two time slots to obtain the estimated channels. During the first time slots, $h_{11} = \dfrac{y_{t_1,1}}{s}, h_{12} = \dfrac{y_{t_1,2}}{s}$. During the second time slots, $h_{21} = \dfrac{y_{t_2,1}}{s}, h_{22} = \dfrac{y_{t_2,2}}{s}$. Now, I use this idea combined with traditional numerical methods to estimate the MIMO channels.

Traditionally, the linear least square channel estimates $\left(\hat{\bullet}\right)$ can be calculated as [10]

$$\hat{\mathrm{h}} = \begin{bmatrix} \hat{h}_{11} & \hat{h}_{12} \\ \hat{h}_{21} & \hat{h}_{22} \end{bmatrix} = (\mathrm{S}^H\mathrm{S})^{-1}\mathrm{S}^H\mathrm{y} \qquad (3)$$

where $\left(\bullet\right)^H$ and $\left(\bullet\right)^{-1}$ denote the complex-conjugate (Hermitian) transpose and the inverse, respectively. The mean-squared error (MSE) for the time-domain case is defined by

$$\left(\tilde{\bullet}\right)\ \mathrm{MSE} = E\left[\left(\mathrm{h} - \hat{\mathrm{h}}\right)^H \left(\mathrm{h} - \hat{\mathrm{h}}\right)\right] = 2\sigma^2 Tr((\mathrm{S}^H\mathrm{S})^{-1}) \qquad (4)$$

where we assume white noise with auto-correlation matrix $\mathrm{R}_z = E\left[zz^H\right] = 2\sigma^2\mathrm{I}_{N_t\text{-}L+1}$, I_n denotes the identity matrix of dimension $n \times n$, and $Tr\left(\bullet\right)$ denotes the trace of a matrix.

The minimum mean-squared error (MMSE) is equal to

$$\mathrm{MMSE} = \frac{2\sigma^2 L}{N_t - L + 1},\ \ \text{iff } \mathrm{S}^H\mathrm{S} = \begin{bmatrix} \mathrm{S}_1{}^H\mathrm{S}_1 & \mathrm{S}_2{}^H\mathrm{S}_1 \\ \mathrm{S}_1{}^H\mathrm{S}_2 & \mathrm{S}_2{}^H\mathrm{S}_2 \end{bmatrix} = (N_t - L + 1)\mathrm{I}_{2L} \qquad (5)$$

The training sequences $\mathbf{s}_1$ and $\mathbf{s}_2$ that satisfy (5) are considered as optimal sequences. So, (5) tells us that the optimal sequences have an impulse-like auto-correlation sequence and zero cross correlation.

I can propose a method to encode the training symbols with zero correlation. I transmit the two same consecutive training sequences $\mathbf{s}_1$ at each transmitter. I send $[S_1 \quad -\tilde{S}_1^*]$ at one transmitter and $[S_1 \quad \tilde{S}_1^*]$ at another transmitter. The operation denoted by $(\tilde{\bullet})$ refers to time-reversing a sequence. The $(\bullet)^*$ denotes the complex conjugate of the sequences. According to (2), the received signals during two time slots can be expressed as

$$\begin{bmatrix} \mathrm{y}_1 \\ \mathrm{y}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -\tilde{\mathrm{S}}_1^* & \tilde{\mathrm{S}}_1^* \\ \mathrm{S}_1 & \mathrm{S}_1 \end{bmatrix}}_{\mathrm{S}} \begin{bmatrix} h_{11}(L) & h_{12}(L) \\ h_{21}(L) & h_{22}(L) \end{bmatrix} + z \qquad (6)$$

If the sequence $\mathbf{s}_1$ is symmetric about its center with impulse-like auto correlation, I can get $\mathrm{S}^H\mathrm{S} = (N_t - L + 1)\mathrm{I}_{2L}$ which is the condition to achieve the MMSE and optimal sequence in (5).

VI. Conclusion

This report introduces studied various methods to design the training sequences and estimate the channels for wired MIMO communication systems. I am able to simplify the channel estimation problem by designing the optimal training sequences an impulse-like auto-correlation sequence and zero cross correlation. This method is easy to implement and can get the MMSE.

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