

# **Channel Estimation for Wired MIMO Communication Systems**

Literature Survey

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## **Abstract**

This report investigates channel modeling and estimation for a wired multiuser multicarrier communications system. The special case of a multiple-input multiple-output (MIMO) channel is considered where the different users transmit at the same time and over the same bandwidth. In the report, I will introduce data transmission and training systems. The report also will present a MIMO channel model and Multicarrier Modulations. Then, three typical channel estimation methods are presented and analyzed.

## **I. Introduction**

Communication systems that use multiple transmitters and receivers are often called multiple-input multiple-output (MIMO) systems. MIMO systems can provide both high data reliability and high data rates for a home network. The bonded Asymmetric Digital Subscriber Line (ADSL) is a wired MIMO communication system. By taking multiple ADSL connections and combining them (bonding) into a single 'virtual' connection you can achieve a high speed, resilient connection using a cost effective medium. The current challenges for MIMO systems are still the transmission power, bandwidth, and computational complexity and channel capacity. To estimate an unknown channel is a very important and necessary work before transmitting the real signals since the channel is commonly time-varying. The channel estimation can be performed by either sending a known training/pilot sequence or using cyclic statistics of the received signal.

In this survey, section II introduces some key techniques and describes the multicarrier data transmission and reception for MIMO systems. Section III describes MIMO channel models and provides the Shannon capacity of MIMO channels. Section IV presents and analyzes three channel estimation methods which are based on the simplified estimation, linear interpolation and linear precoding respectively. Section V discusses and compares these methods. Section VI concludes and summarizes this report.

## **II. Background**

All transmission channels are fundamentally analog and thus may exhibit a wide variety of transmission effects. The modulation is to convert a stream of input bits into equivalent analog signals that are suitable for the transmission line. A primary impairment in communications is inter-symbol interference (ISI) which is caused by the memory in the channel. To combat ISI, a receiver usually uses an equalizer to compensate for the spreading in time and distortion in frequency caused by the channel. One technique to avoid ISI, without sacrificing the transmission rate, is Multicarrier Modulation (MCM). In order to obtain the MCM, we can divide broadband channel into narrowband subchannels which have their own center carrier frequencies. There is no ISI in subchannels if each subchannel has the ideal sampling and constant gain. Because of its robustness to multipath, and the ease of implementing it using the fast Fourier transform (FFT), the MCM concept is growing rapidly in practical importance. It has been implemented in several wireline and wireless high-speed data communications standards (ADSL, IEEE 802.11). The discrete multitone modulation (DMT) is a MCM application in the wired communication system. And another increasingly popular multicarrier modulation technique in wireless communications is orthogonal frequency division multiplexing (OFDM).

A Multicarrier Modulation transmitter is shown below:

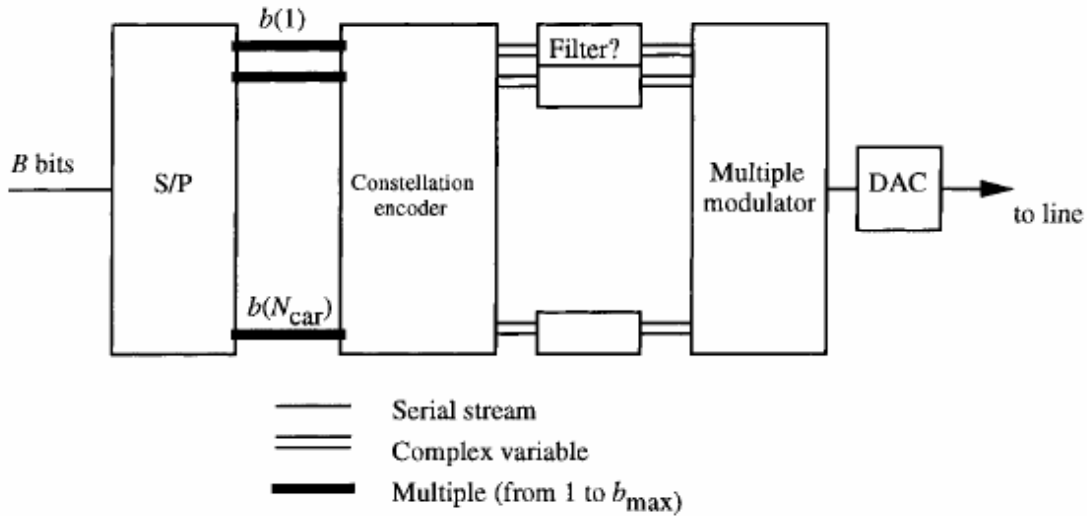


Figure 1 Multicarrier Modulation Transmission

The receiver is the mirror image of the transmitter. The input of the S/P converter is a sequence of symbols of  $B$  bits each; the output for each symbol is  $N_{car}$  groups of  $b(n)$  bit each. That is  $B = \sum_{n \leq N_{car}} b(n)$ . The groups of  $b(n)$  are then constellation-encoded, perhaps filtered, and then modulated onto  $N_{car}$  subcarriers.

### III. MIMO Channel Modeling

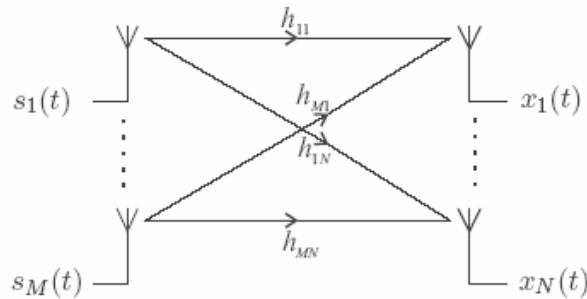


Figure 2 MIMO with M transmitters and N receivers

The relation between the input and output signals of a Multiple-Input Multiple-Output (MIMO) link is represented in the equivalent discrete time base-band model by the complex vector notation  $r = Hs + n$  where  $s$  is the  $(M \times 1)$  complex transmitted signal

vector,  $r$  is the  $(N \times 1)$  complex received signal vector,  $H$  is the  $(N \times M)$  complex channel matrix, and  $n$  is the  $(N \times 1)$  complex noise vector. Further,  $\sigma^2$  denotes the average transmission power;  $M$  and  $N$  are the number of transmitters and receivers, respectively.

#### IV. MIMO Channel Estimation

Although ISI can be avoided by MCM, the phase and gain of each subchannel is needed for coherent symbol detection. An estimate of these parameters can be obtained with pilot/training symbols at the expense of bandwidth. Another way called blind channel estimation to take into account cyclic statistics of the received signal or subspace decomposition of the correlation matrix of the pre-FFT received blocks.

##### 1. Simplified Channel Estimation for OFDM systems [1]

Considering an OFDM system with two transmitters and two receivers, at time  $n$ , a data block  $\{b[n,k]:k=0,1,\dots\}$  is transformed into two different signals  $\{t_i[n,k]:k=0,\dots,K-1 \text{ and } i=1,2\}$  at the transmit diversity processor, where  $K$ ,  $k$ , and  $i$  are the number of subchannels of the OFDM systems, subchannel (or tone) index, and antenna index, respectively. The signal from each receiver can be expressed as

$$r[n,k] = \sum_{i=1}^2 H_i[n,k] t_i[n,k] + w[n,k] \quad (1)$$

where  $H_i[n,k]$  is the frequency response at the  $k$ th tone of the  $n$ th block corresponding to the  $i$ th transmitter which can be expressed as  $H_i[n,k] = \sum_{l=0}^{K_0-1} h_i[n,l] W_K^{kl}$ ,  $K_0$  is the number of nonzero taps of the channel impulse response.

$w[n,k]$  denotes the additive complex Gaussian noise and is assumed to be zero mean with variance  $\sigma_n^2$ . Hence, to obtain  $H_i[n,k]$ , we only need to estimate  $h_i[n,l]$ . if the transmitted

signals  $t_i[n,k]$ 's, for  $i=1,2$  are known,  $\tilde{h}_i[n,l]$ , the temporal estimation of  $h_i[n,l]$ , can be

$$\text{found by [7]} \quad \begin{pmatrix} Q_{11}[n] & Q_{21}[n] \\ Q_{12}[n] & Q_{22}[n] \end{pmatrix} \begin{pmatrix} \tilde{h}_1[n] \\ \tilde{h}_2[n] \end{pmatrix} = \begin{pmatrix} p_1[n] \\ p_2[n] \end{pmatrix} \quad (2)$$

where  $\tilde{h}_i[n]$  is the temporal estimation of channel parameter vector, defined as

$$\tilde{h}_i[n] \triangleq (\tilde{h}_i[n,0], \dots, \tilde{h}_i[n, K_0 - 1])^T \quad (3)$$

and  $q_{ij}[n,l], Q_{ij}[n], p_i[n,l]$  and  $p_i[n]$  are defined as

$$\begin{aligned} q_{ij}[n,l] &\triangleq \sum_{k=0}^{K-1} t_i[n,k] t_j^*[n,k] W_K^{-kl} \\ Q_{ij}[n] &\triangleq (q_{ij}[n, l_1 - l_2])_{l_1, l_2=0}^{K_0-1} \\ p_i[n,l] &\triangleq \sum_{k=0}^{K-1} r[n,k] t_i^*[n,k] W_K^{-kl} \\ p_i[n] &\triangleq (p_i[n,0], \dots, p_i[n, K_0 - 1])^T \end{aligned}$$

For systems with constant modulus modulation,  $\mathbf{Q}_{11}[n] = \mathbf{Q}_{22}[n] = \mathbf{KI}$  and, therefore

$$\begin{aligned} \tilde{h}_1[n] &= \frac{1}{K} (p_1[n] - Q_{21}[n] \tilde{h}_2[n]) & \tilde{h}_1[n] &= \frac{1}{K} (p_1[n] - Q_{21}[n] \tilde{h}_2[n]) \\ \tilde{h}_2[n] &= \frac{1}{K} (p_2[n] - Q_{12}[n] \tilde{h}_1[n]) & \tilde{h}_2[n] &= \frac{1}{K} (p_2[n] - Q_{12}[n] \tilde{h}_1[n]) \end{aligned} \quad (4)$$

From the above equations, if  $\tilde{h}_2[n]$  is known, then  $\tilde{h}_1[n]$  can be estimated without any matrix inversion. However, neither  $\tilde{h}_1[n]$  or  $\tilde{h}_2[n]$  is known. Denote  $\hat{h}_i[n]$ 's for  $i=1,2$  the robust estimation of channel parameter vectors at time  $n$ , i.e.,

$$\tilde{h}_i[n] = \sum_{k \geq 0} f_k \tilde{h}_i[n-k] \quad (5)$$

where  $f_k$ 's ( $k \geq 0$ ) are the coefficients of the robust channel estimator [7], [8] and their Fourier transform denoted by  $\Phi(\omega)$ . If robust estimation of channel parameter vectors at time  $n-1$ ,  $\hat{h}_1[n-1], \hat{h}_2[n-1]$  and are used to substitute  $\tilde{h}_1[n]$  and  $\tilde{h}_2[n]$  in the right sides of (4), then

$$\begin{aligned}\tilde{h}_1[n] &= \frac{1}{K}(p_1[n] - Q_{21}[n]\hat{h}_2[n-1]) \\ \tilde{h}_2[n] &= \frac{1}{K}(p_2[n] - Q_{12}[n]\hat{h}_1[n-1])\end{aligned}\quad (6)$$

The substitution reduces the computational complexity of the channel estimation.

However, it may also cause some performance degradation but it can be negligible.

## 2. Linear Interpolation [2]

At the receiver, after FFT transformation, the signal of the  $k$ th subcarrier can be written

$$\text{as } Y_k = X_k H_k + W, 0 \leq k \leq K-1 \quad (7)$$

where  $X_k$ ,  $H_k$ , and  $W_k$  are the transmitted signal, the channel and zero-mean Gaussian white noise with variance  $\sigma_w^2$ , respectively, and  $K$  is the number of subcarriers. For channel estimation, the  $P$  pilot tones are uniformly inserted into the transmitted signal for channel estimation and  $L$  which is the interval of pilot subcarriers is  $K/P$ .

The estimate of the channel at pilot subcarriers based on least square (LS) estimation is

$$\text{given by } \hat{H}_{pL} = \frac{Y_{pL}}{X_{pL}}, p = 0, 1, \dots, P-1 \quad (8)$$

For  $L$  is greater than 2, an interpolation technique is necessary in order to estimate channel at null or data subcarriers by using the channel information at pilot subcarriers.

The channel estimation at the  $pL+l$  th subcarrier using linear interpolation is given by

$$\hat{H}_{pL+l} = \frac{L-l}{L} \hat{H}_{pL} + \frac{l}{L} \hat{H}_{(p+1)L}, 1 \leq l \leq L-1 \quad (9)$$

Note that the channel information located on the right side beyond the  $(P-1)L$  th pilot subcarrier cannot be obtained from above. For this case, edge interpolation is used as

$$\hat{H}_{(P-1)L+l} = -\frac{l}{L} \hat{H}_{(P-2)L} + \frac{L+l}{L} \hat{H}_{(P-1)L}, 1 \leq l \leq L-1 \quad (10)$$

So, the average mean square error (MSE) of estimation is given as

$$\bar{\varepsilon} = \frac{1}{L} \varepsilon_p + \left( \frac{L-1}{L} - \frac{L-1}{K} \right) \varepsilon_L + \frac{L-1}{K} \varepsilon_G \quad (11)$$

where  $\varepsilon_p$  denotes the MSE of estimation for pilot subcarriers,  $\varepsilon_L$  for the linear interpolation, and  $\varepsilon_G$  for the edge interpolator.

### 3. Linear Precoding by blind estimations [3]

Existing blind channel estimation methods for MCM systems can be classified as statistical and deterministic. The statistical methods explore the cyclostationarity that the cyclic prefix induces to the transmitted signal. They recover the channel using cyclic statistics of the received signal [11] or subspace decomposition of the correlation matrix of the pre-FFT received blocks [12]. This approach belongs to statistical class. Considering an OFDM system and apply at its input a linear precoding block that performs the following task. It transforms the  $i$  th OFDM block of  $N$  information symbols

$\mathbf{d}_{i,k}$

$$s_{i,k} = \frac{1}{\sqrt{1+|A|^2}} (d_{i,k} + (-1)^k A d_{i,T}), k = 0, \dots, N-1 \quad (12)$$

where  $T$  and  $A$  are both predefined numbers, assumed to be known to the transmitter and receiver,  $T$  is an integer in  $[0, N-1]$ , and  $A$  is a pure imaginary number with  $|A| < 1$ .

The coded block  $\{s_{i,k}, k=0, \dots, N-1\}$  goes through the regular OFDM transmission steps.

The  $i$ th received OFDM block after removal of the cyclic prefix (CP) and FFT is

$$\begin{aligned} y_{i,k} &= H(k) s_{i,k} + v_{i,k} \\ &= \frac{1}{\sqrt{1+|A|^2}} H(k) (d_{i,k} + (-1)^k A d_{i,T}) + v_{i,k} \\ & \quad k = 0, \dots, N-1 \end{aligned} \quad (13)$$

where  $H(k)$  is the complex gain of the  $k$ th subcarrier, and  $v_{i,k}$  models the noise. Now, consider the correlation of the signals on the  $k$ th and  $T$ th subcarriers, as

$$z_{k,T} \triangleq E[y_{i,k} y_{i,T}^*]$$

$$= \begin{cases} \frac{(-1)^k A + (-1)^{k+T} |A|^2}{1 + |A|^2} \sigma_d^2 H^*(T) H(k), k = 0, \dots, N-1, k \neq T \\ \sigma_d^2 H^*(T) H(T) + \sigma_v^2, k = T \end{cases} \quad (14)$$

Base on (14), an estimation of the channel  $H(k)$  within the complex constant  $\sigma_d^2 H^*(T)$ ,

can be obtained as

$$\hat{H}(k) \triangleq \begin{cases} \frac{1 + |A|^2}{(-1)^k A + (-1)^{k+T} |A|^2} z_{k,T}, k = 0, \dots, N-1, k \neq T \\ z_{k,T}, k = T \end{cases} \quad (15)$$

Since  $A$  is known to the receiver, the required scaling in (15) is feasible.

## V. Summary

I summarized and compared these methods as below: Estimation

Algorithm	Type	Advantage	Disadvantage
Simplified Estimation[1]	I	The computational complexity	Performance degradation but negligible
Linear Interpolation[2]	I	MSE on Comb-type channel estimation	Block-type estimator for indoor channels
Linear Precoding[3]	II	Converges fast Good for fast-varying channels	Introduce a bias to each carrier

Table1 Comparison of Channel Methods

- Type I: Sending Training/Pilot Sequence method at the transmitter
- Type II: Statistical Blind Channel estimation at the receiver

## VI. Conclusion

This report introduces three typical channel estimation methods. It is intuitional to estimate an unknown channel by sending a known sequence. But blind channel estimation methods avoid the use of pilot symbols, which makes them good candidates for achieving high spectral efficiency.



In the next step, I will propose some improved methods based on above algorithms with specific wired communications. For example, I can make the interpolation length  $L$  adaptive and use the other interpolation method rather than linear one. As for precoding algorithms, I can propose the better transformation formula. I will analyze the bound on the mean square error (MSE) or reduce the computational complexity for my proposed algorithm.

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