Compressive Sensing for Multimedia Communications in Wireless Sensor Networks

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Abstract

Compressive Sensing is an emerging data acquisition scheme with the potential to reduce the number of measurements required by the Nyquist sampling theorem to acquire sparse signals. If a signal is sparse or compressible in a certain basis, compressive sensing computes inner products with a set of basis functions instead of uniformly sampling. In this paper, we examine the performance of compressive sensing for 2-D images in terms of complexity and quality of reconstruction, and consider the benefits of its application to imaging over Wireless Sensor Networks, where stringent constraints exist on energy and bandwidth consumption. We show that we can operate at very low data rates with reduced complexity and still achieve good image quality.

I. INTRODUCTION

The large amount of data needed to acquire a certain signal at the Nyquist sampling rate, especially image and video signals, makes compression an inevitable step before storage or transmission. This double penalty of acquiring and compressing large amounts of information can be eliminated for sparse signals using Compressive Sensing (CS) in which we only acquire relevant components for later reconstruction. The huge savings in energy, memory and processing overhead that CS can achieve proves to be of great practical importance for many applications. Undoubtedly, Wireless Sensor Networks (WSN) tops the list of potential profiters. In fact, the increasing appeal for remote sensing has increased the pressure on WSN in terms of the increasing number of applications to be supported. Simple numerical data acquisition systems such as environmental conditions monitoring and more complex multimedia ones like border surveillance using visual monitoring are now target applications for WSN. Hence, the problems of energy and bandwidth, which are scarce resources in WSN, are exacerbated [1].

II. COMPRESSIVE SENSING

A. Signal Representation & Sparsity

Consider the simple case of a real-valued, finite-length, 1-D, discrete-time signal $\mathbf{x}$ viewed as an $N \times 1$ column vector in $\mathbb{R}^N$. Any signal in $\mathbb{R}^N$ can be expressed as a linear combination of $N \times 1$ basis vectors $\{\psi_i\}_{i=1}^N$ as

$$\mathbf{x} = \sum_{i=1}^{N} s_i \psi_i \qquad \text{or} \qquad \mathbf{x} = \Psi \mathbf{s} \tag{1}$$

where Ψ is the $N \times N$ basis matrix formed by concatenating the column vectors $\{\psi_i\}_{i=1}^N$, and $\mathbf{s}$ is the $N \times 1$ column vector of weighting coefficients given by $s_i = \langle \mathbf{x}, \psi_i \rangle = \psi_i^T \mathbf{x}$ [2].

Vector **x** is said to be K-sparse if it can be expressed only by K basis vectors such that $K \ll N$; i.e. only K of the s_i in (1) are nonzero. This interest in sparse signals comes about from many natural and manmade signals that are compressible in a certain domain: images in the DCT and wavelet domains and audio signals in a localized Fourier domain [2].

Signal sparsity is the foundation of transform coding which is employed in the inefficient but popular sample-then-compress framework. This well-established approach requires acquisition of the full N-sample signal x irrespective of how small the desired K is. Then the encoder must compute all of the N transform coefficients $\{s_i\}$, even though it will discard all but K of them. Finally, the encoder faces the overhead of encoding the locations of the largest K coefficients [2], [3], [4]. Obviously, this is a very wasteful procedure that involves doing far more work than needed.

An alternative is the theory of compressive sensing that exploits signal sparsity [5], bypasses the sampling process and directly acquires a compressed form of the signal by measuring inner products between the signal and a set of functions. By doing so, measurements are no longer point samples, but rather random sums of samples taken across the entire signal.

B. Measurement Matrix

Consider the linear measurement process that computes $M < N$ inner products between **x** and a collection of vectors $\{\phi_j\}_{j=1}^{M}$ via $y_j = \langle \mathbf{x}, \phi_j \rangle$. Let the $M \times 1$ vector **y** be the measurements array y_j and the $M \times N$ matrix Φ be the concatenation of the measurement vectors ϕ_j^T as rows. Substituting in (1), we get:

$$\mathbf{y} = \Phi\mathbf{x} = \Psi\Phi\mathbf{s} = \Theta\mathbf{s} \tag{2}$$

where $\Theta = \Psi\Phi$ is an $M \times N$ matrix that achieves dimensionality reduction [4]. The measurement matrix Φ is given in [2], [5], [6] to be a random matrix whose entries are independent and identically distributed random variables drawn from a zero-mean, $\frac{1}{N}$-variance Gaussian distribution. As a result, the M measurements in **y** are simply randomly weighted linear combinations of the elements in **x**.

C. Signal Reconstruction

Signal recovery takes the measurement vector **y** and the known measurement matrix Φ and reproduces the original signal **x**. But since $M < N$, there are infinitely many solutions $\mathbf{x}'$ that satisfy $\Phi\mathbf{x}' = \mathbf{y}$ [2], [5], [7]. But the magic of CS is that the choice of Φ is in such a way that sparse signals can be almost exactly recovered when $M \geq O(K \log(\frac{N}{K}))$ measurements are made [5], [6]. Perfect reconstruction of K-sparse signals can be achieved using ℓ_1 optimization

$$\hat{\mathbf{s}} = \arg\min \|\mathbf{s}'\|_1 \qquad \text{such that} \qquad \Theta\mathbf{s}' = \mathbf{y} \tag{3}$$

This is a convex optimization problem that reduces to a linear program [5], [6]. If the underlying signal is a 2-D image, the recovery scheme translates into one based on the discrete gradient. Let x_{ij} denote the pixel in the i^{th} row and j^{th} column in the $N \times N$ image **x**. Then the discrete gradient is $D_{ij}x = (D_{h,ij}\ D_{v,ij})$ where

$$D_{h,ij} = \begin{cases} x_{i+1,j} - x_{i,j} & i < N \\ 0 & i = N \end{cases} \qquad \text{and} \qquad D_{v,ij} = \begin{cases} x_{i,j+1} - x_{i,j} & j < N \\ 0 & j = N \end{cases}$$

The total variation of **x** is the sum of the magnitudes of the discrete gradient at every point

$$TV(\mathbf{x}) = \sum_{i,j} \sqrt{(D_{h,ij}x)^2 + (D_{v,ij})^2} \tag{4}$$

And the reconstruction problem in (3) becomes [8]:

$$\hat{\mathbf{s}} = \arg\min TV(\mathbf{s}') \qquad \text{such that} \qquad \Theta\mathbf{s}' = \mathbf{y} \tag{5}$$

III. SYSTEM IMPLEMENTATION MODEL

In order to assess the performance of CS, we implemented the block diagram shown in Figure 1. The input x is one of three 256×256 grayscale test images: Lena, Barbara and Peppers. The projection is done onto a measurement matrix whose elements are generated by gathering 256 samples of the Fourier coefficients of x along each of L radial lines in the frequency plane [8], [9]. L was varied between 20 and 135 in steps of 5 thus generating measurement vectors of size M ranging from $5,000$ to $30,000$. The resulting vector of M measurements is real-valued. A need for quantization

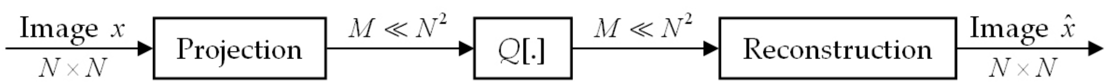

Fig. 1: Simulated System Block Diagram

arises from the fact that, for the WSN application, these measurements have to be transmitted over a digital communication link between a sensor node and the central node where data fusion occurs. Two floating point quantization formats were implemented and the quality of their reconstructed images were evaluated and compared: 16-bit quantization with an exponent length of 9, denoted as $[16\ 9]$ quantization, as well as the less precise 8-bit quantization with a 6-bit exponent and denoted as $[8\ 6]$ quantization.

Reconstruction is then performed on the quantized measurements using the ℓ_1-MAGIC toolbox [8] by solving the total variation minimization problem in (5).

IV. Simulation Results

A. Image Quality

In this section, simulation results that demonstrate the performance of CS in terms of reconstructed image quality are presented. The Structural Similarity Index (SSIM) was the figure of merit used to assess the quality of reconstruction [10]. A subset of the reconstructed images is shown in Figure 2 along with the corresponding SSIM values.

Fig. 2: CS with [8 6] quantization applied to the three test images with resulting SSIM. (a), (b) and (c) are the originals. (d), (e) and (f) are the reconstructed images from 5,000 measurements. (g), (h) and (i) are the reconstructed images from 30,000 measurements.

The quantization loss in quality is examined in Figure 3. While no distortion is captured for the

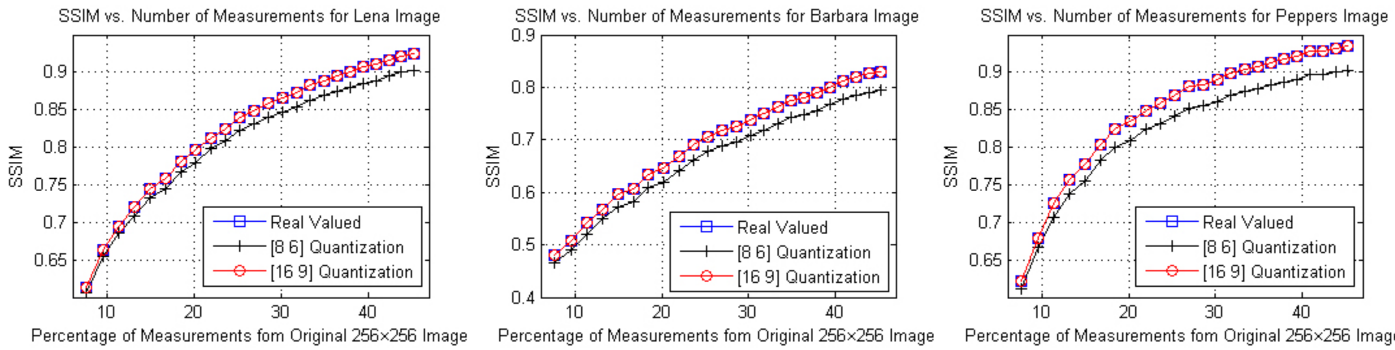

Fig. 3: SSIM vs. number of measurements for Lena (left), Barbara (middle) and Peppers (right).

16-bit quantization due to its high precision, a small degradation in quality is noticed for the 8-bit quantized images which causes the SSIM index to drop by no more than 0.03.

In Figure 4, the SSIM index for the three images is compared. This plot shows the unevenness of CS reconstruction in terms of quality for the different original images. Unlike other compression schemes, the performance of CS will degrade as the complexity in the signal structure increases. This is obvious for the case of the Barbara image where the SSIM index is lower than that for Lena and Peppers by an average of 0.13. This is not the case for JPEG.

For both Figure 3 and 4, an increase in the SSIM value occurs as the size of CS measurements

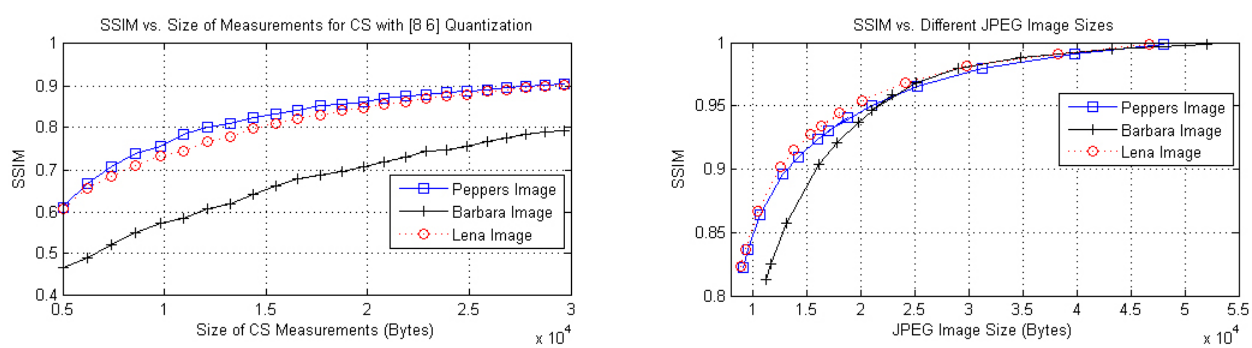

Fig. 4: SSIM vs. image size for CS (left) and JPEG (right) for the three test images.

increases. This will impose a tradeoff between image quality and energy consumption for a sensor node to process and transmit larger measurement sizes.

Figure 5 compares the recovered image quality between CS and JPEG compression. The original image size was 64 $Kbytes$ for all three test images, and the JPEG compression ratio was varied from 10% to 75%. The CS measurement sizes were matched with the resulting JPEG image sizes in order to provide a fair comparison. JPEG yielded better quality for all three images.

B. Complexity

Even though JPEG performs better in terms of image quality, using CS remains a powerful solution for imaging over WSNs. CS off-loads all the complexity to the central node. Unlike JPEG where compression should be done at the sensor to reduce the size of data to be transmitted, CS only requires the sensor node to perform the trivial acquisition step [11] using simple and efficient hardware such as the Single Pixel Camera [4]. The complex task of reconstruction is done at the highly powerful central node. Furthermore, CS is more computationally efficient when compared to JPEG2000 which can achieve double the compression ratios of JPEG. In fact, JPEG2000's additional complexity with respect to JPEG ranges from 38% to 264% for the encoder [12] making it a non-implementable

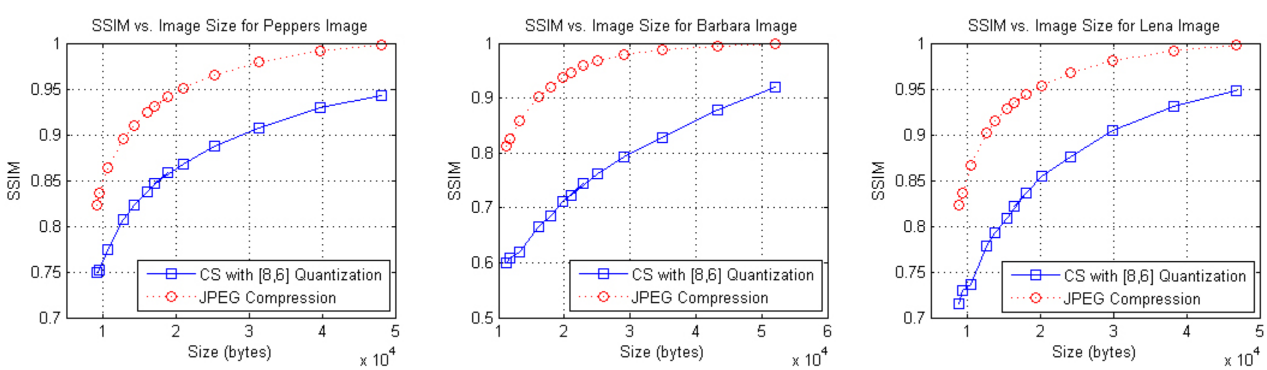

Fig. 5: SSIM vs. image size comparison between CS and JPEG for Peppers (left), Barbara (middle) and Lena (right).

solution in WSN.

V. Conclusion

Compressive sensing is a revolutionary scheme for data acquisition that combines sensing, compression and processing. It promises to substantially increase the performance and capabilities of wireless sensor networks for multimedia applications while reducing their cost, complexity and energy requirements.

References

[1] E. Gurses and O. Akan, "Multimedia communication in wireless sensor networks," *Annals of Telecommunications*, vol. 60, no. 7-8, pp. 799–827, July-August 2005.

[2] R. Baraniuk, "Compressive sensing [lecture notes]," *IEEE Signal Processing Magazine*, vol. 24, no. 4, pp. 118–121, July 2007.

[3] M. Wakin, J. Laska, M. Duarte, D. Baron, S. Sarvotham, D. Takhar, K. Kelly, and R. Baraniuk, "An architecture for compressive imaging," *Proc. IEEE International Conference on Image Processing*, pp. 1273–1276, 8-11 Oct. 2006.

[4] M. Duarte, M. Davenport, D. Takbar, J. Laska, T. Sun, K. Kelly, and R. Baraniuk, "Single-pixel imaging via compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 83–91, March 2008.

[5] D. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289–1306, April 2006.

[6] E. Candes, "Compressive sampling," *International Congress of Mathematics*, vol. 3, pp. 1433–1452, 2006.

[7] J. R. R. Baraniuk and M. Wakin, "Tutorial on compressive sensing," *Proc. Information Theory and Applications Workshop*, February 2008.

[8] E. Candes and J. Romberg, "L1-magic code and documentation," Oct. 2005.

[9] E. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information," *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 489–509, Feb. 2006.

[10] Z. Wang, A. Bovik, H. Sheikh, and E. Simoncelli, "Image quality assessment: from error visibility to structural similarity," *IEEE Transactions on Image Processing*, vol. 13, no. 4, pp. 600–612, April 2004.

[11] J. Romberg, "Imaging via compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 14–20, March 2008.

[12] I. Moccagatta and M. Coban, "Computational complexity evaluation of jpeg-2000," *Proc. of the Society of Photo-Optical Instrumentation Engineers*, vol. 4115, pp. 521–534, Dec. 2000.