Compressive Sensing for Multimedia Communications in Wireless Sensor Networks

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MDDSP Project Literature Survey

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Abstract

The promising scheme of compressive sensing (CS) has overthrown the long-established paradigm for data acquisition that prevailed for a long time. The revelation by Candès, Romberg, and Tao [1] and Donoho [2] that a very small number of projections of a sparse signal into a random set of basis functions can be used to fully reconstruct that signal has led to a serious reassessment of the efficiency of the work required by the Nyquist sampling theorem [3]. This document is a literature review on the topic of CS where we will be examining the theoretical foundations behind this technique as well as one of its hardware architectures in order to pave the way for a study on the benefits of CS in the framework of wireless sensor networks.

I. INTRODUCTION

The large amount of data needed to acquire a certain signal at the Nyquist sampling rate, especially image and video signals, makes compression an inevitable step in order to store or transmit that data. This double penalty of acquiring a large amount of information, added to the overhead of compression can be removed for sparse signals using CS where we only acquire relevant components for later reconstruction. The huge savings in energy, memory and processing overhead that can be achieved as a result of CS proves to be of great importance for many applications, thus taking CS from the idealism of theory to the reality of implementable solutions. Undoubtedly, the prevailing technology of Wireless Sensor Networks (WSN), tops the list of potential profiters. In fact, the increasing appeal for remote sensing has increased the pressure on WSN in terms of the increasing number of applications to be supported. Simple numerical data acquisition systems such as environmental conditions monitoring and more complex multimedia ones like border surveillance using visual monitoring are now target applications for WSN.Hence, the problems of energy and bandwidth, which are scarce resources in WSN, are exacerbated [4].

II. COMPRESSIVE SENSING

A. Signal Representation & Sparsity

[5] and [6] consider the simple case of a real-valued, finite-length, 1-D, discrete-time signal $\mathbf{x}$ viewed as an $N \times 1$ column vector in $\mathbb{R}^N$. Any signal in $\mathbb{R}^N$ can be expressed as a linear combination of $N \times 1$ basis vectors $\{\psi_i\}_{i=1}^N$ as

$$\mathbf{x} = \sum_{i=1}^{N} s_i \psi_i \qquad \text{or} \qquad \mathbf{x} = \Psi \mathbf{s} \tag{1}$$

where Ψ is the $N \times N$ basis matrix formed by stacking the the vectors $\{\psi_i\}_{i=1}^N$ as columns, and $\mathbf{s}$ is the $N \times 1$ column vector of weighting coefficients given by $s_i = \langle \mathbf{x}, \psi_i \rangle = \psi_i^T \mathbf{x}$ [5].

$\mathbf{x}$ is said to be K-sparse, if it can be expressed only by K basis vectors such that $K \ll N$, i.e. only K of the s_i in (1) are nonzero. This interest in sparse signals comes about from many natural and manmade signals that are compressible in a certain domain: images in the DCT and wavelet domains and audio signals in a localized Fourier domain [5].

Signal sparsity is the foundation of transform coding which is employed in the inefficient but popular sample-then-compress framework. This well-established approach requires acquisition of the full N-sample signal x irrespective of how small the desired K is, then the encoder must compute all of the N transform coefficients $\{s_i\}$, even though it will discard all but K of them. Finally, the encoder faces the overhead of encoding the locations of the largest K coefficients [3], [5], [7]. Obviously, this is a very wasteful procedure that involves doing way more work than needed.

The alternative is the theory of compressive sensing that exploits signal sparsity [2], bypasses the sampling process and directly acquires a compressed form of the signal by measuring inner products between the signal and a set of functions. By doing so, our measurements are no longer point sample, but rather random sums of samples taken across the entire signal.

B. Measurement Matrix

Consider the linear measurement process depicted in Figure 1 that computes $M < N$ inner products between $\mathbf{x}$ and a collection of vectors $\{\phi_j\}_{j=1}^{M}$ via $y_j = \langle \mathbf{x}, \phi_j \rangle$. Let the $M \times 1$ vector y be the measurements array y_j and the $M \times N$ matrix Φ be the concatenation of the measurement vectors ϕ_j^T as rows. Substituting in (1), we get:

$$\mathbf{y} = \Phi \mathbf{x} = \Psi \Phi \mathbf{s} = \Theta \mathbf{s} \tag{2}$$

where $\Theta = \Psi\Phi$ is an $M \times N$ matrix that achieves dimensionality reduction [7]. The measurement matrix Φ is given in [2], [8] to be a random matrix. [5] proposes a matrix whose entries are independent and identically distributed (iid) random variables drawn from a zero-mean, $\frac{1}{N}$-variance Gaussian distribution, i.e. white noise. As a result, the M measurements in y are simply randomly weighted linear combinations of the elements in x.

C. Signal Reconstruction

This step must take the measurement vector **y** and the random measurement matrix Φ and reproduce the original signal **x**. But since $M < N$, there are infinitely many solutions $\mathbf{x}'$ that satisfy $\Phi\mathbf{x}' = (y)$ [5], [2], [6]. But the magic of CS is that the choice of Φ is in such a way that sparse signals can be almost exactly recovered [7]. It has been shown in [2], [8] that, when $M \geq O(K \log(\frac{N}{K}))$ measurements are made, perfect reconstruction of K-sparse signals can be achieved using ℓ_1 optimization

$$\hat{s} = \arg\min \|s'\|_1 \qquad \text{such that} \qquad \Theta\mathbf{s}' = \mathbf{y} \tag{3}$$

This is a convex optimization problem that reduces to a linear program [2], [8].

III. Compressive Imaging

In [3] and [7], a new imaging architecture is proposed to optically acquire random projections of a scene without the need to first collect the corresponding pixels. The single-pixel imaging

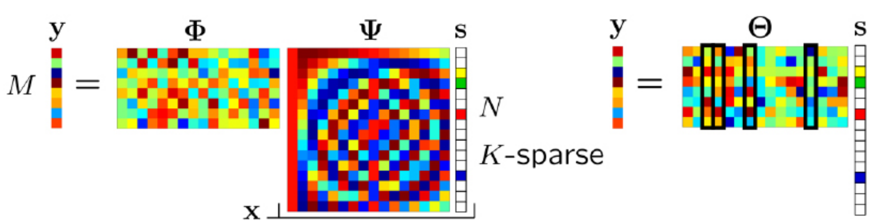

Fig. 1: (Left) CS measurement process with 4-sparse coefficient vector s. (Right) Measurement process in terms of the product matrix $\Theta = \Phi\Psi$ with the four columns corresponding to nonzero s_i highlighted. Measurement vector y is a linear combination of these four columns. (Image from [5])

system measures inner products between the scene under view and a set of test functions, rather than measuring pixel samples. This approach is the fusion of digital micro-mirror devices (DMD) with the theory and algorithms of compressive sensing.

A. Architecture

As shown in Figure 2, the CS camera is very simple to implement and consists of a DMD, two lenses and a single photon detector (the ”single-pixel”) and an analog-to-digital (A/D) converter. The light field from the desired scene is focused by biconvex lens 1 onto the DMD consisting of a 2-D array of N microscopic mirrors that can be independently rotated by $\pm 12^o$. Thus, light falling on each of the mirrors can be reflected either towards another lens in order to make the pixel appear bright (corresponding to a 1 at that pixel in ϕ_m) or away from it making it dark (corresponding to a 0 at that pixel in ϕ_m). The reflected light is collected by biconvex lens 2 and focused onto the photon detector which integrates the product $x[n]\phi_m[n]$ by summing the received intensities to produce $y[m]$ as its output voltage. So, the system sequentially measures the inner products $y[m] = \langle x, \phi_m \rangle$, where x is an N-pixel sampled version of the incident light and ϕ_m is a set of 2-D test functions determined by the orientation of the mirrors [7]. Finally, the A/D converter digitizes the output voltage making it suitable for transmission or storage.

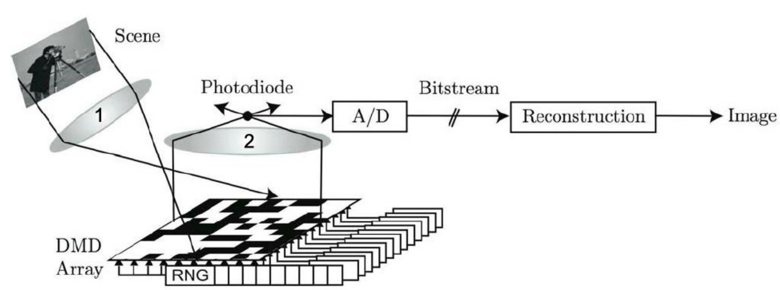

Fig. 2: Compressive Imaging Camera Block Diagram. (Image from [3])

In order to compute CS measurements as specified in [5] by $y = \Phi x$, Φ is chosen to be random using a pseudo-random number generator and thus incoherent with any fixed basis, as imposed by CS theory. Hence, in order to obtain the measurement vector y of size M, this process of randomly reorienting the N mirrors and measuring $y[m]$ has to be repeated M times. The suggested system directly acquires the reduced set of M incoherent projections of an N-pixel image x without first acquiring its N pixel values [3] thus reducing the size, complexity and cost of the data acquisition unit.

B. Advantages

The proposed compressive imaging architecture offers numerous attractive features. The system includes a single photon detector that would enable imaging at wavelengths where silicon based imaging technologies such as CMOS and CCD (charge-coupled devices) are blind or too expensive [7]. In addition, encryption is a byproduct that is ensured by the use of a pseudo-random seed to generate the basis, making the measurements meaningless to undesired observers [9]. Most importantly, for the intended application of WSN, the asymmetry of this approach is its most practical feature. In fact, most of the processing is offloaded from data acquisition to data reconstruction [7], i.e. from the sensors to the fusion center, thus minimizing power consumption at the nodes

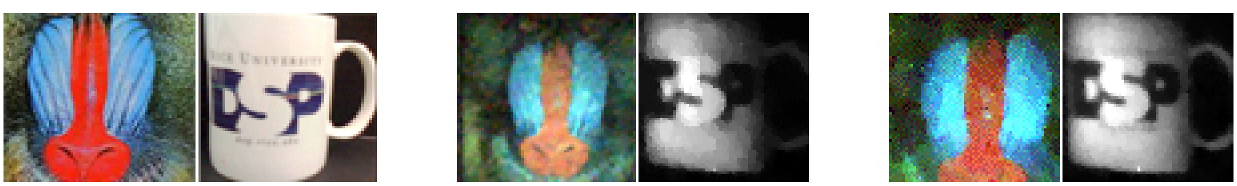

Fig. 3: (Leftmost two) Original images 64×64 pixels. (Center two) Single-pixel camera reconstructed images with $M = 800$ (20%). (Rightmost two) Single-pixel camera reconstructed images with $M = 1600$ (20%). (Images from http://www.dsp.ece.rice.edu/cscamera/)

IV. Distributed Compressed Sensing

In WSN, correlation between different signals is very likely. Signals collected by neighboring nodes often contain some joint structure that allows intelligent processing [9]. Distributed Compressed Sensing (DCS) exploits inter and intra-correlation between sparse signals. In DCS, each node acquires signals independently and projects them to random basis functions; it then transmits the result to a central node that takes care of jointly reconstructing the signals [10]. Three joint sparsity models (JSM) are defined in [10]; JSM-1 models the case when all the signals share a common sparse component and at the same time each signal has a second sparse component that is unique to it. In this case each signal x_i has the following representation $x_i = z + z_i$, where z is the common component for all the signals and z_i is the unique component. JSM-2 models the case when signals are projected into the same sparse index set of basis functions and in this case the signal x_i would be written as $x_i = \Psi\theta_i$, where Ψ represents the common basis and θ_i refers to the signal coefficients in this basis. The last model is JSM-3 where signals share a common component that is not sparse and each signal has its own unique sparse component.

A. Incoherent Measurements & Joint Reconstruction

Sensor j acquires the N-sample signal x_j during a time interval T and computes M_j measurements. The resulting vector is $y_j = x_j\Phi_j$, where Φ_j is an $M_j \times N$ measurement matrix for sensor j. Since reconstruction will not happen at the sensor node and since Φ_j is needed for reconstruction, a pseudo-random measurement matrix is created and the seed is passed to the reconstruction center; the seed will be the node ID [9]. Then each measurement value $y_{j,m}$ in the vector y_j will be quantized and sent along with a timestamp, the index m, and the node ID to the central node. At the receiver, Φ_j is generated from the seed and reconstruction starts with the reception of the measurements M_j.

Different reconstruction methods can be used with different JSMs. Reconstruction in case of JSM-1 is done using a single execution of a program that seeks the sparsest components $[z; z_1; ...; z_j]$ from measurements of different sensors. For JSM-2, a greedy pursuit algorithm is used to reduce the number of measurements needed for independent recovery. Finally, for JSM-3, the common component is estimated first by averaging out the individual components which will be estimated later by subtracting the common component from the signal [9].

B. Advantages of DCS

DCS has many desirable features. First, it provides a simple and universal encoding scheme whereby the sensor nodes only need to compute projections on random basis function sets; these random projections are universal since they are incoherent with any fixed basis function [9]. CS has been shown to be robust to quantization and noise hence DCS enjoys these characteristics. DCS is progressive and error resilient in the sense that better reconstruction and robustness to packet loss are possible with more incoming measurements to the central node.

V. Plans for Implementation

Running simulations to assess the performance of compressive sensing will be done using MATLAB, where we will be relying on a set of existing codes for reconstruction available online at Rice University as well as code we will need to generate to implement different functionalities. A widely used tool to solve optimization operations in compressive sensing is SeDuMi (Self-Dual-Minimization) which is an add-on to MATLAB capable of solving optimization problems with linear, quadratic and semi-definiteness constraints [11]. We intend to implement CS on still images to explore the quality to complexity tradeoff for different sizes and transforms. We will also inspect the enhancements introduced by CS to the energy and bandwidth requirements in WSN.

VI. Conclusion

Compressive sensing is a revolutionary scheme for data acquisition combining sensing, compression as well as processing. Its benefits span a wide spectrum of applications in communications and signal processing. In the framework of WSN, CS promises to reduce cost and energy consumption as well as introduce novel sensing architectures.

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