Optimum Passive Beamforming in Relation to Active-Passive Data Fusion

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Literature Survey

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Abstract

One goal of active-passive data fusion is to combine the complementary information collected by two types of sonar sensors to better perform signal processing. Active sensors, for example, give a good estimate of range, while passive sensors are most efficient at estimating the bearing and radiated spectrum of a contact. If it is assumed that a data fusion framework has collected prior information about the state of a contact, either by active sonar state estimates or by previous passive sonar state estimates, then the opportunity arises to direct the resources of a passive beamformer toward areas of high probability density. The modified beamformer can then provide a better estimate of the position and spectrum of the contact. This paper seeks to find an optimum way of directing the resources of a passive horizontal line array when trying to estimate the direction of arrival (DOA), or bearing, of a contact. Various approaches will be investigated. The resulting refinement in DOA estimation will be compared to popular conventional beamforming (CBF) and adaptive beamforming (ABF) techniques.

I. Introduction

DATA fusion has become an increasingly popular topic in a number of fields (see [1] and the references therein). One of its main goals is to use information from multiple sensors, possibly of different type, to better perform signal processing. The sonar community, for example, has used data fusion in applications such as tracking [2], seafloor mapping [3], ranging, and detection [4]. Additionally, the integration of active and passive sonar systems has long been known to improve performance [5] because of the complimentary information the two systems provide. Specifically, active sensors give a good estimate of range, while passive sensors are most efficient at estimating the bearing and radiated spectrum of a contact. It is therefore natural to apply data fusion to active and passive sonar systems.

Various types of beamformers have been used for passive sonar. The conventional (delay-and-sum) beamformer (CBF) has long been known due to its simple formulation, but its precision is limited by the length of the array. Adaptive beamforming (ABF) can provide much greater precision, but it is sensitive to disagreement between the actual signal and its model (often referred to as mismatch). The most popular form of ABF, originally introduced by Capon [6], is called minimum variance distortionless response (MVDR). This data-dependent beamformer has been analyzed for well over thirty years and is known to have two negative effects due to steering vector (and thus bearing) mismatch: it can lower the output signal power and it can raise the output noise power [7]. Many authors have developed techniques to increase the robustness of MVDR by implementing diagonal loading, white noise gain constraints, and additional linear point constraints (see, e.g., [8] and the references therein). Since diagonal loading can be easily implemented by adding a scaled identity matrix to the array's output cross spectral matrix (CSM) it has arguably become the most popular. It is admittedly *ad hoc*, though, because the level of diagonal loading is a tuning parameter that is often arrived at iteratively.

Only recently have some authors proposed a class of adaptive beamformers that directly address the steering vector uncertainty [9], [10], [11], [12]. These methods belong to the extended class of diagonal loading approaches [13]. Alternatively, Yudichak *et al.* ([14], [15]) suggest using data fusion to densely steer beams in areas of high prior probability density to alleviate the effects of mismatch. This method, when combined with the extended diagonal loading approaches, could provide an improved method for direction of arrival (DOA), or bearing, estimation.

Hence, this paper will focus on improving passive beamforming in relation to active-passive data fusion. Although the approaches introduced here could easily be generalized to apply to many types of passive arrays, the focus will be on DOA estimation for underwater sonar using a passive horizontal line array (HLA).

II. Background

A. Minimum Variance Distortionless Response (MVDR) Adaptive Beamforming

The MVDR beamformer was originally derived by Capon in 1969 [6]. Suppose we have a sampled array output given by $\mathbf{x}_n$, a replica vector $\mathbf{a}(\theta)$, and an estimated CSM, $\mathbf{R}_x$, given by

$$\mathbf{R}_x = \frac{1}{K}\sum_{i=1}^{K}\mathbf{x}_i\mathbf{x}_i^H \tag{1}$$

where i is the snapshot number and K is the number of snapshots used to approximate the CSM. The optimization criteria is to minimize the output power while maintaining a distortionless response in the look direction. The array element weights are found from the solution of

$$\min_{\mathbf{w}} \mathbf{w}^H\mathbf{R_x}\mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H\mathbf{a}(\theta) = 1 \tag{2}$$

to be

$$\mathbf{w} = \frac{\mathbf{R}_{\mathbf{x}}^{-1}\mathbf{a}(\theta)}{\mathbf{a}(\theta)^H\mathbf{R}_{\mathbf{x}}^{-1}\mathbf{a}(\theta)}. \tag{3}$$

The MVDR beamformer is very sensitive to DOA mismatch. In Figure 1 you can see the effect known as squinting in which greater than unity response appears to one side of the contact when the steering vector is not perfect matched to the true DOA.

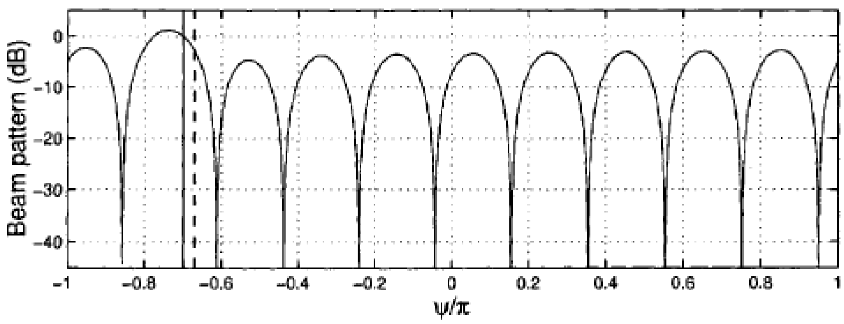

Figure 1. The "squinting" effect. The solid line is where the beamformer is steered to and the dotted line is the true DOA of the contact. At high SNR this effect can become so strong that a null of the beampattern is placed where the contact is. (Figure 6.26 from [7].)

B. Prior Information

Let us assume that a data fusion framework has collected prior information about the state of a contact, either by active sonar state estimates or by previous passive sonar state estimates. Additionally, assume that the only measurement is the bearing from the array to the contact and that it is in the form of a one-dimensional continuous random variable, ϕ, with probability density function (PDF) $p(\phi)$. The generalization to multiple dimensions is trivial and therefore will not be considered at this time.

III. Bayesian Data Fusion Framework

Before attempting to optimize these beamformers in relation to active-passive data fusion let us first discuss how these new measurements will be incorporated into the state estimate of a contact. The information that is provided by each measurement of the HLA is given by a likelihood function $L(\Phi|\phi)$, where $L(\Phi|\phi)$ represents the probability that the measurement Φ would occur given that the contact is actually at ϕ. By a simple application of Bayes' rule, a new state estimate, $p(\phi|\Phi)$ (the posterior PDF), can be formed by combining the previous state estimate, $p(\phi)$ (the prior PDF), and the likelihood function, $L(\Phi|\phi)$:

$$p(\phi|\Phi) = \frac{L(\Phi|\phi)p(\phi)}{\int_0^\pi L(\Phi|\phi')p(\phi')d\phi'}. \tag{4}$$

This is a straightforward technique used by other authors, e.g. [16]. A measure for the amount of refinement given by $p(\phi|\Phi)$ can be given by the difference in *differential entropy* [17] between the prior and posterior PDF. The differential entropy is given by

$$H = \int_0^\pi p(\phi)log_{10}(p(\phi))d\phi \tag{5}$$

and the difference in differential entropy by

$$\Delta H(\theta) = H_{prior} - H_{posterior}(\theta). \tag{6}$$

Here, the argument θ of $H_{posterior}$ is referring to the actual DOA of the contact. The overall effectiveness of a passive beamformer can then be expressed as the expected value of the difference in differential entropy:

$$< \Delta H >= \int_0^\pi p(\theta)\Delta H(\theta)d\theta. \tag{7}$$

This technique was motivated in [14]. Although a positive value of $< \Delta H >$ indicates a refinement in the expected DOA estimate, it does not necessarily indicate refinement towards the actual

DOA. The expected absolute error in DOA estimate,

$$< \Delta\phi >= \int_0^{\pi} p(\theta) \left| \arg\max_{\phi} \{p(\phi|\Phi,\theta)\} - \theta \right| d\theta, \tag{8}$$

gives a better indication of this [15]. In this case $p(\phi|\Phi,\theta)$ refers to the posterior PDF resulting from a measurement when the contact is actually at the DOA θ.

IV. Passive Beamforming Approaches

Now that a method for measuring the effectiveness of a beamformer design has been developed, various new designs can be proposed.

A. Cued Beamforming

In [14], [15] a highly intuitive approach for concentrating beams in areas of high prior probability density was proposed. These *cued beams* were steered within a certain number of standard deviations from the mean of a Gaussian prior PDF. The basic idea behind this was that there will be less of a chance for steering vector mismatch if the beams are closely spaced. Assuming that the number of cued beams equals the number of standard MVDR beams, a greater refinement in bearing could be obtained through an equal expenditure of computational resources. Although advantages have been seen in this technique, a continual spacing of maximum response axes (MRAs), based on the values of $p(\phi)$, would allow for the advantage of a full coverage of bearing. That is, MRAs should continually change from dense spacing in areas of high prior probability to sparse spacing in areas of low prior probability. Another weakness in [14], [15] was that the beamformer output was used as the likelihood function. This method is admittedly *ad hoc* and was meant only to reflect the intuitive idea that it is more likely for a contact to be radiating in the directions where the beamformer output is strongest.

B. Bayesian Beamforming

Other authors, such as [16], have suggested likelihood functions that are more based in theory. Through various approximations, they arrive at a posterior PDF given by

$$p(\phi|\Phi) = cp(\phi)\exp\left\{K\gamma(\mathbf{a}(\theta)\mathbf{R}_{\mathbf{x}}^{-1}\mathbf{a}(\theta))^{-1}\right\} \tag{9}$$

where the constant, c, assures that the PDF sums to 1 and the parameter γ is a function of the number of array elements and the SNR. They then form their beamformer weights as a sum of MVDR beamformers, which are weighted according to the posterior PDF of each look direction. The authors additionally diagonally load $\mathbf{R}_{\mathbf{x}}$ because they have found that it improves performance. Since the inverse CSM, $\mathbf{R}_{\mathbf{x}}^{-1}$, already needs to be calculated for the MVDR beamformer weights, the Bayesian beamformer only requires an additional sum, exponential, and normalization. It is therefore nearly as efficient as the MVDR beamformer. Although the Bayesian beamformer was shown to perform as well as or better than other common approaches, it has a few tuning parameters that cannot be exactly computed. The parameter γ, for example, is a function of SNR, which is in general not know. Additionally, the optimal amount of diagonal loading is not generally known. The final drawback of this design is the assumption about the Gaussianity of the signals and noise. Although this is generally a decent approximation, it is not valid in all cases.

C. Robust Capon Beamforming

The MVDR beamformer is also know as the Capon beamformer. As previously discussed, the MVDR beamformer has long been known to suffer from sensitivity to mismatch. Only recently, though, have some authors derived beamformers that directly account for the steering vector uncertainty [9], [10], [11], [12]. Li *et al.* [9] have proven that although the beamformers derived in [10], [11] were arrived at differently than in their own approach, the solutions are identical. Li *et al.* also take into account a scaling ambiguity and show that their approach is computationally more efficient than the approaches in [10], [11]. In fact, their robust Capon beamformer (RCB) is only slightly more computationally complex than the standard Capon beamformer. Its added complexity comes from an eigendecomposition of $\mathbf{R_x}$ and the solution of a Lagrange multiplier problem by a Newton's method. The optimization criterion of the RCB is to minimize the output power given a steering vector within an ellipsoidal uncertainty set. The weights of the RCB are found from the solution, $\mathbf{a}$, of

$$\min_{\mathbf{a}} \mathbf{a}^H \mathbf{R_x}^{-1} \mathbf{a} \quad \text{subject to} \quad (\mathbf{a}-\mathbf{a}_0)^H \mathbf{C}^{-1} (\mathbf{a}-\mathbf{a}_0) \leq 1 \tag{10}$$

and substitution into (3) to be

$$\mathbf{w} = \frac{(\mathbf{R_x} + \frac{1}{\lambda}\mathbf{I})^{-1}\mathbf{a}_0}{\mathbf{a}_0^H (\mathbf{R_x} + \frac{1}{\lambda}\mathbf{I})^{-1} \mathbf{R_x} (\mathbf{R_x} + \frac{1}{\lambda}\mathbf{I})^{-1}\mathbf{a}_0} \tag{11}$$

where $\mathbf{a}_0$ is the mean steering vector, $\mathbf{C}$ is the covariance matrix of the steering vectors in the ellipsoid, and λ is the Lagrange multiplier solution to the problem (see [9] for intermediate steps). The RCB has been shown to be robust to steering vector mismatch as it outperforms the standard and diagonally loaded Capon beamformer in a variety of scenarios [9].

V. Performance Analysis

The computational complexity of each design discussed is comparable. Some, obviously, are slightly more efficient. Table I lists the complexity of each and the assumptions made in their design.

Simulated data generated by the Sonar Simulation Toolset (SST) [18], [19] will be used to assess the effectiveness of these designs, along with newly proposed designs. Specifically,

Design	Computational Complexity	Assumptions Made
MVDR Beamforming	Low	Exact knowledge of steering vector
Cued Beamforming	Low (equal to MVDR)	Knowledge of a prior PDF
Bayesian Beamforming	Low	Knowledge of a prior PDF and SNR
Robust Capon Beamforming	Medium	Steering vector uncertainty set is known

Table I
COMPARISON OF DIFFERENT BEAMFORMERS

the behavior of the expected difference in differential entropy will be evaluated as a function of various prior PDFs, contact SNRs, and beamformer design parameters. Additionally, the expected absolute error in DOA estimation will be analyzed.

VI. CONCLUSION

In the context of data fusion, the need to form an optimal likelihood function is quite apparent. One approach would be to continue to use the beamformer output as the likelihood function. This would allow one to focus on the design of the beamformer. A possible technique would be to use the robust Capon beamformer proposed in [9] combined with the cued beamforming approach of [14], [15]. Since the RCB is designed based on a steering vector uncertainty set, one could vary the uncertainty set for each beam in relation to its spacing from adjacent beams. This would ensure a low amount of scalloping loss (loss in beamformer gain). Although a fine bearing estimate would not be achieved for contacts in areas of low prior probability, they would be detected by at least one wide beam. After a few updates within the data-fusion framework the DOA estimate would become refined.

Instead of directly using the beamformer output as the likelihood function, the beampattern of each beam could also be taken into account. In other words, when the beamformer output is used directly only one value is obtained at each MRA, but since the beampattern of each beam can be calculated, a weighted average of each beampattern could be also be used. This would provide a more finely sampled likelihood function and take more into account than the beamformer output by itself. The most probabilistically valid approach, though, would be to combine some form of the beamformer output with assumptions about the distributions of the signal and noise (similarly to [16]).

It is not clear which of these designs will work best in a data fusion framework. This literature survey will continue with a performance analysis (described in Sec. V) of the existing and new beamformer designs discussed.

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