Signals and Systems

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Signals

- **Continuous-time signals are functions of a real argument**

  $x(t)$ where t can take any real value

  $x(t)$ may be 0 for a given range of values of t

- **Discrete-time signals are functions of an argument that takes values from a discrete set**

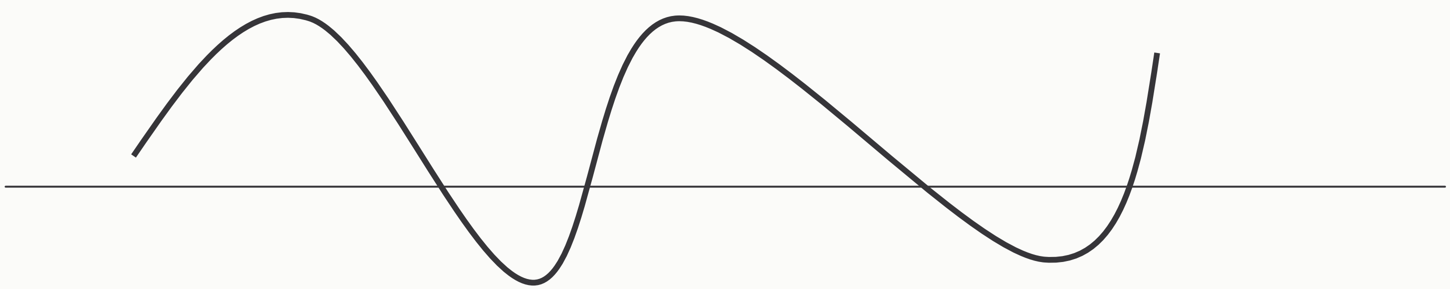
  $x[n]$ where $n \in \{...-3,-2,-1,0,1,2,3...\}$

  Integer index n instead of time t for discrete-time systems

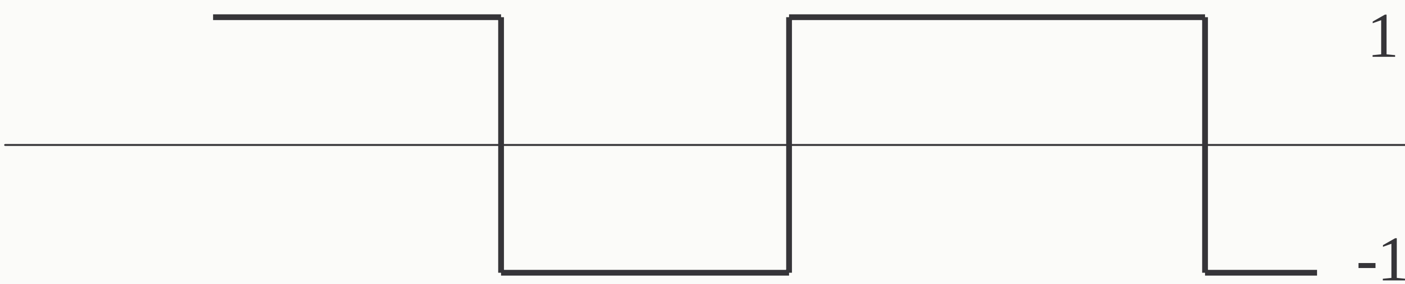
- **Values for** x **may be real, complex, or some other data type**

Analog vs. Digital

- **Amplitude of an analog signal can take any real or complex value at each time/sample**



- **Amplitude of a digital signal takes values from a discrete set**



Systems

- **A system is a transformation from one signal (called the input) to another signal (called the output or the response).**
- **Continuous-time systems with input signal x and output signal y (a.k.a. the response):**

  $y(t) = x(t) + x(t-1)$

  $y(t) = x^2(t)$
- **Discrete-time system examples**

  $y[n] = x[n] + x[n-1]$

  $y[n] = x^2[n]$

Audio Compact Discs

- **Human hearing is from about 20 Hz to 20 kHz**
- **Sampling theorem: sample analog signal at rate of more than twice highest analog frequency**
  - Apply a lowpass filter to pass frequencies up to 20 kHz; e.g. a coffee filter passes water (small particles) through but not coffee grounds (large particles)
  - Lowpass filter needs 10% of maximum passband frequency to roll off to zero (2 kHz rolloff in this case)
  - Sampling at 44.1 kHz captures analog frequencies that are less than 22.05 kHz

Signal Processing Systems

- Speech synthesis and speech recognition
- Audio CD players
- Audio compression (MP3, AC3)
- Image compression (JPEG, JPEG 2000)
- Optical character recognition
- Video CDs (MPEG 1)
- DVD, digital cable, and HDTV (MPEG 2)
- Wireless video (MPEG 4/H.263)

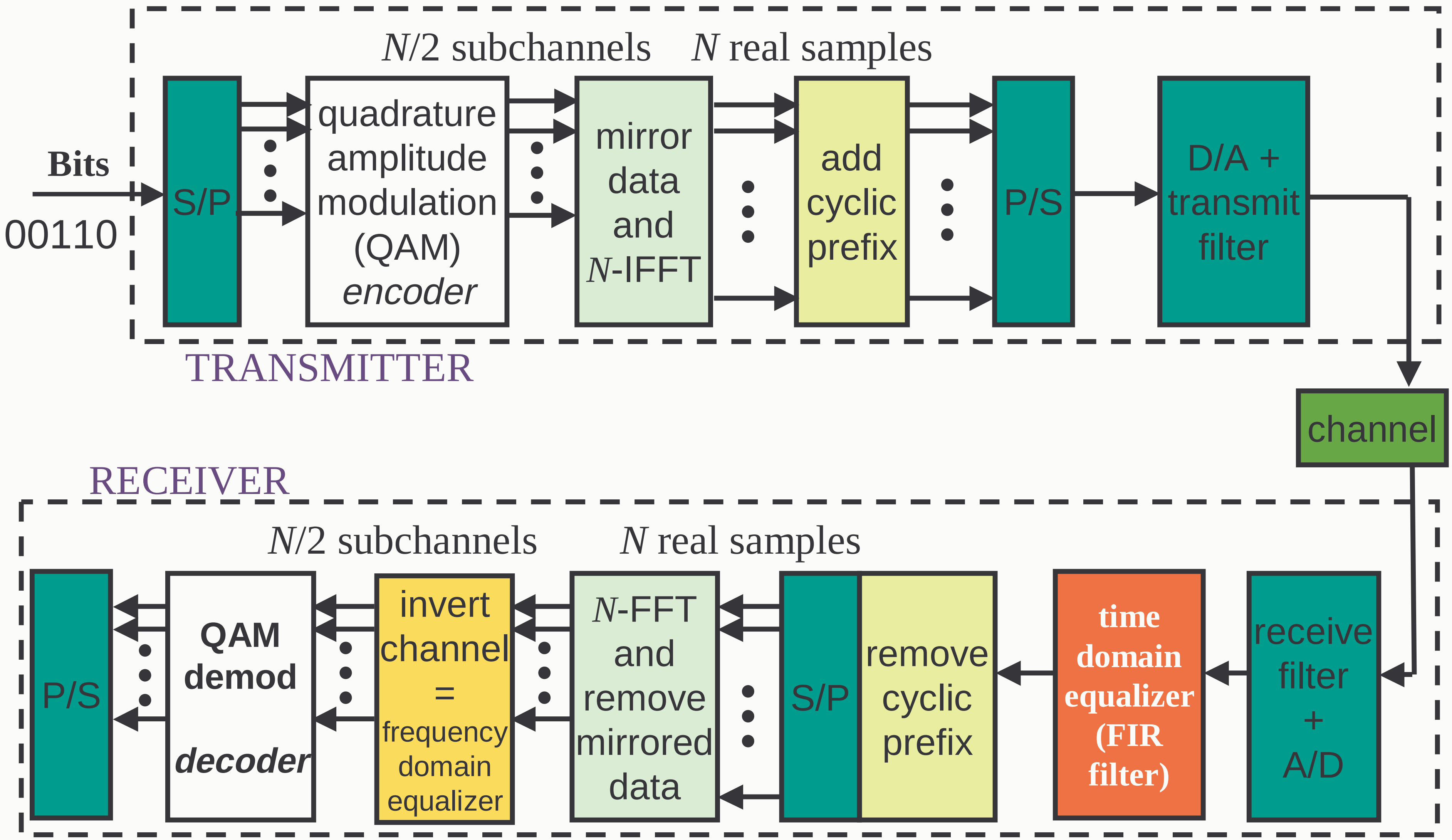
Communication Systems

- **Voiceband modems (56k)**
- **Digital subscriber line (DSL) modems**
  - ISDN: 144 kilobits per second (kbps)
  - Business/symmetric: HDSL and HDSL2
  - Home/asymmetric: ADSL and VDSL
- **Cable modems**
- **Cell phones**
  - First generation (1G): AMPS
  - Second generation (2G): GSM, IS-95 (CDMA)
  - Third generation (3G): cdma2000, WCDMA

Analog

Digital

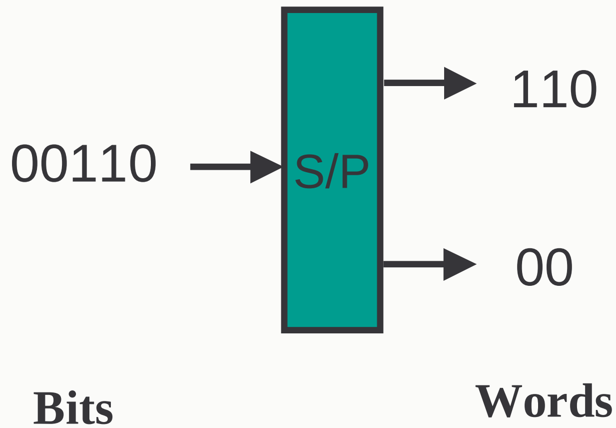
ADSL Modem



N/2 subchannels
N real samples
Bits
00110
S/P
quadrature amplitude modulation (QAM) encoder
mirror data and N-IFFT
add cyclic prefix
P/S
D/A + transmit filter
TRANSMITTER
channel
RECEIVER
N/2 subchannels
N real samples
P/S
QAM demod decoder
invert channel = frequency domain equalizer
N-FFT and remove mirrored data
S/P
remove cyclic prefix
time domain equalizer (FIR filter)
receive filter + A/D

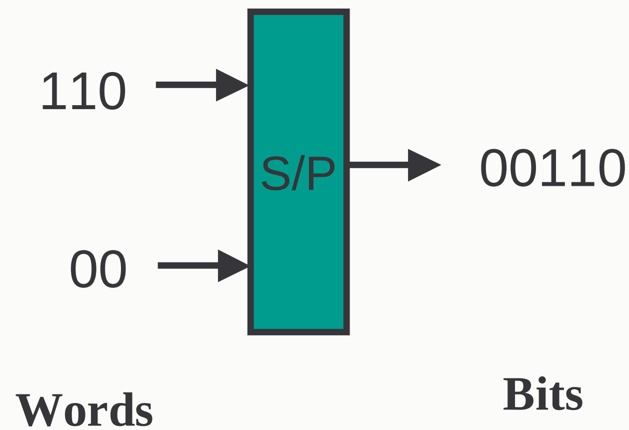
Bit Manipulations

- Serial-to-parallel converter



- Example of one input bit stream and two output words

- Parallel-to-serial converter



- Example of two input words and one output bit stream

Amplitude Modulation by Cosine

- Multiplication in time: convolution in Fourier domain

$$y(t) = f(t)\cos(\omega_0 t)$$

$$Y(\omega) = \frac{1}{2\pi} F(\omega) * \pi\left(\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right)$$

- Sifting property of Dirac delta functional

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} \delta(\tau)\, x(t-\tau)\, d\tau = x(t)$$

$$x(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} \delta(\tau - t_0)\, x(t-\tau)\, d\tau = x(t-t_0)$$

- Fourier transform property for modulation by a cosine

$$Y(\omega) = \frac{1}{2} F(\omega+\omega_0) + \frac{1}{2} F(\omega-\omega_0)$$

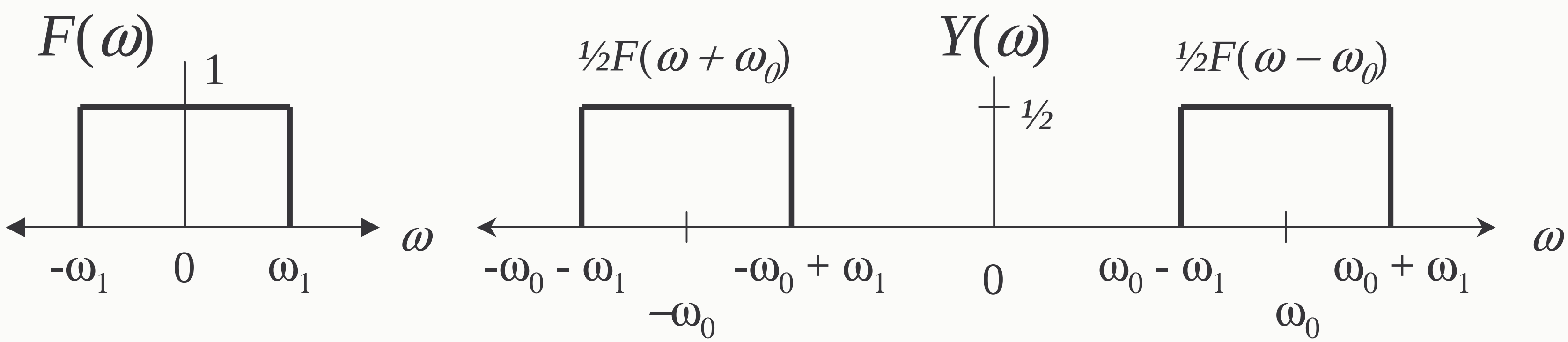
Amplitude Modulation by Cosine

- **Example:** $y(t) = f(t)\cos(\omega_0 t)$

  Assume $f(t)$ is an ideal lowpass signal with bandwidth ω_1

  Assume $\omega_1 << \omega_0$

  $Y(\omega)$ is real-valued if $F(\omega)$ is real-valued



- **Demodulation: modulation then lowpass filtering**
- **Similar derivation for modulation with** $\sin(\omega_0 t)$

Amplitude Modulation by Sine

- **Multiplication in time is convolution in Fourier domain**

$$y(t) = f(t)\sin(\omega_0\, t)$$

$$Y(\omega) = \frac{1}{2\pi} F(\omega) * j\pi\left(\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right)$$

- **Sifting property of the Dirac delta functional**

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} \delta(\tau)\, x(t-\tau)\, d\tau = x(t)$$

$$x(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} \delta(\tau - t_0)\, x(t-\tau)\, d\tau = x(t-t_0)$$

- **Fourier transform property for modulation by a sine**

$$Y(\omega) = \frac{j}{2} F(\omega+\omega_0) - \frac{j}{2} F(\omega-\omega_0)$$

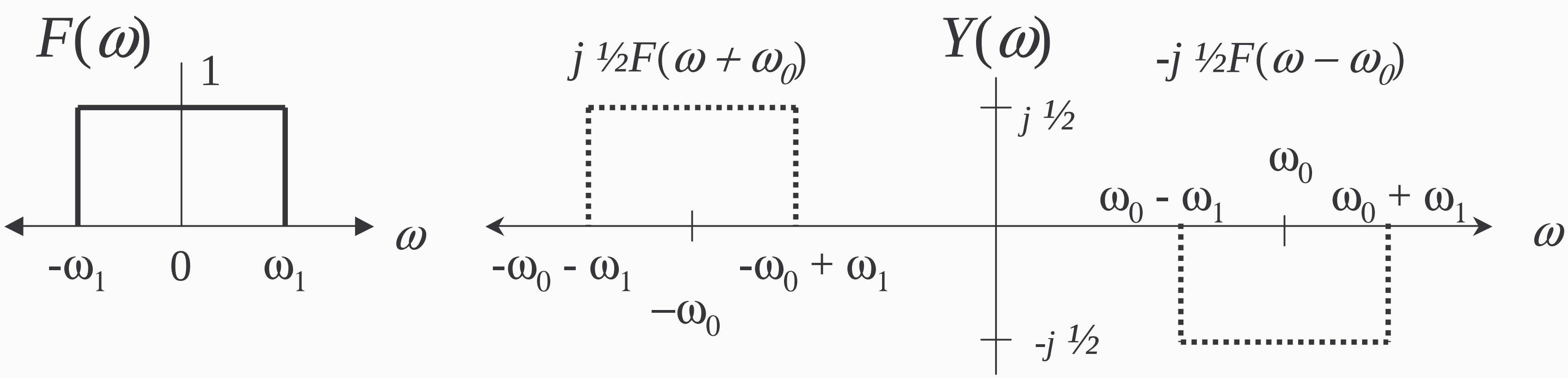
Amplitude Modulation by Sine

- **Example:** $y(t) = f(t)\sin(\omega_0 t)$

  Assume $f(t)$ is an ideal lowpass signal with bandwidth ω_1

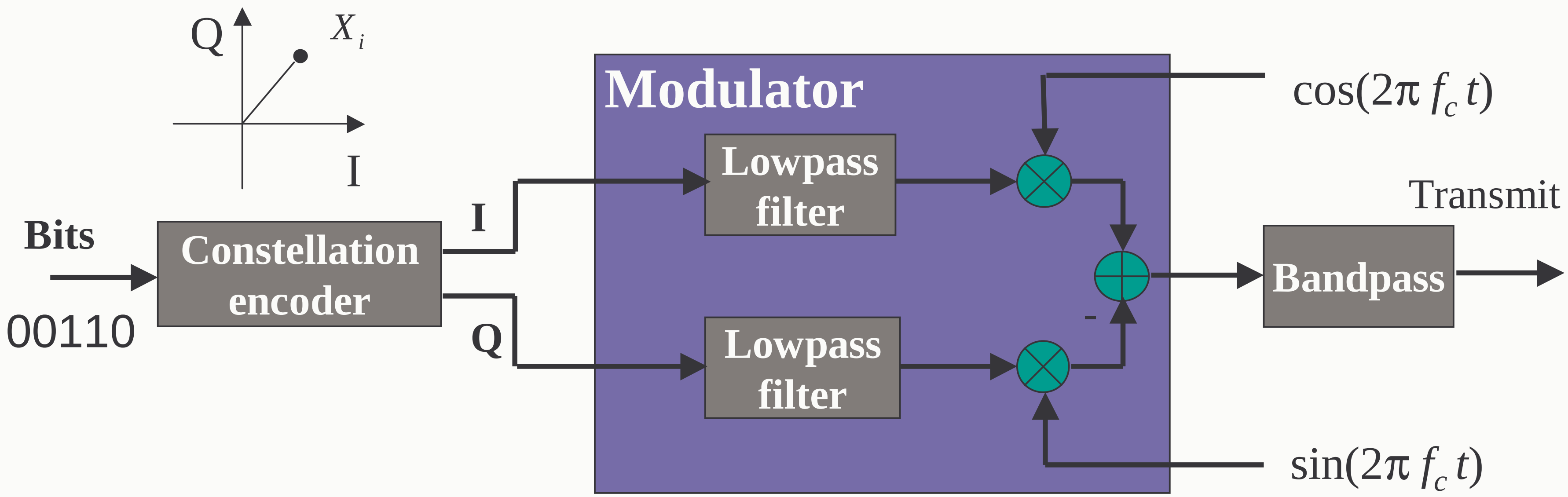
  Assume $\omega_1 << \omega_0$

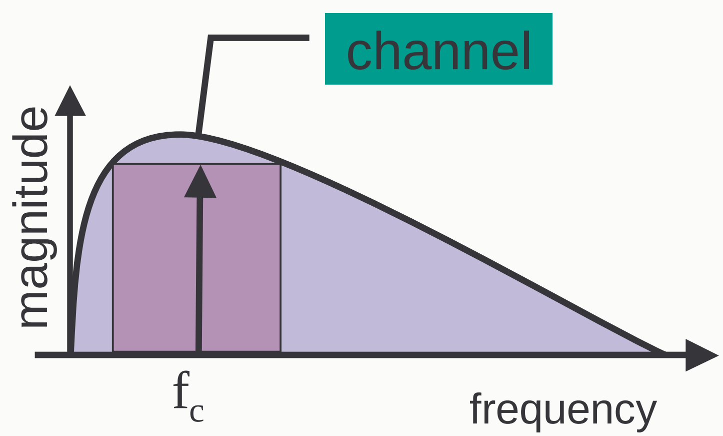
  $Y(\omega)$ is imaginary-valued if $F(\omega)$ is real-valued



- **Demodulation: modulation then lowpass filtering**

Quadrature Amplitude Modulation





- One carrier
- Single signal, occupying the whole available bandwidth
- Symbol rate is bandwidth of the signal being centered on the carrier frequency