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PROJECT: T1E1.4 Standards Project (HDSL2)

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TITLE: A 512-State PAM TCM Code for HDSL2

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ABSTRACT:

In this contribution a 512-state PAM TCM code for HDSL2 is presented. This code can achieve a 5.1dB coding gain with 217μsec decoding latency or a 5.0dB coding gain with only 124μsec latency. The coset encoder is a rate 1/2 feed-forward convolutional encoder and the Viterbi decoder can be implemented very easily. Together with the Tomlinson-Harashima precoder and the OPTIS transmission, more than 6.0dB coded margin can be achieved even under severe mixed crosstalk environments.

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1. Introduction

In this contribution we present a 512-state PAM TCM code that is compatible with the newly proposed OPTIS transmission system [4,5]. Assuming Tomlinson-Harashima precoding, this code can achieve 5.1dB coding gain with a latency of 217μsec or 5.0dB coding gain with a latency of only 124μsec. Together with the OPTIS transmission system, more than 6dB coded performance margin can be achieved. Details of the code and its performance are given in the following sections.

2. Encoder

As shown in Figure 1, the 1.552Mbps source bit sequence is first equally distributed into three 517 1/3Kbps sequences: x_0, x_1, and x_2. The x_0 bit is encoded by a rate 1/2 512-state feed-forward convolutional encoder, shown in Figure 2, to generate coset bits y_0 and y_1. Two generator polynomials are required to uniquely specify a rate 1/2 convolutional encoder. In Figure 2 the generator polynomial for bit y_0 is denoted (in octal numbers) as $g_0 = 0556$ and the generator polynomial for bit y_1 is denoted as $g_1 = 1461$. The x_1 and x_2 bits are connected to the index bits y_2 and y_3 directly. The 16PAM constellation bits-to-symbol mapping is shown in Figure 3.

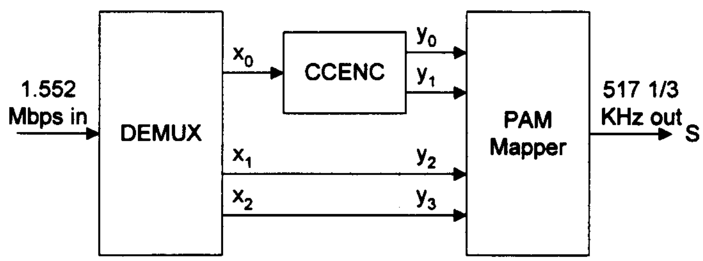

Figure 1: Encoder Block Diagram.

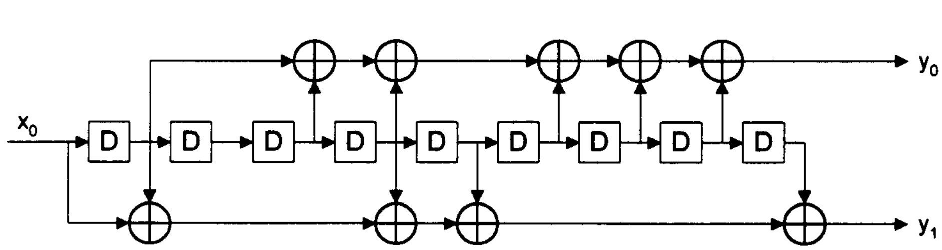

Figure 2: 512-State Feed-Forward Concolutional Encoder.

-15	-13	-11	-9	-7	-5	-3	-1	1	3	5	7	9	11	13	15	
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	y_0
0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	y_1
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	y_2
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	y_3

Figure 3: 16PAM Bits-to-Symbol Mapping.

3. Transmitter/Channel/Receiver

It is assumed that the Tomlinson-Harashima precoder is used, which implies constellation expansion should be factored in the decoder operations. It is also assumed that the noise at the decoder input is additive white Gaussian.

4. Code Performance

The code performance can be evaluated both theoretically and through simulations. A Viterbi decoder with finite trace-back depth is assumed. In Section 5 the decoder complexity and feasibility will be addressed.

4.1 BER Upper Bound

For TCM codes, union upper bounds can be derived based on the pairwise squared Euclidean distances between all possible codewords (see [2] and [3] for references). Let Δ be the distance between the constellation points (Δ = 2 in Figure 3). Let d_{free} be the free distance of the code normailzed by Δ^2 and SNR_dB be the SNR level in dB and E_s be the average PAM symbol energy ($E_s = 256/3$ in our precoded case). The upper bounds for the first event error probability, P_e, and the bit error probability, P_b, are given by:

Equation 1: $$P_e \le \frac{1}{2} \sum_{j=d_{free}} n_j \cdot \mathrm{erfc}\left(\frac{1}{2\cdot\sqrt{2}} \cdot \sqrt{\frac{j\cdot\Delta^2}{E_s}\cdot 10^{0.05\cdot SNR\_dB}}\right),$$

Equation 2: $$P_b \le \frac{1}{6} \sum_{j=d_{free}} e_j \cdot \mathrm{erfc}\left(\frac{1}{2\cdot\sqrt{2}} \cdot \sqrt{\frac{j\cdot\Delta^2}{E_s}\cdot 10^{0.05\cdot SNR\_dB}}\right),$$

where n_j is the average number of codewords at squared distance $j\cdot\Delta^2$ from a specific codeword with the first branch in error and e_j is the average total number of information bit differences in those codewords, including the uncoded bits. The average is taken over all possible codewords. A list of the numbers n_j and e_j is called the distance spectrum of the code.

For the 512-state feed-forward code in Figure 2, the first five terms of the distance spectrum is shown in Table 1.

512-state code with $g_0 = 0556$, $g_1 = 1461$ free distance is $16\cdot\Delta^2$.		
normalized distance	n_j	e_j
16	2	2
17	0	0
18	44	274
19	0	0
20	248	2468

Table 1: Distance Spectrum of the 512-State PAM TCM Code.

In Figure 4 the BER bound is compared with the simulation results. It is evident that the bound is very tight at BER levels below 1E-4. Both curves reach 1E-7 BER at SNR = 22.6dB.

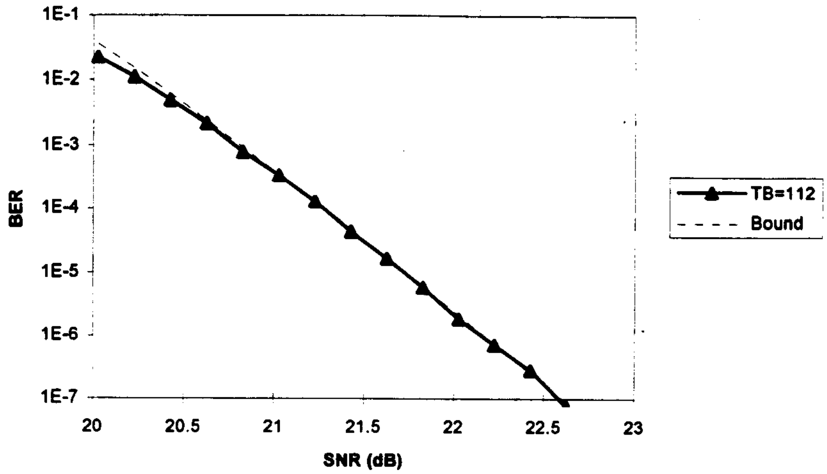

Figure 4: BER Performance of the 512-State Code – Bound vs. Simulation.

4.2 Finite Trace-Back Depth

In this section the effect of finite trace-back (TB) depth to the BER performance is presented. Figure 5 shows the simulated BER vs. trace-back depth at two SNR levels.

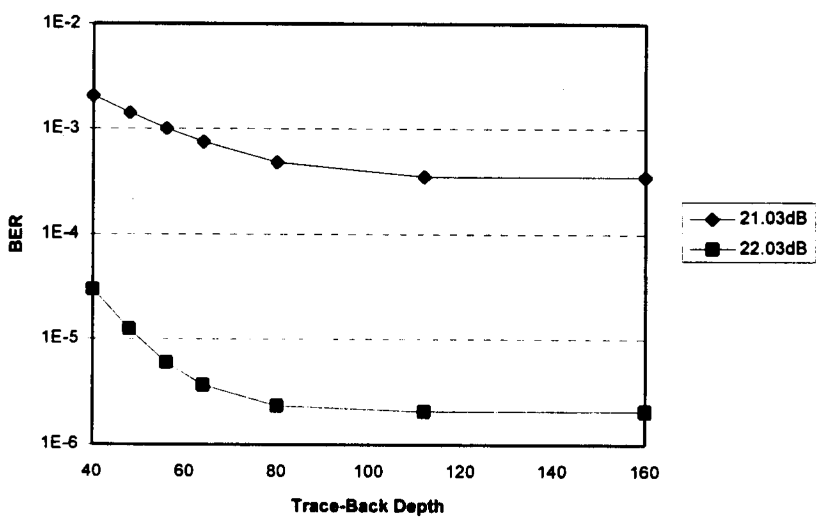

Figure 5: Trace-Back Depth vs. BER.

Based on Figure 5, the improvement by using longer trace-back depth diminishes beyond TB = 80. The performance degradation due to shorter trace-back depths is estimated in Table 2 using TB = 160 as a reference.

Trace-Back (symbols)	Decoding Latency	Degradation
40	77.32μsec	0.53dB
48	92.78μsec	0.38dB
56	108.25μsec	0.23dB
64	123.71μsec	0.1dB
80	154.64μsec	0.03dB
112	216.49μsec	0dB
160	309.28μsec	0dB

Table 2: Estimated Degradation vs. Trace-Back Depth.

5. Decoder Complexity

Since the encoder is in feed-forward form, the state transition follows very simple rules and the read/write operations of the Viterbi decoder can be easily partitioned into parallel processing units. For each ACS unit, the 1/2 code rate enables particularly simple implementations.

The majority of the decoder circuit is composed of a few blocks of RAM's that can be packed tightly. The trace-back buffer size dominates the memory requirements. With TB = 112, the required buffer size is $512 \cdot 112 = 57344$ bits, or 7 kilobytes. By adding the path metric registers, the total memory size is less than 10 kilobytes.

6. Optimality of the Code

Exhaustive searches of all linear feed-forward Z/4Z partitioned TCM codes with up to 2048 states have been conducted. The best codes are chosen based on their BER bound values (Equation 2) at SNR = 22.8dB. Table 3 shows the search results with various constraint lengths. For comparison purpose, the parallel transition bound is used as a reference (shown as "limit" in the last row).

# states	gen. poly.	SNR @ BER=10^{-7}	coding gain rel. to 27.7dB(*)	d_{free}	n_j	e_j
32	g0 = 10 g1 = 45	23.58dB	4.12dB	13	12,28,56,126,236	50,168,436,1122,2458
64	g0 = 032 g1 = 135	23.32dB	4.38dB	14	8,32,66,84,236	48,236,510,930,2504
128	g0 = 052 g1 = 341	23.07dB	4.63dB	14	4,8,14,56,136	16,24,110,460,1484
256	g0 = 336 g1 = 755	22.83dB	4.87dB	16	14,0,108,0,484	88,0,928,0,5470
512	**g0 = 0556 g1 = 1461**	**22.60dB**	**5.10dB**	**16**	**2,0,44,0,248**	**2,0,274,0,2468**
1024	g0 = 1512 g1 = 2461	22.44dB	5.26dB	16	2,0,4,28,68	2,0,20,258,632
2048	g0 = 2202 g1 = 4105	22.26dB	5.44dB	16	2,0,0,16,12	2,0,0,48,132
limit	N/A	21.46dB	6.24dB	16	2,0,0,0,0	2,0,0,0,0

(*): The 27.7dB number is roughly the required SNR level for uncoded 8PAM to reach SER = 1E-7 and has been used as a reference point for margin calculations in the other contributions [4,5,7].

Table 3: Optimal Linear Feed-Forward Z/4Z TCM Codes.

The optimal codes with up to 128 states are equivalent to the previously known systematic feedback codes published in the literature [1]. By equivalence we mean the codes have exactly the same set of codewords, and hence their first event error probability will be the same. However, their BER performance can be different.

The codes with 256 or more states are new. The 512-state code is equivalent to a systematic feed-back code found by Chris Heegard [6]. In a previous contribution [7] two feed-forward PAM TCM codes are used in the simulations. The corresponding new codes listed here are slightly better.

As the constraint length increases, the incremental coding gain decreases. This is clearly shown in Figure 6. Before reaching 512 states, the incremental coding gains are about 0.23~0.26dB. After 512 states the incremental coding gains reduce to about 0.16~0.18dB. It is noticed that the 512-state code is about 1.15dB away from the parallel transition limit at the BER = 1E-7 level. Based on the hardware complexity vs. coding gain trade-off, the 512-state code appears to be a good choice.

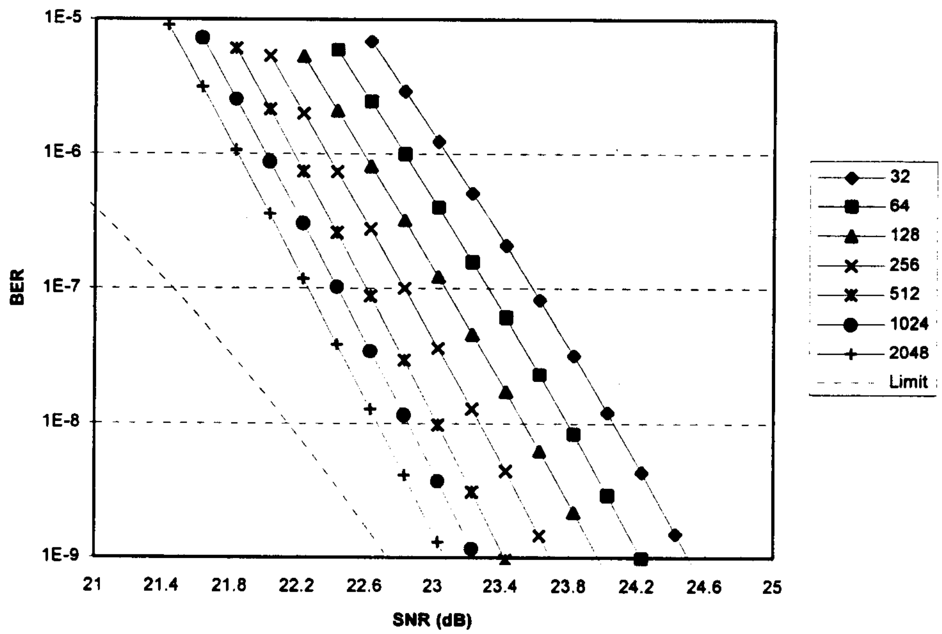

Figure 6: BER Bounds of FFD PAM TCM Codes.

7. Conclusion

A 512-state PAM TCM code has been presented. Its performance is confirmed by theoretical bounds as well as by simulations. The code can achieve 5.1dB coding gain with a 217μsec decoding latency or 5.0dB coding gain with a 124μsec latency. The decoder can be easily implemented due to the nature of the rate 1/2 feed-forward encoder and the regular Viterbi decoder structure. With the Tomlinson-Harashima precoder and the OPTIS transmission, more than 6.0dB coded margin can be achieved with less than 217μsec decoding latency even in severe mixed crosstalk environments [4,8].

8. References

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