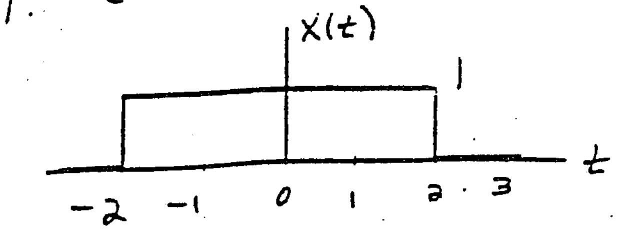
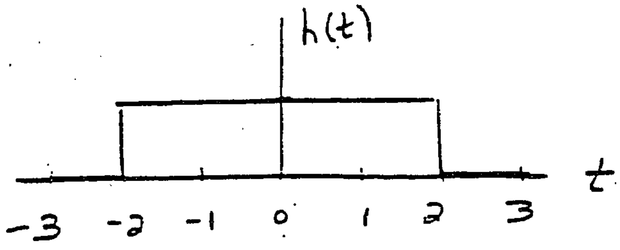
Prof. Evans

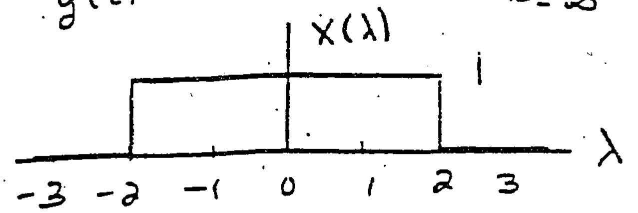
Convolution of Two Rectangular Pulses

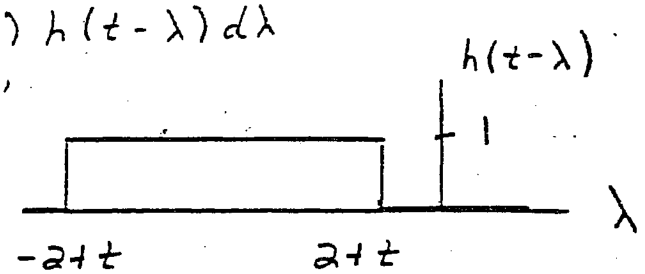
1. Continuous-Time Convolution

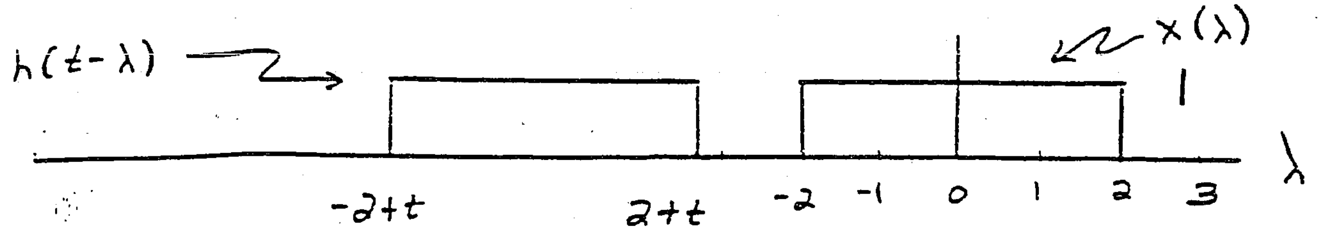




$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$



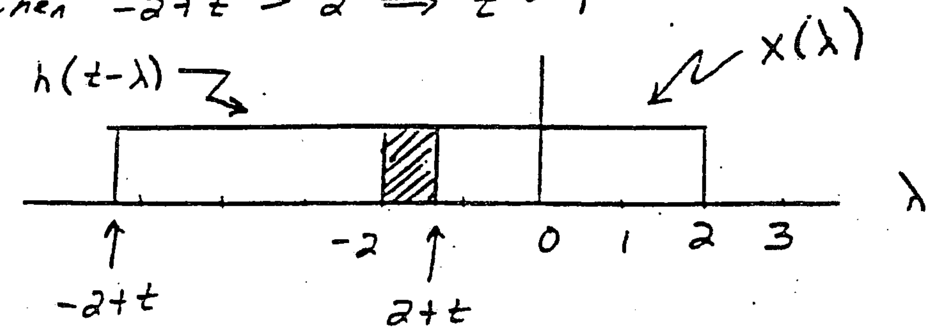




There is no overlap when $2+t < -2 \Rightarrow t < -4$

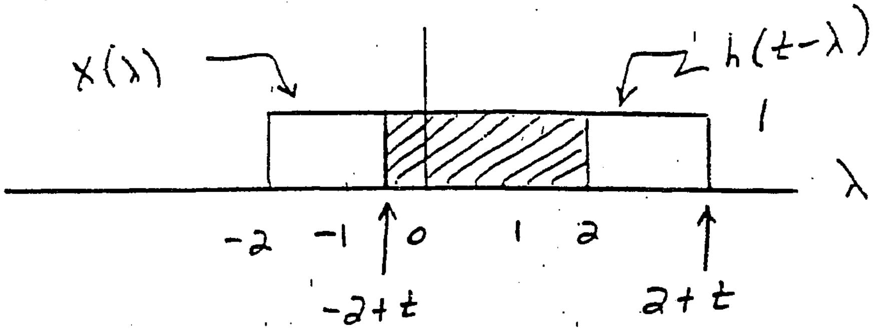
There is no overlap when $-2+t > 2 \Rightarrow t > 4$

For $-4 \le t \le 0$, there is overlap from $\lambda = -2$ to $\lambda = 2+t$



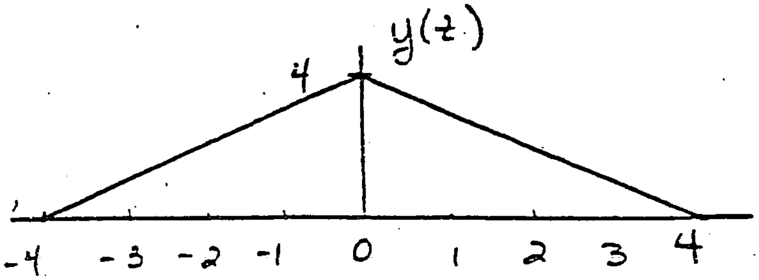
$$y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda = \int_{-2}^{2+t} 1 \cdot d\lambda = \Big[\lambda\Big]_{-2}^{2+t} = t+4$$

For $0 < t \leq 4$, there is overlap from $\lambda = -2+t$ to $\lambda = 2$



$$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,d\lambda = \int_{-2+t}^{2} 1 \cdot d\lambda = [\lambda]_{-2+t}^{2} = 4-t$$

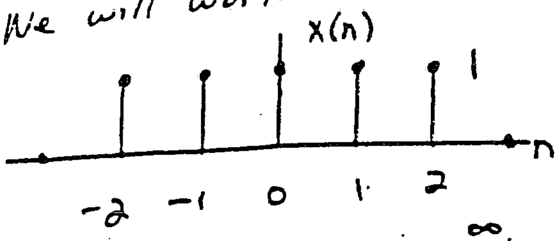
$$y(t) = \begin{cases} 0 & \text{for } t < -4 \\ 4+t & \text{for } -4 \leq t \leq 0 \\ 4-t & \text{for } 0 < t \leq 4 \\ 0 & \text{for } t > 4 \end{cases}$$

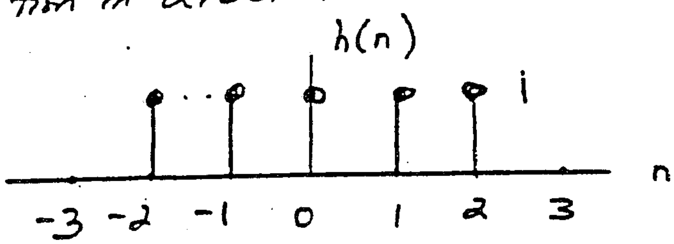


To check a convolution result, check the value of $y(t)$ at the endpoints of each interval — they should agree.

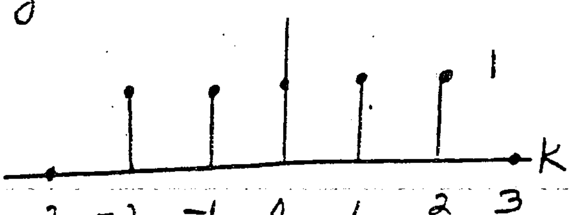
2. Discrete-Time Convolution

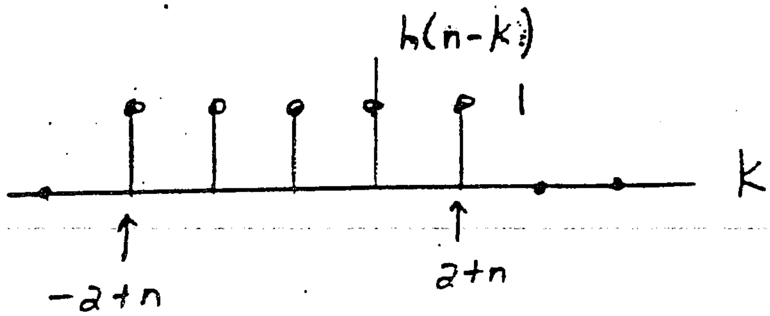
We will work a similar convolution in discrete time.

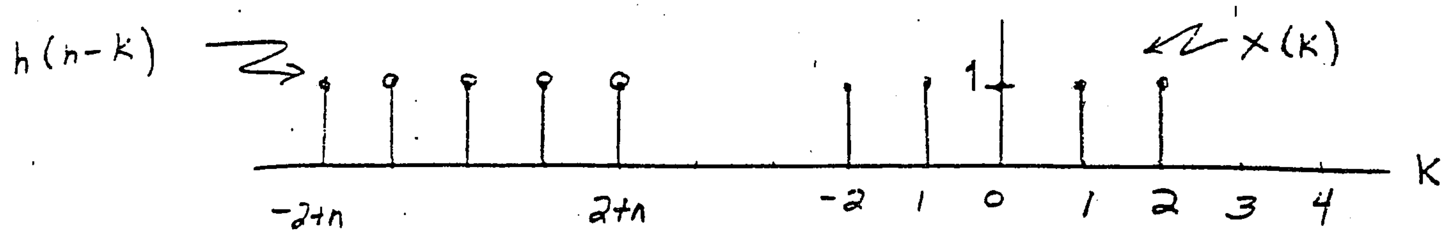




$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k),$$ so k has the same role as λ



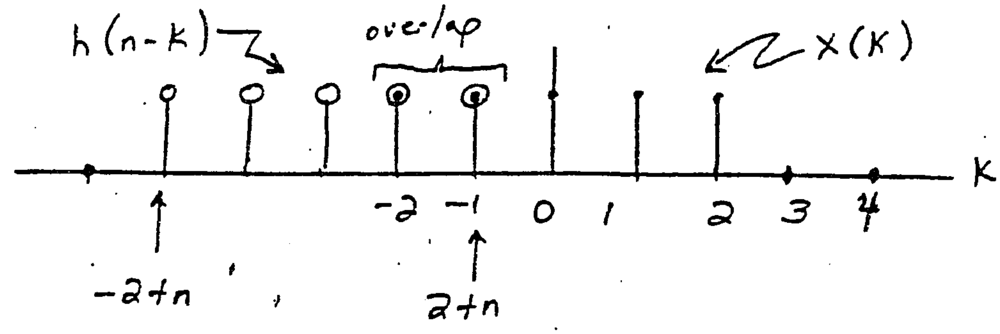




There is no overlap when $2+n < -2 \Rightarrow n < -4$

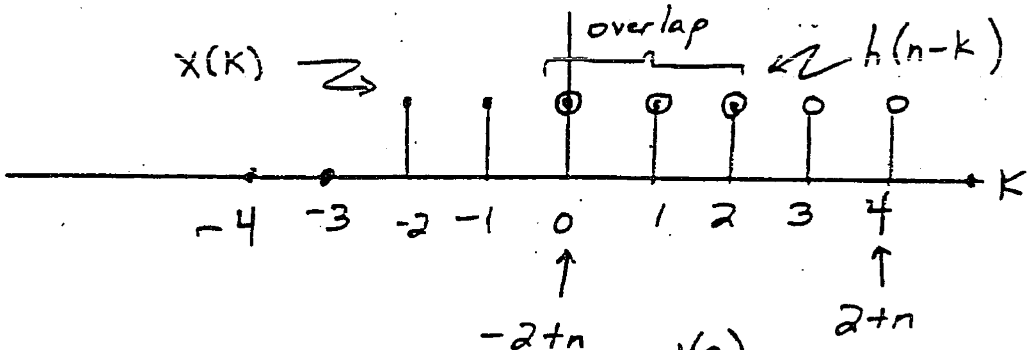
There is no overlap when $-2+n > 2 \Rightarrow n > 4$

For $-4 \leq n \leq 0$,
there is overlap
from $K=-2$ to
$K = 2+n$

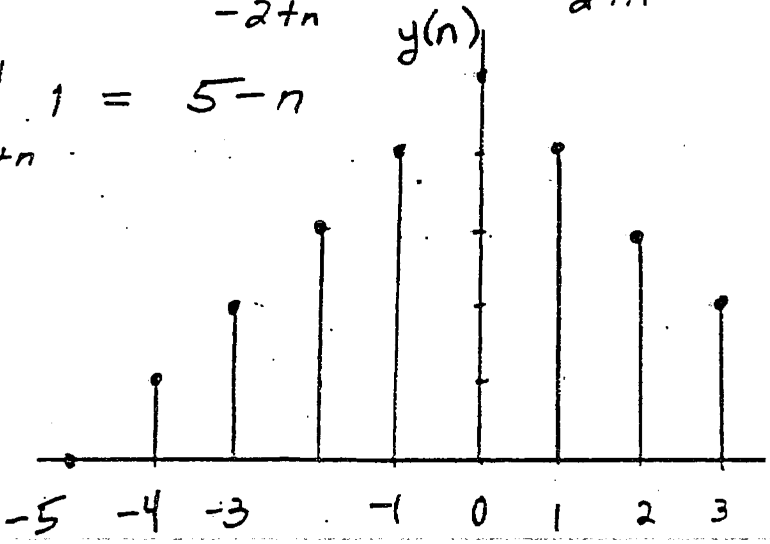


$$y(n) = \sum_{K=-\infty}^{\infty} x(K)\,h(n-K) = \sum_{K=-2}^{2+n} 1 = n+5$$

For $1 \leq n \leq 4$,
there is overlap
from $K=-2+n$
to $K=2$



$$y(n) = \sum_{K=}^{\infty} x(k)\,h(n-K) = \sum_{k=-2+n}^{2} 1 = 5-n$$



$$y(n) = \begin{cases} 0 & \text{for } n < -4 \\ 5+n & \text{for } -4 \leq n \leq 0 \\ 5-n & \text{for } 1 \leq n \leq 4 \\ 0 & \text{for } n > 4 \end{cases}$$