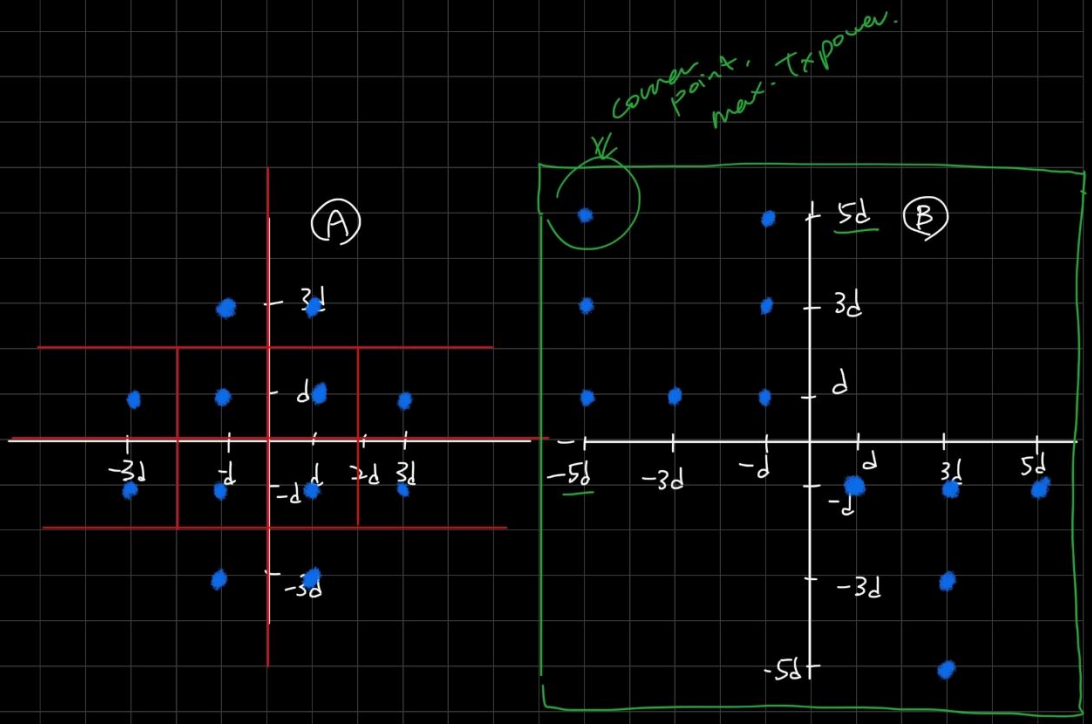
DSP Midterm 2 Fall 2019, Q2.2



a) peak Tx power

⇒ if square / rectangula ⇒ usually corner points

⇒ if constellation = symmetric

⇒ only examine 1 quadrant

(A) symmetric constellation, look at quadr. I

3 points: $(d + jd)$

$(d + j3d)$ ⇒ power?

$(3d + jd)$

Power = $(a^2 + b^2)$ for complex $(a+jb)$

$(d+jd) \Rightarrow (d^2 + d^2) = 2d^2$

$(d+j3d) \Rightarrow (d^2 + (3d)^2) = 10d^2$

$(3d+jd) \Rightarrow 10d^2$

peak Tx power ⇒ $10d^2$

(B) NOT symmetric

find rectangle, ⇒ find cornerpoint.

cornerpoint is @ $(-5d + j5d)$

thus, peak Tx Power $= (5d)^2 + (-5d)^2$

$= 25d^2 + 25d^2$

$= 50d^2$

b) avg. Tx power.

⇒ if there's symmetry, just find avg. power for 1 quadrant.

⇒ otherwise brute force.

for constellation (B) NO symmetry,

our points:

$(-5d + j5d)$
$(-5d + j3d)$
$(-5d + jd)$
$(-3d + jd)$
$(-d + jd)$
$(-d + j3d)$
$(-d + j5d)$
$(d - jd)$
$(3d - jd)$
$(3d - j3d)$
$(3d - j5d)$
$(5d - jd)$

Calculate power

calculate power with...

... 5d: $7(5d)^2$

... 3d: $7(3d)^2$

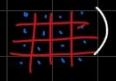
... 1d: $10(1d)^2$

$$\text{avg. power} = \frac{7(5d)^2 + 7(3d)^2 + 10(1d)^2}{(\text{total \# points} = 12)}$$

$$= \frac{7(25d^2) + 7(9d^2) + 10(d^2)}{12}$$

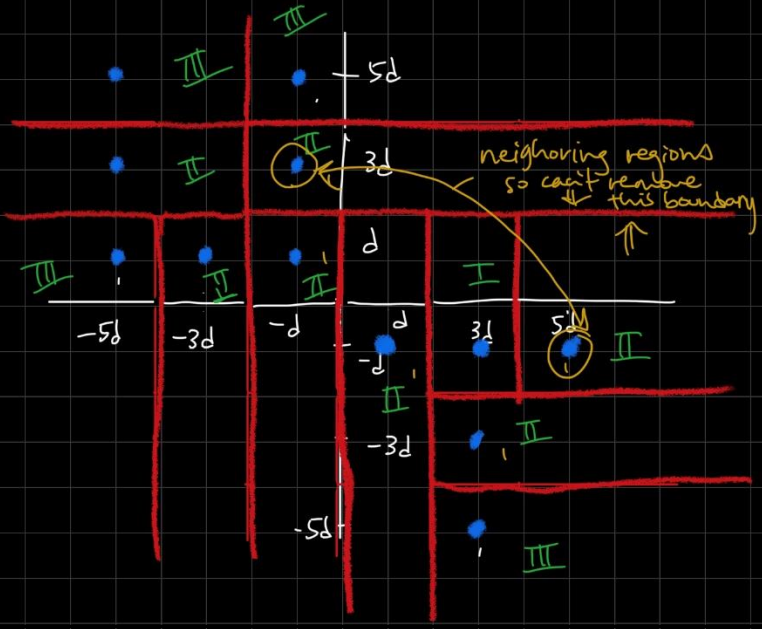
$$= \text{avg power Tx} = 20.67d^2$$

c) draw decision region (B) (in red lines)

① draw all decision regions between every point (ie. )

② erase boundary if NO neighboring points / regions

⇒ try to reduce Type I region (4 sides) as much as possible!



Type 1 : 4 sides
low $P_1(c)$
(bad)

Type 2 : 3 sides
mid $P_2(c)$
(ok)

Type 3 : 2 sides
high $P_3(c)$
(good!)

d) # of Type I ? ⇒ count based off decision region in part (C)

1 point

e) # Type II — 7 points

f) # Type III — 4 points

double check it adds 12 (for 12 AM)

g) $P(e) = ?$

$P(e) = 1 - P(c)$ = probability of symbol error

$P(c) = ?$

$= (\%\ \text{Type I points})\, P_1(c) + (\%\ \text{Type II points})\, P_2(c) + (\%\ \text{Type III points})\, P_3(c)$

$\%\ \text{Type I points} = \frac{\#\ \text{Type I points}}{\text{total \# points}}$

$$\Rightarrow P(c) = \frac{1}{12} \cdot P_1(c) + \frac{7}{12} \cdot P_2(c) + \frac{4}{12} \cdot P_3(c)$$

these eqns provided in Lecture 15 (QAM Tx) slides

h) $\frac{d}{\sigma} = ?$ in terms of SNR.

$$SNR = \frac{\text{signal power}}{\text{noise power}} = \frac{\text{avg signal power}}{\text{noise power}}$$

avg. signal power = from (b) = avg Tx power

$= 20.67 d^2$

noise power $= \sigma^2$

$$SNR = \frac{20.67 d^2}{\sigma^2} = 20.67 \left(\frac{d}{\sigma}\right)^2$$

$$\sqrt{\frac{SNR}{20.67}} = \frac{d}{\sigma}$$

(i) for this 12 QAM, you estimate $-5d - jd$ what is decoded Txed constellation point using ...

one your decision region for (B) ?

two smallest Euclidean distance ?

which gives shortest line between $(-5d - jd)$ and other constellation points?