The University of Texas at Austin Dept. of Electrical and Computer Engineering Midterm #1

Date: October 14, 2016

Course: EE 445S Evans

Name:

Last,

First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. *Please disable all wireless connections on your computer system(s).*
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Problem	Point Value	Your score	Торіс
1	28		Filter Analysis
2	24		Sampling
3	27		Filter Design
4	21		Potpourri
Total	100		

Problem 1.1 Filter Analysis. 28 points.

A discrete-time linear time-invariant (LTI) filter is described by the following transfer function

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}$$

where b_0 , b_1 , b_2 and b_3 are the filter coefficients.

- (a) Give a formula in discrete time for the impulse response h[n]. Plot h[n]. 3 points.
- (b) Is the filter a finite impulse response (FIR) or infinite impulse response (IIR) filter? Why? 3 points.
- (c) Give a formula in discrete time for the output y[n] in terms of the input x[n] including the initial conditions. *3 points*.
- (d) Draw the block diagram of the filter relating input x[n] and output y[n]. 6 points.

- (e) Give a formula for the discrete-time frequency response of the filter. 3 points.
- (f) Give all possible conditions on the coefficients for the filter to have constant group delay. 6 points.
- (g) Using only values of +1 and -1, give values for the filter coefficients for a lowpass magnitude response. *4 points*

Problem 1.2 Sampling. 24 points.

Consider a two-sided continuous-time cosine signal with frequency f_0 in Hz given by

 $x(t) = \cos(2\pi f_0 t)$

(a) Plot the continuous-time Fourier transform of x(t). 6 points.

(b) Plot the continuous-time Fourier transform of the result of sampling x(t) at a sampling rate of f_s assuming that the Sampling Theorem has been satisfied, i.e. $f_s > 2 f_0$. 6 points.

(c) Plot the continuous-time Fourier transform of x(t) sampled at a sampling rate of f_s assuming that $f_0 < f_s < 2 f_0$, which would not satisfy the Sampling Theorem. 6 *points*.

(d) In part (c), give a formula for the continuous-time frequency that would result after trying to reconstruct x(t) from its sampled version. 6 points.

Problem 1.3 Filter Design. 27 points.

People suffering from tinnitus, or ringing of the ears, hear a tone in their ears even when the environment is quiet. The tone is generally at a fixed frequency in Hz, denoted as f_c .

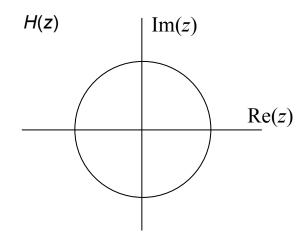
Filtering music to remove as much as possible an octave of frequencies from f_1 to f_2 that contains f_c as its center frequency can provide relief of tinnitus symptoms.

This problem will ask you to design a sixth-order discrete-time infinite impulse response (IIR) filter to remove the octave of frequencies. The **sampling rate** is f_s where $f_s > 4 f_2$.

(a) Give formulas for f_1 and f_2 in terms of f_c given that $f_c = \frac{1}{2}(f_1 + f_2)$. 6 points.

- (b) Give formulas for discrete-time frequencies ω_1 , ω_c and ω_2 that correspond to continuous-time frequencies f_1 , f_c and f_2 , respectively. *3 points*.
- (c) Give formulas in terms of ω_1 , ω_c and ω_2 for the pole and zero locations for the sixth-order discrete-time IIR filter. Assume that the gain is one. *12 points*.

(d) Draw the pole-zero diagram. 6 points.



Problem 1.4. Potpourri. 21 points.

- (a) Oversampling generally gives a higher signal quality but at a higher implementation complexity. If we increase the sampling rate by a factor of K, analyze the increase in implementation complexity for finite impulse response (FIR) filtering in terms of
 - 1. Multiplication operations per second. 3 points.
 - 2. Memory reads per second. 3 points.
- (b) In lab #2, you implemented a cosine generator on the digital signal processing board in lab using a lookup table to store one period of values for

$$x[n] = \cos(\omega_0 n)$$

- 1. Assuming the sampling theorem has been satisfied, i.e. $f_s > 2 f_0$, give the range of values that ω_0 can take. Please be sure to include negative, zero and positive frequencies. *3 points*.
- 2. What is the discrete-time period in samples for x[n]? 3 points.
- 3. Describe a way to use a smaller lookup table to save memory. 3 points.
- (c) If discrete-time signal $\cos(\omega_0 n)$ is input to a squaring block, what discrete-time frequencies will appear on the output? *6 points*.