The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: March 24, 2021 Course: EE 445S Evans

Name: ____
Last, First

Please sign your name below to certify that you did not receive any help, directly or indirectly, on this test from another human other your instructor, Prof. Brian L. Evans, and to certify that you did not provide help, directly or indirectly, to another student taking this exam.

(please sign here) ____

- **Take-home exam** is scheduled for Wednesday, Mar. 24, 2021, 10:30am to 11:59pm.
  - The exam will be available on the course Canvas page at 10:30am on Mar. 24, 2021.
  - Your solutions can be on notebook paper, or on the test and your own paper, or whatever. This means that you won't have to print the test to complete the test.
  - Please include this cover page signed by you with your solution and upload your solution as a single PDF file to the course Canvas page by 11:59pm on Mar. 24, 2021.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. Sources can include course lecture slides, handouts, homework solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Internet access.** Yes, you may fully access the Internet when answering exam questions provided that you comply with the other instructions on this page.
- **Academic integrity.** You shall not receive help directly or indirectly on this test from another human except your instructor, Prof. Evans. You shall not provide help, directly or indirectly, to another student taking this exam.
- **Send questions to Prof. Evans.** You may send questions or concerns about this midterm exam during the test to Prof. Evans via Canvas or by e-mail at bevans@ece.utexas.edu.
- **Contact by Prof. Evans.** Prof. Evans might contact all students in the class during the exam through Canvas announcements. Please periodically monitor those announcements.

Problem	*Point Value*	*Your score*	*Topic*
1	24		IIR Filter Design
2	24		Filter Design Tradeoffs
3	28		FIR Filter Bank Design
4	24		Mystery Nonlinearities
Total	100		

Problem 1.1. *IIR Filter Design*. 24 points.

Consider a second-order discrete-time linear time-invariant (LTI) infinite impulse response (IIR) filter.

A second-order section, also known as a biquad, has <u>two</u> zeros z_0 and z_1 and <u>two</u> poles p_0 and p_1. Its transfer function in the z-domain is

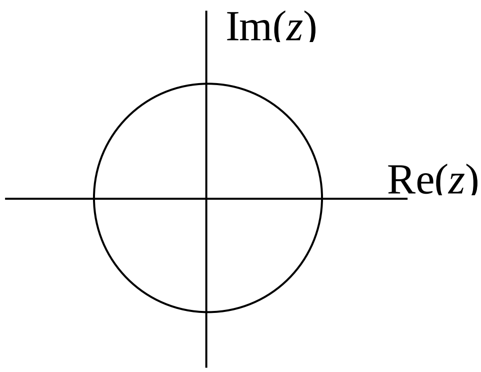
$$H(z) = C\,\frac{(z - z_0)(z - z_1)}{(z - p_0)(z - p_1)}$$

In this problem, **all poles and zeros will be complex-valued but <u>not real-valued</u>.** The imaginary part of the complex number cannot be zero, and the real part of the complex number can be anything.

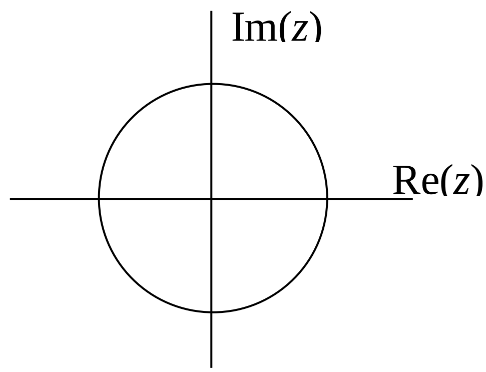
In each part below, design a biquad by placing complex non-real-valued poles and zeros to achieve the indicated frequency selectivity (lowpass, highpass, bandpass, bandstop, allpass or notch) or indicate that no such biquad could be designed. For each filter,

- Please use O to indicate the zero locations and X to indicate pole locations
- Give numeric values for the two poles and two zeros in polar form (i.e. magnitude and phase form)

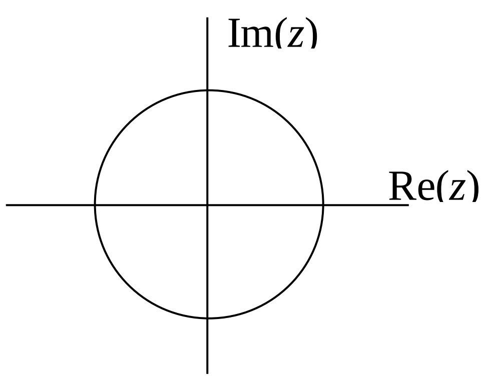
(a) Lowpass filter



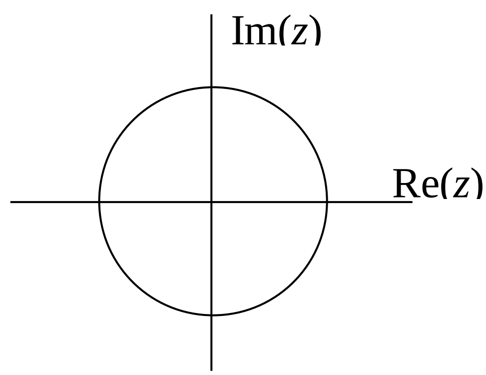
(b) Highpass filter



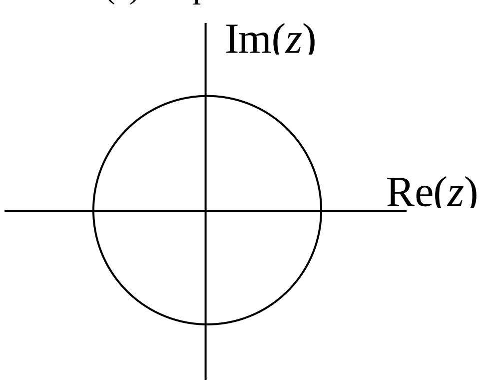
(c) Bandpass filter



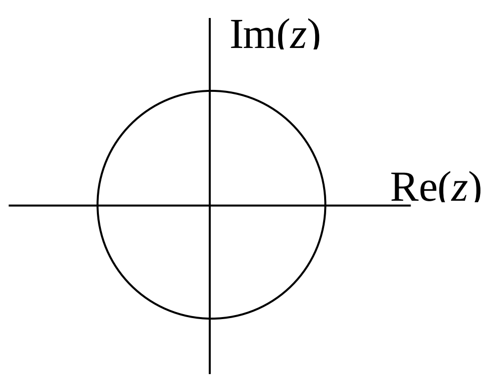
(d) Bandstop filter



(e) Allpass filter

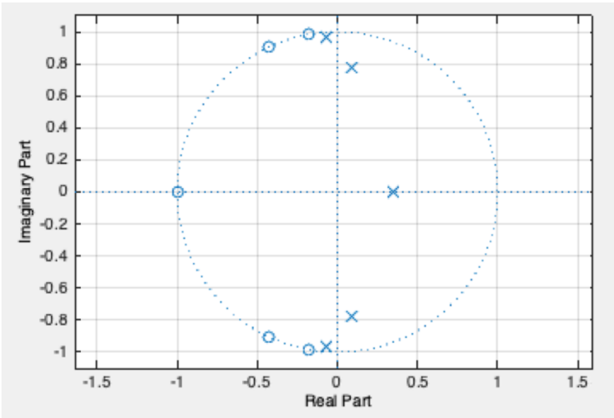


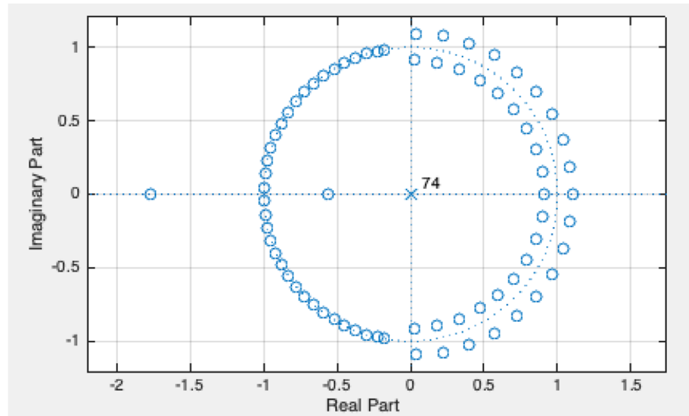
(f) Notch filter



Problem 1.2 *Filter Design Tradeoffs.* 24 points.

Both discrete-time linear time-invariant (LTI) filters below meet the same filter design specifications based on the magnitude response for an audio application.





Design #1
Number of complex/real poles: 5
Number of complex/real zeros: 5
Passband behavior: Rippling
Stopband behavior: Rippling
Design method: Elliptic
Maximum Group Delay: 33 samples

Design #2
Number of complex/real poles: 74
Number of complex/real zeros: 74
Passband behavior: Rippling
Stopband behavior: Rippling
Design method: Parks-McClellan
Maximum Group Delay: 37 samples

Please **answer** the following questions about the filter designs with **justification**. *3 points each.*

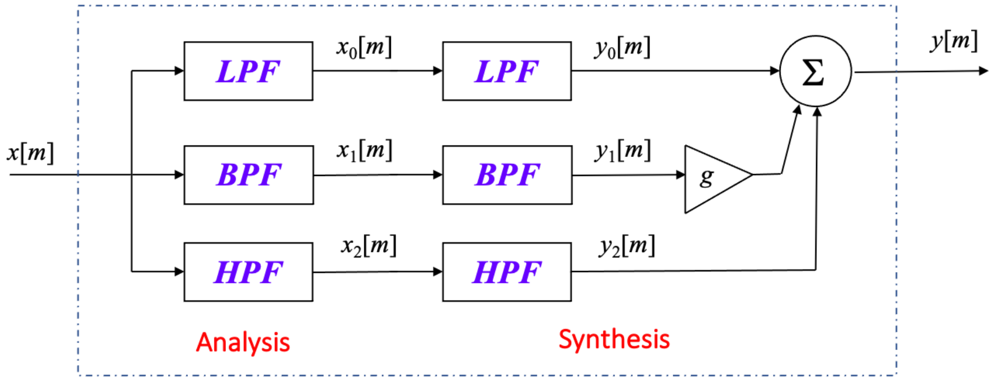
	Design #1	**Design #2**
(a) Finite Impulse Response (FIR) or Infinite Impulse Response (IIR) Filter?		
(b) Filter Order		
(c) Bounded-Input Bounded Output Stable?		
(d) Approximate range of discrete-time frequencies in the passband in rad/sample		
(e) Frequency selectivity (lowpass, highpass, bandpass, bandstop, allpass or notch)		
(f) Number of multiplication operations (use cascade of biquads structure if IIR filter)		
(g) Amount of storage (use cascade of biquads structure if IIR filter)		
(h) Give an advantage of each design, and indicate which design you would choose.		

Problem 1.3 *FIR Filter Bank Design*. 28 points.

A bank of analysis filters divides a signal $x[m]$ into frequency bands for subsequent processing.

A bank of synthesis filters combines the analysis frequency bands into a single signal $y[m]$.

The analysis-synthesis filter bank below has three linear time-invariant (LTI) filters in each bank:



The LPF, BPF, and HPF filters are finite impulse response (FIR) filters with three coefficients each.

Both lowpass filters (LPFs) have impulse response of $h_0[m] = \delta[m] + \delta[m-1] + \delta[m-2]$. The LPF coefficients are [1, 1, 1].

(a) Both bandpass filters (BPFs) are designed by modulating the LPF to shift its frequency response by $\pi/2$ to the right and left: $h_1[m] = \cos\left(\frac{\pi}{2} m\right)\ h_0[m]$. Give the coefficients for $h_1[m]$. Does the BPF have linear phase? Why or why not? *6 points*.

(b) Both highpass filters (HPFs) are designed by modulating the LPF to shift its frequency response by π to the right and left: $h_2[m] = \cos(\pi\, m)\ h_0[m]$. Give the coefficients for $h_2[m]$. Does the HPF have linear phase? Why or why not? *6 points*.

(c) Compute the impulse response $h[m]$ for the overall system with input $x[m]$ and output $y[m]$. The impulse response will include the real-valued gain g. **Hint:** $y[m] = y_0[m] + g\ y_1[m] + y_2[m]$. *9 points*.

(d) Does the overall system have linear phase for all possible values of g? Why or why not? *3 points*.

(e) Compute the value of g that causes the overall system to act like an ideal delay with gain C, i.e. $y[m] = C\ x[m - m_0]$. Please give the values of C and m_0. *4 points*.

Problem 1.4. *Mystery Nonlinearities.* 24 points.

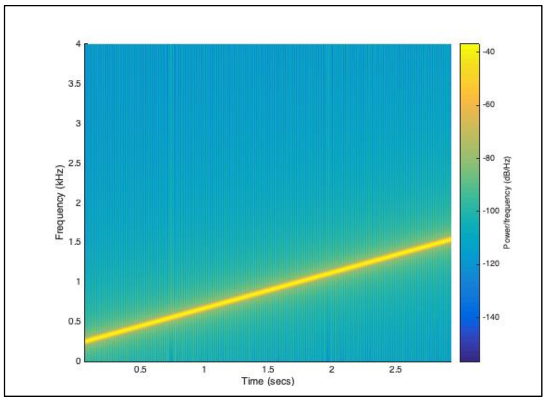
You're trying to determine input-output relationships for discrete-time pointwise nonlinear systems.

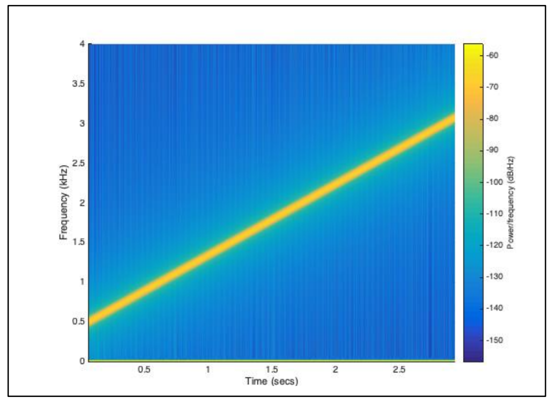
For discrete-time pointwise systems, output at discrete-time *n* only depends on input at discrete-time *n*.

You input a chirp signal and look at the resulting output signal to figure out what the system is doing.

For the analysis, you decide to use the chirp signal from in-lecture assignment #1, which starts around 240 Hz and ends around 1520 Hz, and uses a sampling rate of 8000 Hz.

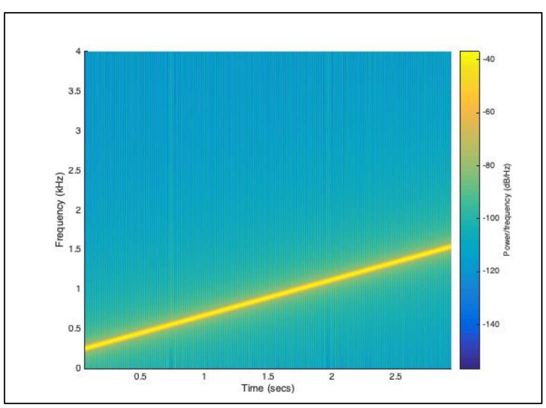
(a) Give a formula for output $y[n]$ in terms of input $x[n]$ by looking at the spectrogram for a chirp input signal (left) and the spectrogram of the output signal (right). *12 points*.

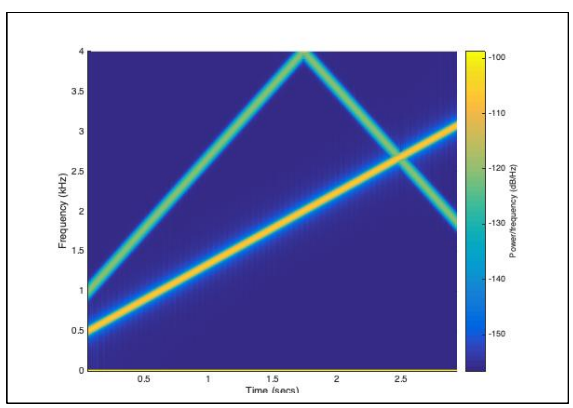




The spectrogram of the output shows a strong DC component at all times, and another component shows a linear increase from around 480 Hz to around 3040 Hz over time.

(b) Give a formula for output $y[n]$ given the input $x[n]$ by looking at the spectrogram for a chirp input signal (left) and the spectrogram of the output signal (right). *12 points*.





The spectrogram of the output shows a strong DC component at all times; a second component that shows a linear increase from around 480 Hz to around 3040 Hz; and a third component that starts around 960 Hz, linearly increases to 4000 Hz, and then linearly decreases to around 1880 Hz.