The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm \#2
Prof. Brian L. Evans
Date: December 5, 2016
Course: EE 445S

Name: $\qquad$
Last,
First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. Disable all wireless access from your standalone computer system.
- Please turn off all smart phones and other personal communication devices.
- Please remove headphones.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- Fully justify your answers unless instructed otherwise. When justifying your answers, you may refer to the Johnson, Sethares \& Klein (JSK) textbook, the Welch, Wright and Morrow (WWM) lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content in your justification.

| Problem | Point Value | Your score | Topic |
| :---: | :---: | :---: | :---: |
| 1 | 21 |  | Interpolation |
| 2 | 27 |  | QAM Communication Performance |
| 3 | 28 |  | Phase Locked Loop (PLL) |
| 4 | 24 |  | Communication System Design |
| Total | 100 |  |  |

Problem 2.1. Interpolation. 21 points.
Interpolation can change the sampling rate of discrete-time signal $x[n]$ through discrete-time operations of upsampling by $L$ and then filtering:

(a) Give a formula for $f_{2}$ in terms of $f_{1}$. 3 points.
(b) Specify the filter's passband frequency $\omega_{\text {pass }}$ and stopband frequency $\omega_{\text {stop }}$ in rad/sample to pass as many frequencies in $x[n]$ as possible and reduce artifacts due to upsampling. 6 points.
(c) If $x[n]$ is represented with $B$ bits, specify the passband tolerance $A_{\text {pass }}$ in dB and the stopband attenuation $A_{\text {stop }}$ in dB. 6 points.
(d) Give an advantage for each type of interpolation filter below. 6 points.
i. Finite impulse response filter.
ii. Infinite impulse response filter.

Problem 2.2 QAM Communication Performance. 27 points.
Consider the two 16-QAM constellations below. Constellation spacing is $2 d$.


Energy in the pulse shape is 1 . Symbol time $T_{\text {sym }}$ is 1 s . The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature ( Q ) axis and dashed lines.

Each part below is worth 3 points. Please fully justify your answers.

|  | Left Constellation | Right Constellation |
| :--- | :---: | :---: |
| (a) Peak transmit power | $18 d^{2}$ |  |
| (b) Average transmit power | $10 d^{2}$ |  |
| (c) Draw the decision regions for the right constellation on top of the right constellation. |  |  |
| (d) Number of type I regions | 4 |  |
| (e) Number of type II regions | 8 |  |
| (f) Number of type III regions | 4 |  |
| (g) Probability of symbol error <br> for additive Gaussian noise <br> with zero mean \& variance $\sigma^{2}$ | $3 Q\left(\frac{d}{\sigma}\right)-\frac{9}{4} Q^{2}\left(\frac{d}{\sigma}\right)$ |  |

(h) Which of the constellations would you advocate using? Why? Please give two reasons. 6 points.

Problem 2.3. Phase Locked Loop (PLL). 28 points.
The discrete-time transmitter below is for pulse amplitude modulation (PAM) with upconversion:

where
$a[n]$ symbol amplitude
$f_{s}$ sampling rate
$g[m]$ pulse shape
$J \quad$ bits/symbol
$L$ samples/symbol
$\omega_{c} \quad$ carrier frequency

The discrete-time PAM receiver below has two downconversion paths, and one feeds into the PLL:

$\theta[m]$ is the carrier phase offset, and $h[m]$ represents a lowpass finite impulse response (FIR) filter.
(a) When the receiver carrier phase matches the transmitter carrier phase, i.e. when $\theta[m]=0$, show that $q[m]$ is zero. 6 points.
(b) Develop a steepest descent algorithm to estimate the carrier phase offset, $\theta[m]$, per the steps below.
i. Give an objective function. 6 points.
ii. Give an update equation for $\theta[m+1]$ in terms of $\theta[m] .9$ points.
iii. Give an initial value of $\theta[m] .3$ points.
iv. What values of the step size, $\mu$, would you use. Why? 4 points.

## Problem 2.4. Communication System Design. 24 points

For $M$-level pulse amplitude modulation systems, the probability of a symbol error in the receiver is

$P_{\text {error }}=\frac{2(M-1)}{M} Q\left(\frac{d}{\sigma} \sqrt{T_{\text {sym }}}\right)$
where
$2 d$ is spacing between adjacent constellation points in Volts,
$\sigma^{2}$ is variance of the noise in the communication channel,
$T_{\text {sym }}$ is symbol time, and
$Q(x)=\frac{1}{\sqrt{2 \pi}} \int_{x}^{\infty} e^{\frac{-y^{2}}{2}} d y$
(a) How would you choose $M, d$ and $T_{\text {sym }}$ for a high-speed communication link with probability of symbol error of $10^{-3}$. 9 points.
(b) How would you choose $M, d$ and $T_{\text {sym }}$ for a low-speed control channel with probability of symbol error of $10^{-7}$. The control channel would allow the feedback of information from receiver to transmitter, such as estimated SNR and channel impulse responses, with high accuracy. 9 points.
(c) Give an optimal encoding for symbols of bits for the 8-PAM constellation below. In what sense is your encoding optimal? 6 points.


