The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

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Date: December 9, 2019 Course: EE 445S

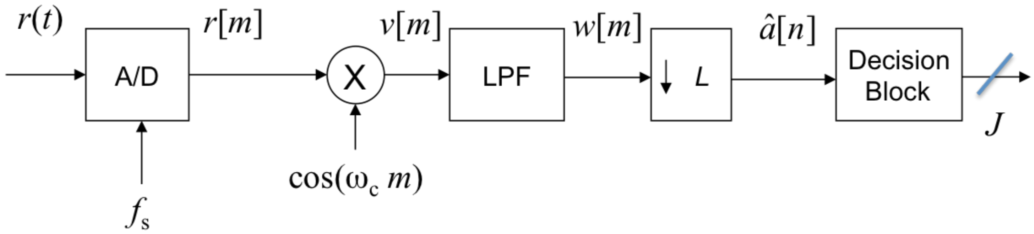
Name: ____
Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all smart phones and other personal communication devices.
- Please remove headphones.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein (JSK) textbook, the Welch, Wright and Morrow (WWM) lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content in your justification.

Problem	**Point Value**	**Your score**	**Topic**
1	24		Bandpass PAM Receiver Tradeoffs
2	30		QAM Communication Performance
3	28		Channel Equalization
4	18		Total Harmonic Distortion
Total	100		

Problem 2.1. *Bandpass Pulse Amplitude Modulation Receiver Tradeoffs*. *24 points*.

A bandpass pulse amplitude modulation (PAM) receiver is described as



where m is the sampling index and n is the symbol index, and has **system parameters**

$a[n]$ transmitted symbol amplitude	$\hat{a}[n]$ received symbol amplitude	
$2d$ constellation spacing	f_s sampling rate	f_{sym} symbol rate
$g[m]$ raised cosine pulse with rolloff α	J bits/symbol	L samples/symbol
M number of levels, i.e. $M = 2^J$	N_g symbol periods in $g[m]$	ω_c carrier freq. in rad/sample

The only impairment is additive thermal noise $w(t)$ modeled as zero-mean Gaussian with variance σ^2. Hence, $r(t) = s(t) + w(t)$ where $s(t)$ is the transmitted bandpass PAM signal.

(a) Give formulas for communication signal quality measures below in terms of system parameters:

i. Bit rate. *3 points*

ii. Probability of symbol error. *3 points*

(b) Draw the spectrum for $r(t)$. What is the transmission bandwidth in Hz? *6 points*.

(c) For the lowpass filter (LPF),

i. What three roles does it play? *3 points*

ii. If its impulse response is equal to $g[m]$, give a formula for its bandwidth. *3 points*

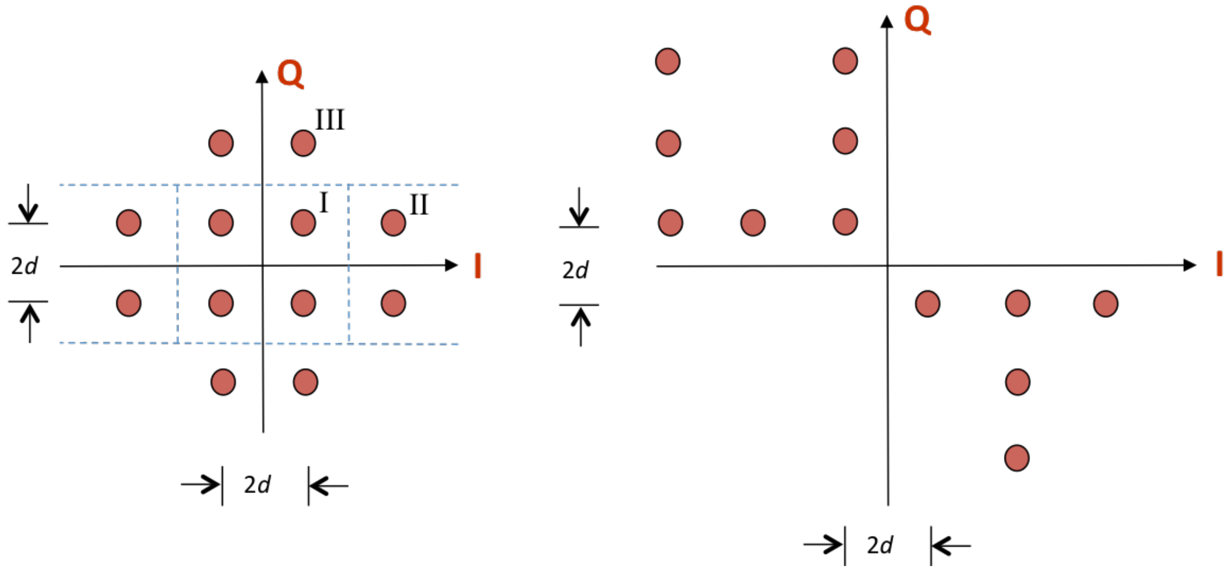
(d) For the cascade of the lowpass filter (LPF) and downsampling by L,

i. How many multiplications per second are required? *3 points*

ii. What would the savings be if the cascade were realized in polyphase form? Why? *3 points.*

Problem 2.2 *QAM Communication Performance. 30 points.*

Consider the two 12-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

Each part below is worth 3 points. **Please fully justify your answers.**

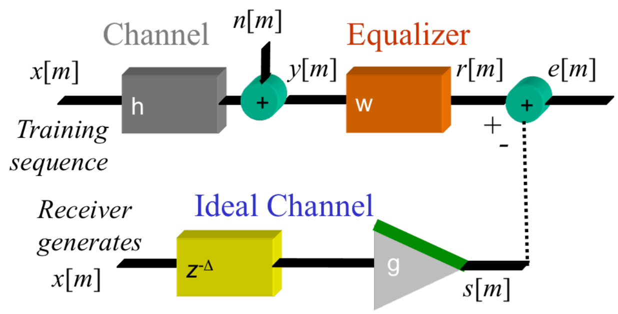
	Left Constellation	**Right Constellation**
(a) Peak transmit power	$10d^2$	
(b) Average transmit power	$\frac{22}{3} d^2 \approx 7.33 d^2$	
(c) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***		
(d) Number of type I regions	4	
(e) Number of type II regions	4	
(f) Number of type III regions	4	
(g) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$3\,Q\left(\frac{d}{\sigma}\right) - Q^2\left(\frac{d}{\sigma}\right)$	
(h) Express d/σ as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{22}{3}\left(\frac{d^2}{\sigma^2}\right)$ $\frac{d}{\sigma} = \sqrt{\frac{3}{22}\text{SNR}} \approx 0.369\sqrt{\text{SNR}}$	

(i) In a 12-QAM receiver for the right constellation, an estimated symbol amplitude is $-5d - jd$. What is the decoded transmitted constellation point using

- Your constellation regions given above. *3 points*

- Smallest Euclidean distance. *3 points*

Problem 2.3. *Channel Equalization*. *28 points*.



In the discrete-time system on the right, the equalizer operates at the sampling rate.

The equalizer is a finite impulse response (FIR) filter with N real coefficients $w_0, w_1, \ldots w_{N-1}$:

$r[m] = w_0\, y[m] + w_1\, y[m-1] + \ldots + w_{N-1}\, y[m - (N-1)]$

Channel model is an FIR filter with impulse response $h[m]$ in cascade with additive noise $n[m]$.

(a) What two training sequences for $x[m]$ could you use? Why? *6 points.*

(b) For one of the training sequences in part (a), describe how you would estimate the delay parameter Δ in the ideal channel model. *3 points.*

(c) For an adaptive FIR equalizer, **derive the update equation** for the vector of FIR coefficients $\vec{w}$ for the objective function $J(e[m]) = |\, e[m]\,|$. Here, $\vec{w} = [\, w_0 \;\; w_1 \;\; \cdots \;\; w_{N-1}\,]$. Please use the fact that $\frac{d}{dx}|x| = \text{sign}(x)$ except at $x = 0$ which we will extend to include $x = 0$. What value should sign(x) take at $x = 0$ for the adaptive update? Let $\vec{w}[m] = [\, w_0[m] \quad w_1[m] \;\; \cdots \;\; w_{N-1}[m] \;\,]$. *12 points.*

(d) Compare your answer in (c) with an adaptive least mean squares (LMS) equalizer. For the LMS approach, use $J(e[m]) = \frac{1}{2} e^2[m]$ which leads to the update equation

$$\vec{w}[m+1] = \vec{w}[m] - \mu\, e[m]\, \vec{y}[m]$$

where $\vec{y}[m] = [\, y[m] \quad y[m-1] \quad \cdots \quad y[m-(N-1)]\,]$. Would you use (c) or (d)? *4 points.*

(e) For your answer in (c), what values of the step size (learning rate) μ would you use? *3 points.*

Problem 2.4. *Total Harmonic Distortion. 18 points*

Total harmonic distortion is a measure of the power in the harmonics of a fundamental frequency.

Design a discrete-time, linear time-invariant (LTI), infinite impulse response (IIR) comb filter to

- **Pass** harmonics of a 1 kHz tone, i.e. 2 kHz, 3 kHz, etc.
- **Not pass** 0 kHz or 1 kHz frequencies.

Assume a sampling rate of 48 kHz.

Please give the transfer function of your design and explain your reasoning to get there.