The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2 **Take-Home Exam** ***Solutions 3.0***

Prof. Brian L. Evans

Date: December 7, 2020 Course: EE 445S

Name: **Scallon,** **Rob**
Last, First

Please sign your name below to certify that you did not receive any help, directly or indirectly, on this test from another human other your instructor, Prof. Brian L. Evans, and to certify that you did not provide help, directly or indirectly, to another student taking this exam.

(please sign here) ____

- **Take-home exam** is scheduled for Monday, Dec. 7, 2020, 10:30am to 11:59pm.
  - The exam will be available on the course Canvas page at 10:30am on Dec. 7, 2020.
  - Your solutions can be on notebook paper, or on the test and your own paper, or whatever. This means that you won't have to print the test to complete the test.
  - Please upload your solution as a single PDF file to the course Canvas page by 11:59pm on Oct 14, 2020.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content for your justification. You may reference homework solutions, exam solutions, lecture slides, textbooks, Internet resources, etc.
- **Internet access.** Yes, you may fully access the Internet when answering exam questions provided that you comply with the other instructions on this page.
- **Academic integrity.** You shall not receive help directly or indirectly on this test from another human except your instructor, Prof. Evans. You shall not provide help, directly or indirectly, to another student taking this exam.
- **Send questions to Prof. Evans.** You may send questions or concerns about this midterm exam during the test to Prof. Evans via Canvas or by e-mail at bevans@ece.utexas.edu.
- **Contact by Prof. Evans.** Prof. Evans might contact all students in the class during the exam through Canvas announcements. Please periodically monitor those announcements.

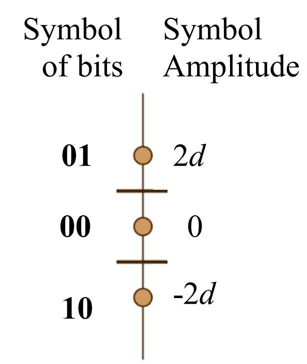
Problem	Point Value	Your score	Topic
1	24		PAM Symbol Error Probability
2	27		QAM Communication Performance
3	24		Interference Cancellation
4	25		Communication System Design Tradeoffs
Total	100		

***Prologue*. Lectures 13 & 14; JSK Ch. 8 & 11; Lab #5; WWM Ch. 17; HW 5.2, 5.3, 6.1, 6.2; Midterm 2.1 F19 & Sp20; Handout P**

Problem 2.1. *PAM Symbol Error Probability. 24 points.*

Consider a three-level baseband pulse amplitude modulation (PAM) system:

$2d$ constellation spacing	f_s sampling rate
f_{sym} symbol rate	$g[m]$ raised cosine pulse with rolloff α
J bits/symbol	L samples/symbol
M number of levels, i.e. $M = 2^J$	N_g symbol periods in $g[m]$
T_s sampling time	T_{sym} symbol time.



3-PAM Constellation

The 3-PAM constellation is shown on the right (i.e. $M = 3$).

This problem is asking you to analyze the 3-PAM system and enter your results in the table below to compare against 2-PAM and 4-PAM systems.

For this problem, assume that $T_{sym} = 1$ s and hence $f_{sym} = 1$ Hz.

(a) What is the bit rate for the 3-PAM system? Why? *4 points.*

Bit rate = $J\, f_{sym}$. With f_{sym} = 1 Hz and $J = \log_2 M = \log_2 3 \approx 1.585$, the bit rate is 1.585 bits/s.

(b) Give a symbol of bits for each symbol amplitude on the 3-PAM constellation. Make sure that the bit encoding for adjacent symbols only differs by one bit (Gray coding). *6 points.*

Shown in bold on the constellation map above.

(c) Derive 3-PAM symbol error probability. Thresholds are at $-d$ and d. *8 points.*

The constellation regions are $(-\infty, -d)$, $(-d, d)$, and (d, ∞).

The region $(-d, d)$ is an inner region, and the others are outer regions. (Lecture Slide 15-11).

Assuming each symbol is equally likely to be transmitted, we use Lecture Slide 15-11 to find

$$\boldsymbol{P(e) = \frac{1}{3} P_I(e) + \frac{2}{3} P_O(e) = \frac{1}{3}\left(2Q\left(\frac{d}{\sigma}\right)\right) + \frac{2}{3}\left(Q\left(\frac{d}{\sigma}\right)\right) = \frac{4}{3} Q\left(\frac{d}{\sigma}\right)}$$

(d) Derive the transmitted maximum power, average power, and peak-to-average power ratio. *6 points.*

The maximum symbol amplitude is 2d Volts.

Maximum (peak) power is proportional to the maximum amplitude squared., which is $4d^2$.

The average power, assuming each symbol is equally likely, is ($4d^2 + 0 + 4d^2$) / 3 = (8/3) d^2.

M	Bit rate	Symbol Error Probability	Peak Power	Average Power	Peak-to-Average Power Ratio
2	1 bit/s	$Q\left(\frac{d}{\sigma}\right)$	d^2	d^2	1.0
3	**1.585 bits/s**	$\boldsymbol{\frac{4}{3} Q\left(\frac{d}{\sigma}\right)}$	$\boldsymbol{4\, d^2}$	$\boldsymbol{\frac{8}{3}\, d^2}$	**1.5**
4	2 bits/s	$\frac{3}{2} Q\left(\frac{d}{\sigma}\right)$	$9\, d^2$	$5\, d^2$	1.8

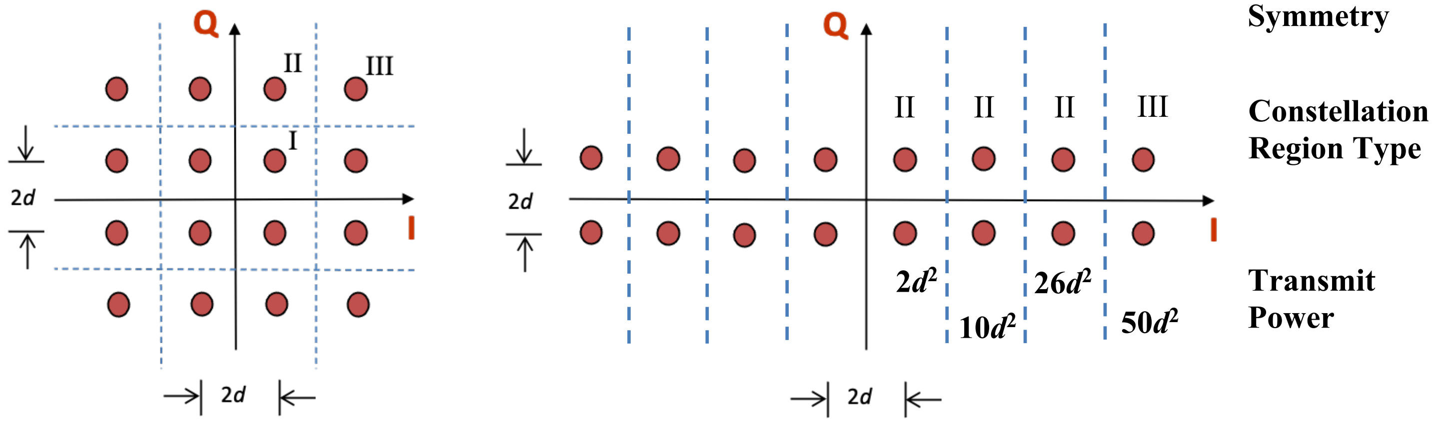
Note: All entries in the above table assume that $T_{sym} = 1$ s and hence $f_{sym} = 1$ Hz.

***Epilogue*. Some communications standards, such as High-Speed Digital Subscriber Line 2 (HDSL2), use PAM constellation sizes that are not powers of two.**

***Prologue*: Lectures 15 & 16; JSK Ch. 16; Lab #6; HW 6.3 & 7.3; Handout P; Midterm 2.2 problems in F14, Sp15, F15, Sp16, F16, Sp17, F17, Sp18, F18, Sp19, F19, Sp20**

Problem 2.2 *QAM Communication Performance. 27 points.*

Consider the two 16-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

Each part below is worth 3 points. **Please fully justify your answers.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$18d^2$	$\mathbf{50d^2}$
(b) Average transmit power	$10d^2$	**((2+10+26+50)/4)** $\mathbf{d^2 = 22d^2}$
(c) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***		
(d) Number of type I regions	4	**0**
(e) Number of type II regions	8	**12**
(f) Number of type III regions	4	**4**
(g) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$3Q\left(\frac{d}{\sigma}\right)-\frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$	$\frac{11}{4}Q\left(\frac{d}{\sigma}\right)-\frac{7}{4}Q^2\left(\frac{d}{\sigma}\right)$
(h) Express d/σ as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR}=\frac{10d^2}{\sigma^2} \Rightarrow \frac{d}{\sigma}=\sqrt{\frac{\text{SNR}}{10}}$	$\text{SNR}=\frac{22d^2}{\sigma^2} \Rightarrow \frac{d}{\sigma}=\sqrt{\frac{\text{SNR}}{22}}$

See below for work

(g) ***Approach #1:*** **Same number of types 1-3 regions as Sp19 2.2 gives same symbol error prob.**

Approach #2: **Lecture slides 15-13 to 15-15.** $P(e) = 1 - P(c)$. **Let** $q = Q\left(\frac{d}{\sigma}\right)$.

$$P(c) = \frac{3}{4}P_2(c) + \frac{1}{4}P_3(c) = \frac{3}{4}(1-q)(1-2q) + \frac{1}{4}(1-q)^2$$

$$P(c) = \frac{3}{4}(1-3q+2q^2) + \frac{1}{4}(1-2q+q^2) = 1 - \frac{11}{4}q + \frac{7}{4}q^2 \text{ and } P(e) = 1 - P(c) = \frac{11}{4}q - \frac{7}{4}q^2$$

(i) For an application, four of the 16 symbols have the most significance and we would like to transmit them with the lowest symbol error probability. Using the **left constellation**, which constellation points would you use to represent these four high-significance symbols? Why? *3 points.*

Type III regions have the lowest symbol error probability among the three types.
Use the four corner points in the left constellation for the four symbols of highest significance.

***Epilogue*. Application for (i). Consider an image in a progressive format as a low-resolution plus a medium-resolution plus a high-resolution image. Since the low-resolution image is the most important part because it is needed by the other two resolutions, the low-res data would be carried by the four highest-significance symbols. The medium- and high-resolution images would be carried by type 2 and 3 symbols, respectively.**

Prologue. JSK Sec. 2.12 & 13.1-13.3; HW 3.1, 5.1, 6.1, 6.2, 7.2; In-Lecture Assignment #4; Lectures 12-14; Lecture Slides 16-6 to 16-8; Midterm #2 Problem 2.3 F14 & 2.4 Sp16

Problem 2.3. *Interference Cancellation. 24 points.*

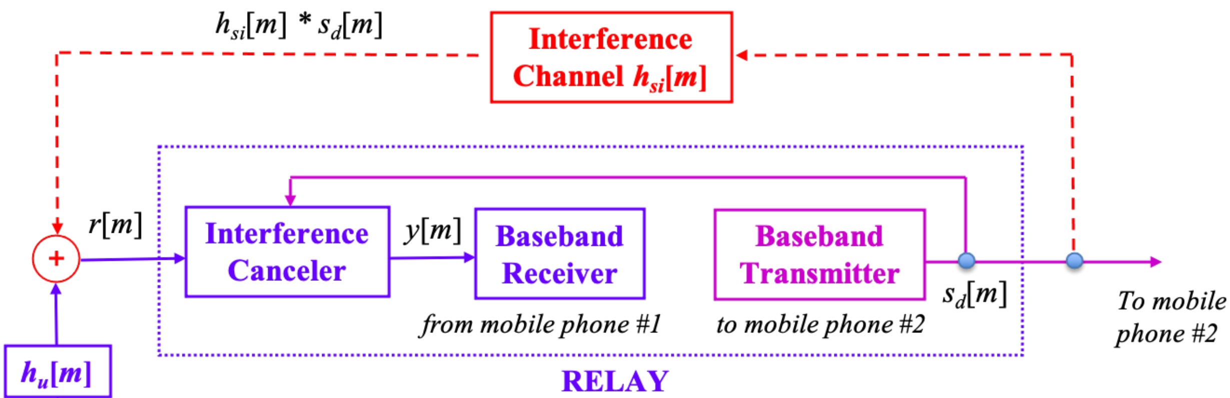
In wireless communication systems, transmitted signals experience a wide variety of impairments by the time they reach a receiver, including attenuation over distance traveled.

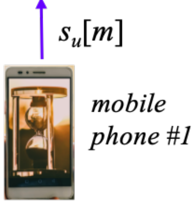
A relay can be deployed to receive and decode a basestation transmission and then re-encode and retransmit the signal so that it arrives at a mobile phone with higher signal power.

A relay can also receive and decode a mobile phone transmission and then re-encode and retransmit the signal so that it arrives at a basestation with higher signal power.

When a relay receives and transmits signals at the same time and in the same frequency band, the propagation of the relay transmission to the relay receiver causes **interference**.

The diagram below gives a baseband model for a relay system serving two mobile phones. Mobile phone #1 is transmitting at the same time that the relay is receiving and transmitting.





The interference canceler does not know $h_{si}[m]$, $h_u[m]$, or $s_u[m]$, and uses knowledge of $s_d[m]$ and $r[m]$ to adapt a finite impulse response (FIR) filter $c[m]$ to subtract interference from $r[m]$:

$$r[m] = \underbrace{h_u[m] * s_u[m]}_{from\ mobile\ phone\ \#1} + \underbrace{h_{si}[m] * s_d[m]}_{interference}$$

$$y[m] = r[m] - c[m] * s_d[m]$$

$$\boldsymbol{y[m] = h_u[m] * s_u[m] + h_{si}[m] * s_d[m] - c[m] * s_d[m] = h_u[m] * s_u[m] + (h_{si}[m] - c[m]) * s_d[m]}$$

(a) What objective function would you use? Why? *8 points.* **Adapt canceler to remove interference in $r[m]$ and hence reduce its power: $J(y[m]) = ½\, y^2[m]$. Best removal when $c[m] = h_{si}[m]$.**

(b) For an adaptive FIR canceler, **derive the update equation** for the vector of FIR coefficients $\vec{c}$ for the objective function in part (a). Here, $\vec{c} = [\, c_0 \;\; c_1 \;\; c_2 \;\; \cdots \;\; c_{N-1} \,]$. *12 points.*
Transmitted baseband relay signal $s_d[m]$ also serves as a reference signal to adapt the interference canceler. A training sequence is not needed. Using steepest descent,

$$\boldsymbol{y[m] = r[m] - c_0\, s_d[m] - c_1\, s_d[m-1] - \cdots - c_{N-1}\, s_d[m-(N-1)]}$$

Let $\vec{s}_d[m] = [\; s_d[m] \quad s_d[m-1] \quad \cdots \quad s_d[m-(N-1)] \;]$

For steepest descent algorithm, let $\vec{c}[m] = [\; c_0[m] \quad c_1[m] \quad \cdots \quad c_{N-1}[m] \;]$:

$$\boldsymbol{\vec{c}[m+1] = \vec{c}[m] - \mu \left. \frac{d}{d\vec{c}} J(y[m]) \right|_{\vec{c}=\vec{c}[m]} = \vec{c}[m] + \mu\, y[m]\, \vec{s}_d[m]}$$

(c) For your answer in (b), what values of the step size (learning rate) μ would you use? *4 points.* **Use a small positive value for the step size μ, such as 0.01 or 0.001, for convergence of the steepest descent algorithm. A step size of zero will prevent any updates. A negative step size and a large positive step size will cause divergence.**

Epilogue
Transmitting and receiving at the same time in the same frequency band is known as *full duplex*. Potential spectral efficiency gain 2x. Subscripts are *d* downlink, *si* self-interference and *u* uplink.

Smart phone picture by Samer Daboul

***Prologue*. Lectures 8, 13 & 14; JSK Ch. 8 & 11; Lab #5; WWM Ch. 17; HW 5.2, 5.3, 6.1, 6.2; Midterm 2.1 Sp 19, F19 & Sp20; Handout P**

Problem 2.4. *Communication System Design Tradeoffs. 25 points*

Describe the effect of increasing each pulse amplitude modulation system design parameter in the first column on the quantity in the other columns: **increase**, **decrease** or leave it **blank** to mean no effect.

When considering the impact of increasing the parameter in the first column, assume that the values of the other parameters are held constant.

After the table, justify your answers for each row. The row for J bits/symbol is given as an example. For the entry, "2d constellation spacing in Volts", consider what happens when d increases.

Parameter	Transmit Power Consumption	Transmission Bandwidth	Bit Rate	Symbol Error Rate	Run-Time Complexity
B bits in A/D output & D/A input			?	?	**increase**
$2d$ constellation spacing in Volts	**increase**			**decrease**	
f_{sym} symbol rate in Hz		**increase**	**increase**	**increase**	**increase**
***J* bits/symbol**	**increase**		**increase**	**increase**	**increase**
L samples/symbol				**decrease**	**increase**
N_g symbol periods in pulse shape				**decrease**	**increase**

***J* bits/symbol.** Increasing J increases number of constellation points, $M = 2^J$. Peak transmit power is proportional to square of max amplitude $(M\text{-}1)d$ and hence increases exponentially in J. Bit rate $J\,f_{sym}$ increases linearly with J. For symbol error probability $\frac{2\,(M-1)}{M}\;Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)$, the constant in front is on the interval [1, 2) and increases slightly with J because $M = 2^J$. In the transmitter, the constellation map has $M = 2^J$ entries and fast symbol decoding in the receiver takes J comparisons.

<u>*B* bits in A/D output & D/A input</u>. Power consumption by a data converter is exponential in *B*.

- ***Transmit power consumption* (i.e. transmitted signal power). As *B* increases, the power consumed in the transmitter increases but does not affect the transmit power consumption.**
- ***Bit rate.* Increasing *B* has no effect unless $B < J$. In that case, the number of levels of resolution in the data converters 2^B would not be enough to represent the 2^J different symbol amplitudes uniquely and increasing *B* until $B = J$ would increase the symbol rate.**
- ***Symbol error rate* would not be affected because the parameters in the symbol error probability expression (J, d, σ, and T_{sym}) remain the same per the problem statement except**
  - **when $B < J$, the number of levels of resolution in the data converters 2^B would not be enough to represent the 2^J different symbol amplitudes uniquely and in that case, increasing *B* until $B = J$ would decrease the symbol error rate.**

- **when the SNR w/r to thermal noise in the receiver analog/RF front end is less than the A/D SNR w/ quantization noise power, and in the case, adding more bits to the A/D converter would decrease the symbol error rate until the two SNRs are equal.**

- ***Run-time complexity* in memory and computation in a software receiver will increase as *B* increases from 8 to 9 bit due to switching from bytes to 16-bit integers to represent the A/D output and from 16 to 17 bits due to switching from 16-bit integers to 32-bit integers or floating-point numbers. Multiplying two 32-bit IEEE floating-point numbers takes much longer than multiplying two 16-bit integers (4x as long on the TI TMS320C6748 DSP processor).**

<u>2*d* constellation spacing in Volts.</u>

- ***Transmit power*. Peak and average transmit power increases with d^2.**
- ***Symbol error rate* $\frac{2\,(M-1)}{M}\; Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)$ decreases with *d* because $Q(x)$ is monotonically decreasing vs. *x*.**

<u>f_{sym} symbol rate in Hz.</u> (Sp20 Midterm #2 Problem 2.1)

- ***Transmission bandwidth* $f_{sym}\,(1 + \alpha)$ increases linearly with f_{sym} where α is the rolloff factor. *Bit rate* $J\, f_{sym}$ increases linearly with f_{sym}.**
- ***Symbol error rate* $\frac{2\,(M-1)}{M}\; Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)$ increases with f_{sym} because $Q(x)$ is decreasing vs. *x* and $T_{sym} = 1/f_{sym}$.**
- ***Run-time complexity* in transmitter and receiver increases linearly with sampling rate $L\, f_{sym}$.**

<u>*L* samples/symbol</u>.

- ***Symbol error rate/probability.* In the receiver, the matched filter correlates the received signal against a known pulse shape of $L\, N_g$ samples, and the increase in the SNR is proportional to $L\, N_g$, which in turn decreases the symbol error probability.**
- ***Run-time complexity* in the transmitter and receiver increases linearly with the sampling rate $L\, f_{sym}$. The pulse shape has $L\, N$ samples. It takes $L\, N_g\, f_{sym}$ multiplications/s for the transmitter pulse shaping filter and receiver matched filter when using an efficient polyphase filter bank implementation.**

<u>N_g symbol periods in pulse shape</u>.

- ***Symbol error rate/probability.* In the receiver, the matched filter correlates the received signal against a known pulse shape of $L\, N_g$ samples to improve SNR. Most of the improvement in SNR is due to the lowpass filtering out-of-band additive noise, and the stopband attenuation can increase with the FIR filter length. When N_g increases, SNR improves and in turn the symbol error probability decreases.**
- ***Run-time complexity* increases with N_g because the transmitter pulse shaping filter and receiver matched filter take $L\, N_g\, f_{sym}$ multiplications/s when using an efficient polyphase filter bank implementation**