The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2 **Take-Home Exam**

Prof. Brian L. Evans

Date: December 7, 2020 Course: EE 445S

Name: ____
Last, First

Please sign your name below to certify that you did not receive any help, directly or indirectly, on this test from another human other your instructor, Prof. Brian L. Evans, and to certify that you did not provide help, directly or indirectly, to another student taking this exam.

(please sign here) ______________________________________

- **Take-home exam** is scheduled for Monday, Dec. 7, 2020, 10:30am to 11:59pm.
  - The exam will be available on the course Canvas page at 10:30am on Dec. 7, 2020.
  - Your solutions can be on notebook paper, or on the test and your own paper, or whatever. This means that you won't have to print the test to complete the test.
  - Please upload your solution as a single PDF file to the course Canvas page by 11:59pm on Oct 14, 2020.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content for your justification. You may reference homework solutions, exam solutions, lecture slides, textbooks, Internet resources, etc.
- **Internet access.** Yes, you may fully access the Internet when answering exam questions provided that you comply with the other instructions on this page.
- **Academic integrity.** You shall not receive help directly or indirectly on this test from another human except your instructor, Prof. Evans. You shall not provide help, directly or indirectly, to another student taking this exam.
- **Send questions to Prof. Evans.** You may send questions or concerns about this midterm exam during the test to Prof. Evans via Canvas or by e-mail at bevans@ece.utexas.edu.
- **Contact by Prof. Evans.** Prof. Evans might contact all students in the class during the exam through Canvas announcements. Please periodically monitor those announcements.

Problem	Point Value	Your score	Topic
1	24		PAM Symbol Error Probability
2	27		QAM Communication Performance
3	24		Interference Cancellation
4	25		Communication System Design Tradeoffs
Total	100		

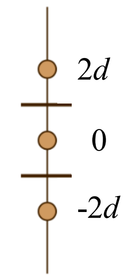
Problem 2.1. *PAM Symbol Error Probability. 24 points.*

Consider a three-level baseband pulse amplitude modulation (PAM) system:

$2d$ constellation spacing	f_s sampling rate
f_{sym} symbol rate	$g[m]$ raised cosine pulse with rolloff α
J bits/symbol	L samples/symbol
M number of levels, i.e. $M = 2^J$	N_g symbol periods in $g[m]$
T_s sampling time	T_{sym} symbol time.

The 3-PAM constellation is shown on the right (i.e. $M = 3$).

Symbol of bits | Symbol Amplitude



3-PAM Constellation

This problem is asking you to analyze the 3-PAM system and enter your results in the table below to compare against 2-PAM and 4-PAM systems.

For this problem, assume that $T_{sym} = 1$ s and hence $f_{sym} = 1$ Hz.

(a) What is the bit rate for the 3-PAM system? Why? *4 points*.

(b) Give a symbol of bits for each symbol amplitude on the 3-PAM constellation. Make sure that the bit encoding for adjacent symbols only differs by one bit (Gray coding). *6 points.*

(c) Derive 3-PAM symbol error probability. Thresholds are at $-d$ and d. *8 points.*

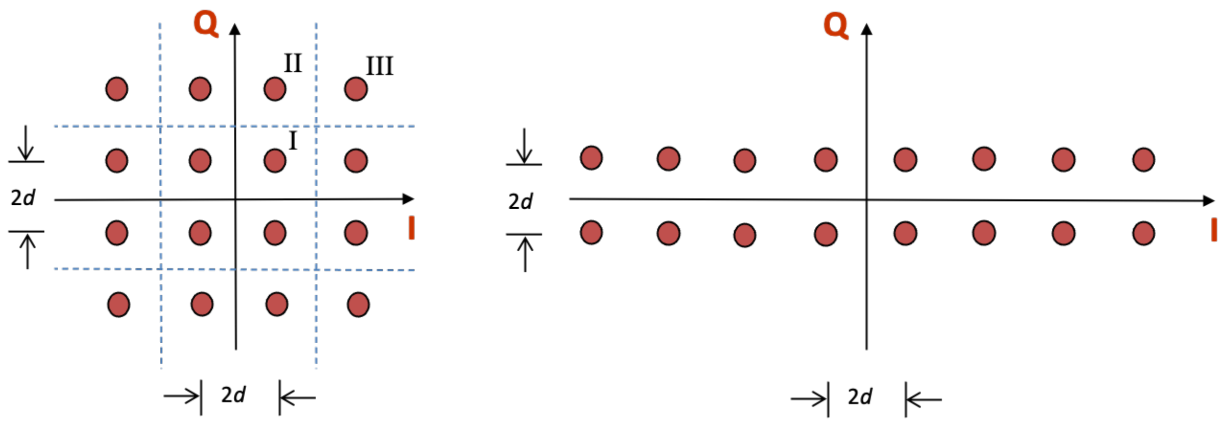
(d) Derive the transmitted maximum power, average power, and peak-to-average power ratio. *6 points.*

M	**Bit rate**	**Symbol Error Probability**	**Peak Power**	**Average Power**	**Peak-to-Average Power Ratio**
2	1 bit/s	$Q\left(\frac{d}{\sigma}\right)$	d^2	d^2	1.0
3					
4	2 bits/s	$\frac{3}{2}\,Q\left(\frac{d}{\sigma}\right)$	$9\,d^2$	$5\,d^2$	1.8

Note: All entries in the above table assume that $T_{sym} = 1$ s and hence $f_{sym} = 1$ Hz.

Problem 2.2 *QAM Communication Performance. 27 points.*

Consider the two 16-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

Each part below is worth 3 points. **Please fully justify your answers.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$18d^2$	
(b) Average transmit power	$10d^2$	
(c) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***		
(d) Number of type I regions	4	
(e) Number of type II regions	8	
(f) Number of type III regions	4	
(g) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$3Q\left(\frac{d}{\sigma}\right)-\frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$	
(h) Express d/σ as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR}=\frac{10d^2}{\sigma^2}$ $\frac{d}{\sigma}=\sqrt{\frac{\text{SNR}}{10}}$	

(i) For an application, four of the 16 symbols have the most significance and we would like to transmit them with the lowest symbol error probability. Using the **left constellation**, which constellation points would you use to represent these four high-significance symbols? Why? *3 points*.

Problem 2.3. *Interference Cancellation. 24 points.*

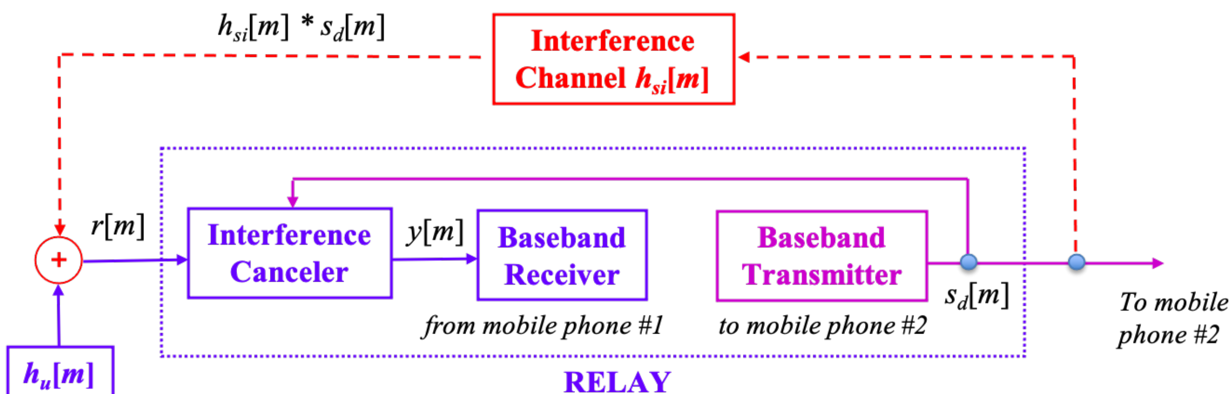
In wireless communication systems, transmitted signals experience a wide variety of impairments by the time they reach a receiver, including attenuation over distance traveled.

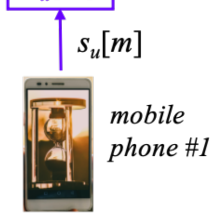
A relay can be deployed to receive and decode a basestation transmission and then re-encode and retransmit the signal so that it arrives at a mobile phone with higher signal power.

A relay can also receive and decode a mobile phone transmission and then re-encode and retransmit the signal so that it arrives at a basestation with higher signal power.

When a relay receives and transmits signals at the same time and in the same frequency band, the propagation of the relay transmission to the relay receiver causes **interference**.

The diagram below gives a baseband model for a relay system serving two mobile phones. *Hint during test*: mobile phone #1 is transmitting at the same time that the relay is receiving and transmitting.





The interference canceler does not know $h_{si}[m]$, $h_u[m]$, or $s_u[m]$, and uses knowledge of $s_d[m]$ and $r[m]$ to adapt a finite impulse response (FIR) filter $c[m]$ to subtract interference from $r[m]$:

$$r[m] = \underbrace{h_u[m] * s_u[m]}_{\text{from mobile phone \#1}} + \underbrace{h_{si}[m] * s_d[m]}_{interference}$$

$$y[m] = r[m] - c[m] * s_d[m]$$

(a) What objective function would you use? Why? *8 points.*

(b) For an adaptive FIR canceler, **derive the update equation** for the vector of FIR coefficients $\vec{c}$ for the objective function in part (a). Here, $\vec{c} = [\, c_0 \;\; c_1 \;\; c_2 \;\; \cdots \;\; c_{N-1} \,]$. *12 points.*

(c) For your answer in (b), what values of the step size (learning rate) μ would you use? *4 points.*

Smart phone picture by Samer Daboul

Problem 2.4. *Communication System Design Tradeoffs. 25 points*

Describe the effect of increasing each pulse amplitude modulation system design parameter in the first column on the quantity in the other columns: **increase**, **decrease** or leave it **blank** to mean no effect.

When considering the impact of increasing the parameter in the first column, assume that the values of the other parameters are held constant.

After the table, justify your answers for each row. The row for J bits/symbol is given as an example.

Parameter	Transmit Power Consumption	Transmission Bandwidth	Bit Rate	Symbol Error Rate	Run-Time Complexity
B bits in A/D output & D/A input					
$2d$ constellation spacing in Volts					
f_{sym} symbol rate in Hz					
***J* bits/symbol**	**increase**		**increase**	**increase**	**increase**
L samples/symbol					
N_g symbol periods in pulse shape					

***J* bits/symbol.** Increasing J increases number of constellation points, $M = 2^J$. Peak transmit power is proportional to square of max amplitude $(M\text{-}1)d$ and hence increases exponentially in J. Bit rate $J f_{sym}$ increases linearly with J. For symbol error probability $\frac{2\,(M-1)}{M}\; Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)$, the constant in front is on the interval [1, 2) and increases slightly with J because $M = 2^J$. In the transmitter, the constellation map has $M = 2^J$ entries and fast symbol decoding in the receiver takes J comparisons.

Hint during test: For "$2d$ constellation spacing in Volts", consider what happens when d increases.