The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2 ***Solutions 2.0***

Date: Dec. 6, 2021 Course: EE 445S Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except your instructor, Prof. Evans, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	Point Value	Your score	Topic
1	21		Baseband PAM System
2	33		PAM vs. QAM Communication Performance
3	28		Decision-Directed Equalization
4	18		Potpourri
Total	100		

Prologue*: Lectures 13 & 14; JSK Sec. 2.10; JSK Ch. 8 & 11; Lab 5; HW 4.2, 4.3, 5.2, 6.1 & 6.2; Midterm 2.1: F17, Sp18, F18, Sp21

Problem 2.1. *Baseband PAM System. 21 points.*

Consider a binary phase shift keying (BPSK) system, a.k.a. a two-level pulse amplitude modulation (2-PAM) system.

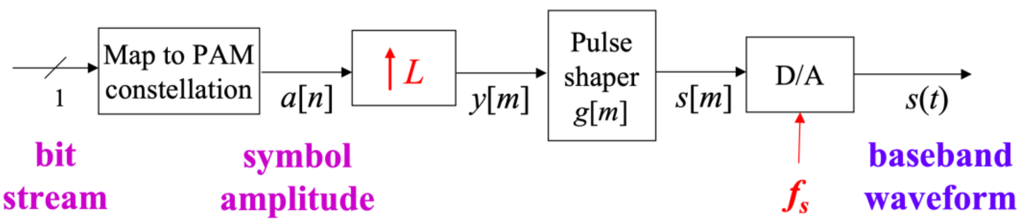
The system parameters are described on the right:

- J = 1 bit/symbol
- L = 4 samples per symbol period
- Pulse shape $g[m]$ is a rectangular pulse of L = 4 samples in duration with amplitude $1/L$.
- A bit of value 0 is mapped to symbol amplitude $-d$, and a bit of value 1 is mapped to symbol amplitude d.

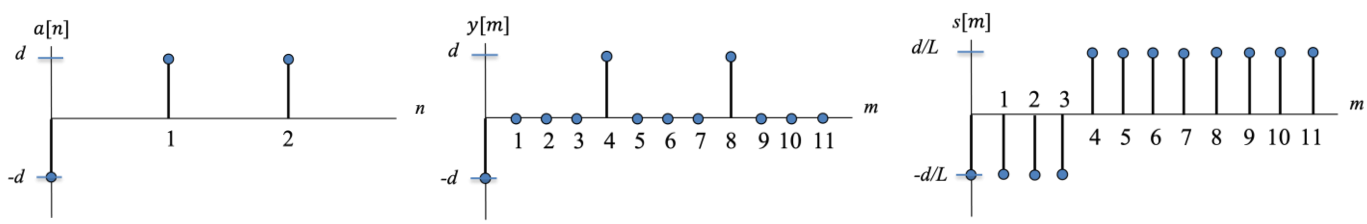
PAM System Parameters

$a[n]$	symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index

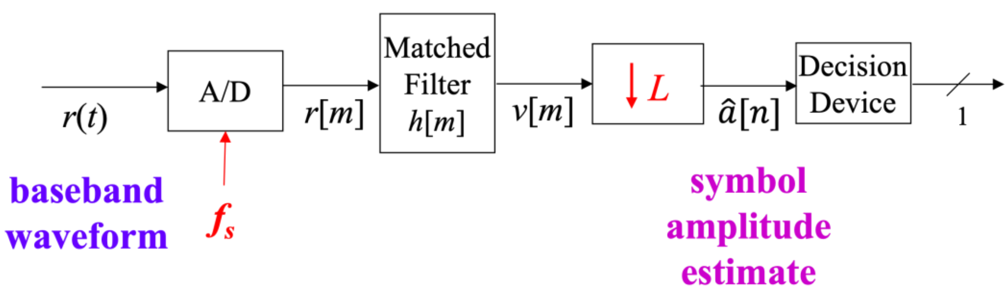
(a) For the **BPSK transmitter** below, the input bit stream is 011. Plot the discrete-time signals $a[n]$, $y[m]$ and $s[m]$. *9 points.*



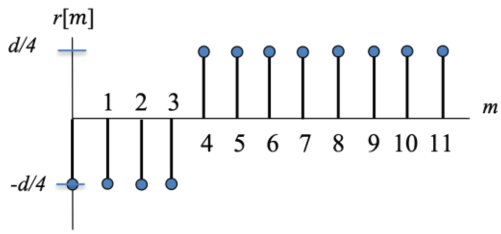
Lecture Slide 13-8

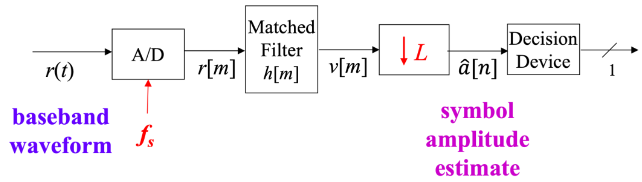


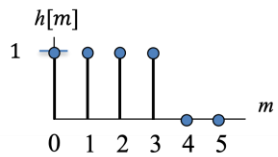
(b) For the **BPSK receiver** below, assume there is no channel distortion or additive noise and assume that $r[m] = s[m]$. Plot the discrete-time signals $r[m]$, $h[m]$, $v[m]$ and $\hat{a}[n]$ based on the BPSK transmitter in (a). *12 points.*



See the next page.

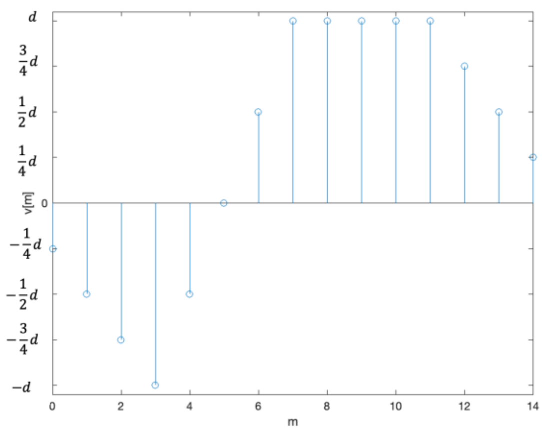




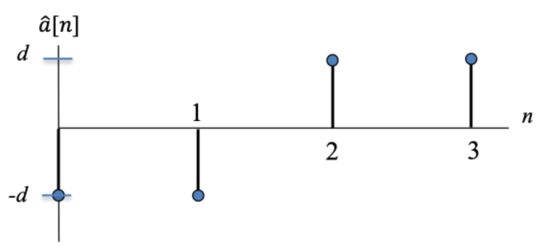


Matched filter

$$h[m] = k\, g^*[L - m]$$




```
r = [ -1 -1 -1 -1 1 1 1 1 1 1 1 1] / 4;
h = [ 1 1 1 1 ];
v = conv(h, r);
M = length(v);
m = 0 : M-1;
stem(m, v);
ylim( [-1.05 1.05] );
xlabel('m');
ylabel('v[m]');
```



<u>Epilogue</u>: Four bits are output by the receiver (0011) even though only three bits were input to the transmitter (011).

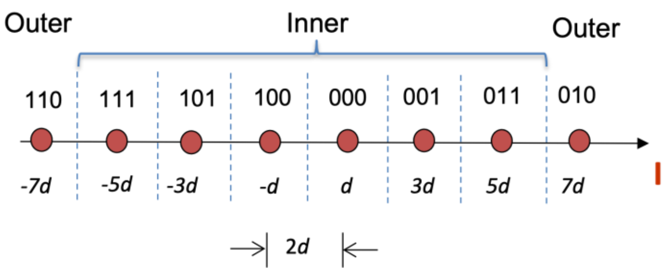
The reason is that we haven't accounted for the group delay through the pulse shaping filter of $\frac{L-1}{2}$ samples or the group delay through the matched filter also of $\frac{L-1}{2}$ samples.

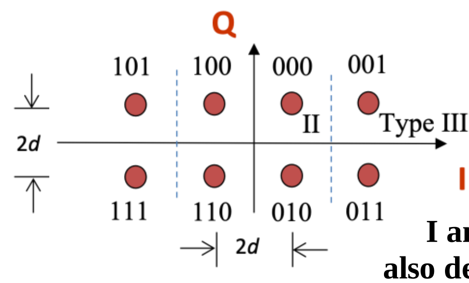
Prior to the downsampler in the receiver, we should discard the first $L-1$ samples.

***Prologue*: Lectures 13-16; JSK Ch. 16; Lab 5 & 6; HW 4.2, 4.3, 5.2, 6.3 & 7.3; Handout P PAM vs. QAM; Midterm 2.2 problems: F14, Sp15, F15, Sp16, F16, Sp17, F17, Sp18, F18, Sp19, F19, Sp20, F20 & Sp21**

Problem 2.2 *PAM vs. QAM Communication Performance. 33 points.*

Consider the 8-PAM (left) and 8-QAM (right) constellations below. Constellation spacing is $2d$.





I and Q axes are also decision region boundaries

Energy in the pulse shape is 1. Symbol time T_{sym} is 1s.

Each part below is worth 3 points. **Please fully justify your answers.**

	8-PAM Constellation	**8-QAM Constellation**
(a) Peak transmit power	$49d^2$	$\mathbf{10d^2}$
(b) Average transmit power	$21d^2$	$\mathbf{6d^2}$
(c) Peak-to-average power ratio	$\frac{49d^2}{21d^2} = \frac{7}{3} \approx 2.33$	$\mathbf{\frac{10d^2}{6d^2} = \frac{5}{3} \approx 1.67}$
(d) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***		
8-PAM inner constellation points	6	
8-PAM outer constellation points	2	
(e) Number of type I QAM regions		**0**
(f) Number of type II QAM regions		**4**
(g) Number of type III QAM regions		**4**
(h) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2. For QAM, the variance is σ^2 in the in-phase component and σ^2 in the quadrature component. For PAM, the variance is $2\sigma^2$ to keep the total noise power the same as in QAM.	$\frac{7}{4} Q\left(\frac{d}{\sqrt{2}\sigma}\right)$	$\boldsymbol{P_c = \frac{1}{2} P_c^{II} + \frac{1}{2} P_c^{III}}$ $\boldsymbol{P_c^{II} = (1-q)(1-2q)}$ $\boldsymbol{P_c^{III} = (1-q)^2}$ $\boldsymbol{P_e = 1 - P_c}$ $\boldsymbol{P_e = \frac{5}{2} Q\left(\frac{d}{\sigma}\right) - \frac{3}{2} Q^2\left(\frac{d}{\sigma}\right)}$
(i) Express the argument of the Q function as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{21d^2}{2\sigma^2}$ $\frac{d}{\sqrt{2}\sigma} = \sqrt{\frac{\text{SNR}}{21}}$	$\mathbf{SNR} = \boldsymbol{\frac{6d^2}{\sigma^2}}$ $\boldsymbol{\frac{d}{\sigma} = \sqrt{\frac{\mathbf{SNR}}{6}}}$

$$\boldsymbol{q = Q\left(\frac{d}{\sigma}\right)}$$

(j) Give a Gray coding for the 8-QAM constellation points on the constellation above. *3 points*.

Gray coding means the bit pattern only differs by one bit in adjacent decision regions to minimize the number of bit errors when a symbol error occurs. One approach is to use the first two bits to encode the quadrant, and third bit to specify which point in the quadrant.

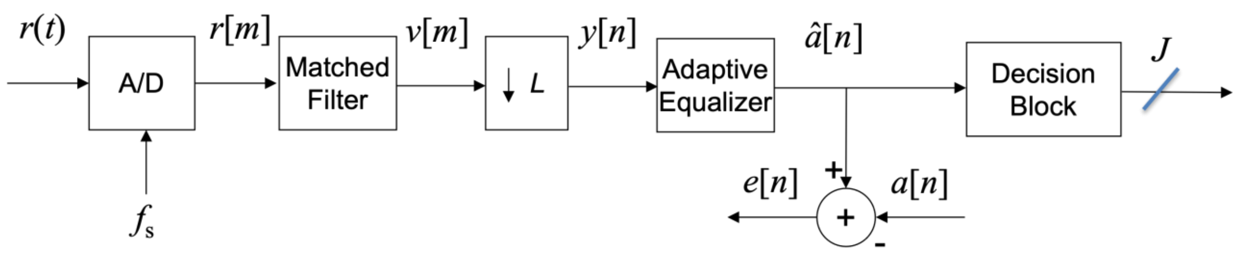
(k) Would you recommend using 8-PAM or 8-QAM? Give two reasons. *3 points.*

Both 8-PAM and 8-QAM would have the same bit rate. Both can be Gray coded. Choose 8-QAM because it has a lower probability of symbol error for the same SNR, and lower maximum power, average power, and peak-to-average-power ratio for the same value of *d*.
(8-QAM baseband Tx has at least 2x implementation complexity as 8-PAM; same for Rx.)

***Prologue*. JSK Sec. 2.12 & 13.1-13.4; HW 5.1, 6.1, 6.2 & 7.2;**
In-Lecture Assignments 3 & 4; Lectures 12-14; Lecture Slides 16-6 to 16-8;
Midterm #2 Prob 2.2 F12; 2.1 Sp13; 2.1 F13; 2.1 Sp14; 2.3 & 2.4 Sp17; 2.3 F17; 2.3 F19; 2.3 Sp20

Problem 2.3. *Decision-Directed Equalization. 28 points.*

Consider the following baseband pulse amplitude modulation (PAM) receiver with an adaptive finite impulse response (FIR) equalizer placed immediately before the decision device:



The adaptive FIR equalizer runs at the symbol rate and has N coefficients. We place the coefficients in a vector

$$\vec{w} = [\, w_0 \;\; w_1 \ldots w_{N-1} \,]$$

Consider adapting the decision-directed equalizer during training. The error signal is $e[n] = \hat{a}[n] - a[n]$ which is the difference between the estimated and transmitted symbol amplitudes.

This is similar to the adaptive least-mean squares (LMS) FIR filter in HW 7.2 except it runs at the symbol rate instead of the sampling rate and uses a different error measure.

For the adaptive FIR equalizer,

(a) Give an objective function $J(n)$. *6 points.*

We want drive the error to zero by driving the square of the error to zero: $J(n) = \frac{1}{2} e^2[n]$

(b) What is the initial value of the adaptive FIR equalizer coefficients you would use? Why? *3 points.*

$\vec{w}[0] = [\, 1\; 0\; 0 \ldots 0 \,]$ **is equivalent to the original receiver block diagram that does not have an adaptive filter. So, if it's not needed, it shouldn't adapt. Using all zeros means equalizer output $\hat{a}[n]$ will initially be zero, but equalizer will still adapt. Any initialization works.**

(c) Derive an update equation for the adaptive FIR equalizer coefficients vector at iteration n

$$\vec{w}[n] = [\, w_0[n] \; w_1[n] \ldots w_{N-1}[n] \,]$$

Compute all derivatives. Simplify the result. *9 points*

$$\vec{w}[n+1] = \vec{w}[n] - \mu \left. \frac{dJ(n)}{d\vec{w}} \right|_{\vec{w}=\vec{w}[n]} \quad \text{where} \quad \frac{dJ(n)}{d\vec{w}} = e[n] \frac{de[n]}{d\vec{w}} = e[n] \frac{d\hat{a}[n]}{d\vec{w}} = e[n]\, \vec{y}[n]$$

where $\hat{a}[n] = w_0\, y[n] + w_1\, y[n-1] + w_2\, y[n-2] + \cdots + w_{N-1}\, y[n-(N-1)]$ **and**

$$\frac{d\hat{a}[n]}{dw_m} = y[n-m] \quad \text{so} \quad \frac{d\hat{a}[n]}{d\vec{w}} = \vec{y}[n] \quad \text{where } \vec{y}[n] = [\, y[n] \;\; y[n-1] \; \ldots y[n-(N-1)] \,]$$

$$\vec{w}[n+1] = \vec{w}[n] - \mu\, e[n]\, \vec{y}[n]$$

PAM System Parameters

$a[n]$	symbol amplitude
$\hat{a}[n]$	estimated symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse response
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index
T_{sym}	symbol time

(d) What range of values would you recommend for the step size μ? Why? *3 points*.

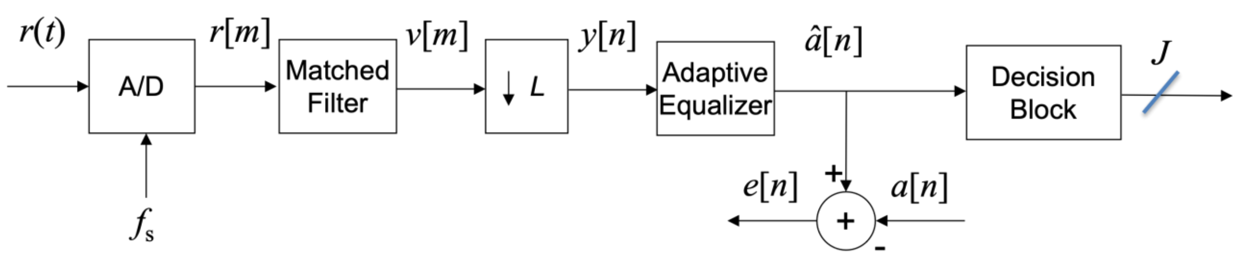
Use small positive values for the step size such as 0.01 or 0.001 for convergence as seen on HW 7.2 on adaptive equalizers. Using a large positive value can lead to instability. Using value of zero would not update the value of the coefficients. Using a negative value would maximize the objective function instead of minimizing it.

(e) Is it possible for the adaptive equalizer to compensate symbol timing error? Why or why not? *7 points*.

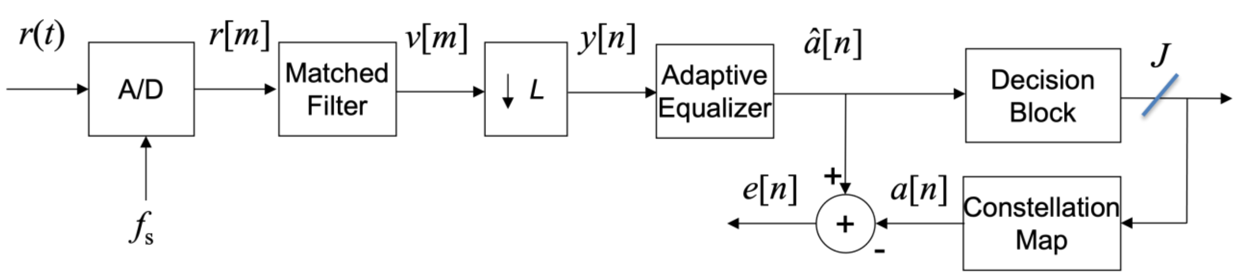
During training, the error (vector) $e[n]$ is a measurement of all the remaining impairments, including symbol timing error.

By driving the error (vector) $e[n]$ to zero, the adaptive equalizer can compensate linear magnitude and phase distortion that varies over the training period. Compensating for phase distortion would include compensating for symbol timing error because compensating symbol timing error can be expressed as a phase shift. (See Fall 2018 Midterm Problem 2.4.)

Epilogue: We can also use decision-directed methods when we're not training. That is, when an unknown message is being received, we can adjust the equalizer from



to the following:



The adjusted version can be "fooled" when there is a symbol error. This version of the decision-directed FIR equalizer described in JSK Section 13.4.

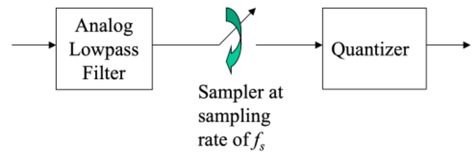
[1] Erkan Acar, "How Error Vector Magnitude (EVM) Measurement Improves Your System-Level Performance", Analog Devices, Technical Article, Dec. 2021.

Prologue: These questions are from the second half of the semester. References are given with each part.

Problem 2.4. *Potpourri*. *18 points.*

Please determine whether the following claims are true or false and **support each answer with a brief justification**. A true or false answer without any justification will not earn any points.

(a) An increase in thermal noise power always causes a decrease in the signal-to-noise ratio (SNR) in the receiver after the analog-to-digital (A/D) converter. *3 points*.



$$\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}}$$

False. **An A/D converter is shown on the right. The analog lowpass filter attenuates noise at frequencies above $\frac{1}{2} f_s$ which will alias after sampling. At the quantizer output, noise power will be the greater of the thermal noise power at the quantizer input and the quantization noise power introduced by the quantizer. If the quantization noise power were greater than the thermal noise power, noise power at the quantizer output will be the quantization noise power, until the thermal noise power increases to exceed the quantization noise power. So, an increase in the thermal noise power at the A/D input does not always cause a decrease in SNR at the A/D output. (Matched filter after the A/D converter will increase SNR by filtering out-of-baseband noise.)**

(b) For an M-level pulse amplitude modulation (PAM) transmitter using a raised cosine pulse shape and a B-bit digital-to-analog (D/A) converter, setting $B = \log_2 M$ will avoid any clipping in amplitude of the input of the D/A converter. *3 points*.

False. **The number of bits B needed to avoid clipping in the transmitter depends on J which is the number of bits in the PAM constellation, d which is half of the distance between constellation points, and $g[m]$ which is the pulse shape. PAM symbol amplitudes are in $[-(M-1)d, (M-1)d]$. For a raised cosine pulse shape with max amplitude of 1, pulse shaping filter output values are in $[-2(M-1)d, 2(M-1)d]$ which requires $\log_2(4(M-1)d)$ bits.** **See lecture slides 13-6 and 15-8, and Spring 2019 Midterm 2.3(b).**

(c) PAM and QAM transmission using the same constellation size and symbol rate will always have the same bit rate. *3 points*. ***True***. **For PAM and QAM, the bit rate in bits/s is $J f_{sym}$ where J is the number of bits/symbol and f_{sym} is the symbol rate in symbols/s. Moreover, $J = \log_2 M$ where M is the constellation size. Hence, the bit rate is $f_{sym} \log_2 M$.** **See lecture slide 13-3.**

(d) The number of constellation points for a PAM system must always be a power of 2. *3 points*.

False. **The High-Speed Digital Subscriber Line 2 standard supports PAM constellation sizes that are not powers of two. Also, Fall 2020 Midterm Problem 2.1 concerned 3-PAM systems.**

(e) In a QAM system, the only way to reduce the symbol error rate is to reduce the symbol rate. *3 points*. ***False.*** **Symbol error rate for 16-QAM is $3Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right) - \frac{9}{4}Q^2\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)$ per lecture slide 15-15. Q is monotonically decreasing (lecture slide 14-22). In addition to reducing the symbol error rate by reducing the symbol rate f_{sym} or equivalently increasing the symbol time T_{sym}, one can increase d, which is half of the distance between constellation points. Increasing d increases transmit power, which in turn increases SNR.**

(f) In a communication system, the sampling rate used in the receiver must always be equal to the sampling rate used in the transmitter. *3 points*. ***False.*** **Consider a transmitter that implements cosine modulation by a carrier frequency f_c in discrete time. Sampling rate must be greater than $2(f_c + W)$ where W is the baseband bandwidth. The receiver will demodulate by first modulating the bandpass signal and the sampling rate must exceed $2(2f_c + W)$. Receiver could use 2x sampling rate, and the number of samples per symbol period L would double. Also, transmitter and receiver usually have different circuits. Symbol timing recovery is used to synchronize the receiver sampling clock with the transmitter sampling clock.**