The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Date: Dec. 6, 2021 Course: EE 445S Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except your instructor, Prof. Evans, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	Point Value	Your score	Topic
1	21		Baseband PAM System
2	33		PAM vs. QAM Communication Performance
3	28		Decision-Directed Equalization
4	18		Potpourri
Total	100		

Problem 2.1. *Baseband PAM System. 21 points.*

Consider a binary phase shift keying (BPSK) system, a.k.a. a two-level pulse amplitude modulation (2-PAM) system.

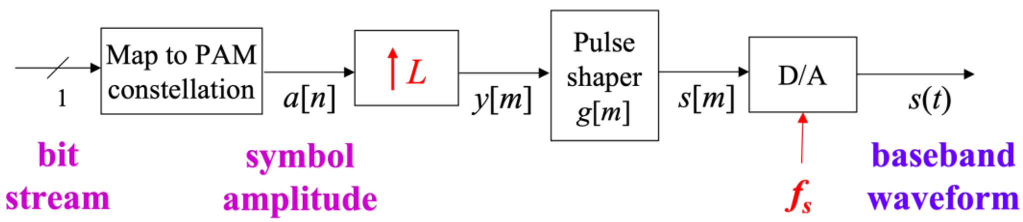
The system parameters are described on the right:

- $J = 1$ bit/symbol
- $L = 4$ samples per symbol period
- Pulse shape $g[m]$ is a rectangular pulse of $L = 4$ samples in duration with amplitude $1/L$.
- A bit of value 0 is mapped to symbol amplitude $-d$, and a bit of value 1 is mapped to symbol amplitude d.

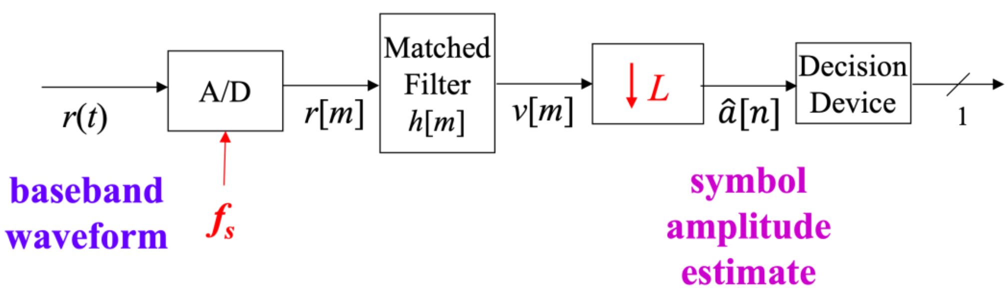
<u>PAM System Parameters</u>

$a[n]$	symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index

(a) For the **BPSK transmitter** below, the input bit stream is 011. Plot the discrete-time signals $a[n]$, $y[m]$ and $s[m]$. *9 points.*

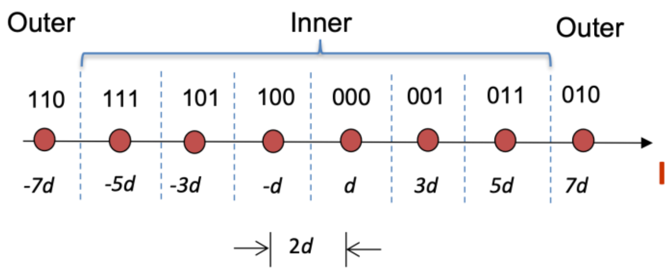


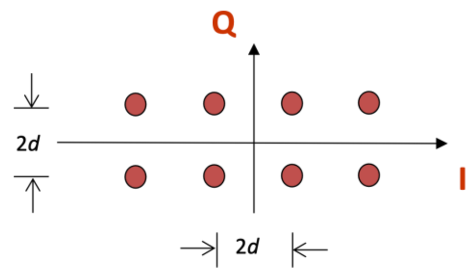
(b) For the **BPSK receiver** below, assume there is no channel distortion or additive noise and assume that $r[m] = s[m]$. Plot the discrete-time signals $r[m]$, $h[m]$, $v[m]$ and $\hat{a}[n]$ based on the BPSK transmitter in (a). *12 points.*



Problem 2.2 *PAM vs. QAM Communication Performance. 33 points.*

Consider the 8-PAM (left) and 8-QAM (right) constellations below. Constellation spacing is $2d$.





Energy in the pulse shape is 1. Symbol time T_{sym} is 1s.

Each part below is worth 3 points. **Please fully justify your answers.**

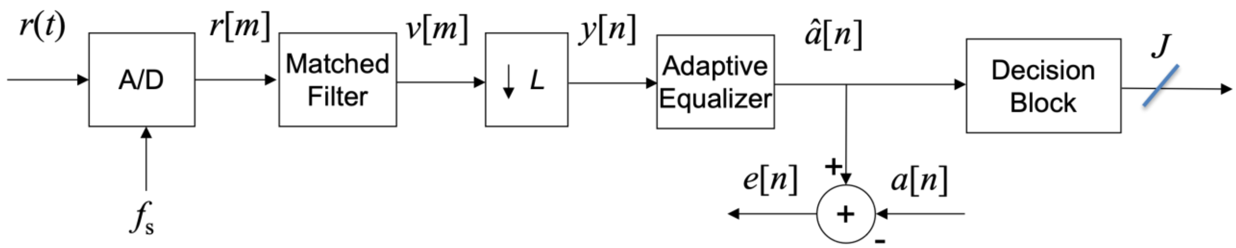
	8-PAM Constellation	**8-QAM Constellation**
(a) Peak transmit power	$49d^2$	
(b) Average transmit power	$21d^2$	
(c) Peak-to-average power ratio	$\frac{49d^2}{21d^2} = \frac{7}{3} \approx 2.33$	
(d) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***		
8-PAM inner constellation points	6	
8-PAM outer constellation points	2	
(e) Number of type I QAM regions		
(f) Number of type II QAM regions		
(g) Number of type III QAM regions		
(h) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2. For QAM, the variance is σ^2 in the in-phase component and σ^2 in the quadrature component. For PAM, the variance is $2\sigma^2$ to keep the total noise power the same as in QAM.	$\frac{7}{4} Q\left(\frac{d}{\sqrt{2}\sigma}\right)$	
(i) Express the argument of the Q function as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{21d^2}{2\sigma^2}$ $\frac{d}{\sqrt{2}\sigma} = \sqrt{\frac{\text{SNR}}{21}}$	

(j) Give a Gray coding for the 8-QAM constellation points on the constellation above. *3 points.*

(k) Would you recommend using 8-PAM or 8-QAM? Give two reasons. *3 points.*

Problem 2.3. *Decision-Directed Equalization. 28 points.*

Consider the following baseband pulse amplitude modulation (PAM) receiver with an adaptive finite impulse response (FIR) equalizer placed immediately before the decision device:



The adaptive FIR equalizer runs at the symbol rate and has N coefficients. We place the coefficients in a vector

$$\vec{w} = [\, w_0 \;\; w_1 \ldots w_{N-1} \,]$$

Consider adapting the decision-directed equalizer during training.

The error signal is $e[n] = \hat{a}[n] - a[n]$ which is the difference between the estimated and transmitted symbol amplitudes.

For the adaptive FIR equalizer,

(a) Give an objective function $J(n)$. *6 points*.

(b) What is the initial value of the adaptive FIR equalizer coefficients you would use? Why? *3 points*.

<u>PAM System Parameters</u>

$a[n]$	symbol amplitude
$\hat{a}[n]$	estimated symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse response
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index
T_{sym}	symbol time

(c) Derive an update equation for the adaptive FIR equalizer coefficients vector at iteration n

$$\vec{w}[n] = [\, w_0[n] \; w_1[n] \ldots \; w_{N-1}[n] \,]$$

Compute all derivatives. Simplify the result. *9 points*

(d) What range of values would you recommend for the step size μ? Why? *3 points*.

(e) Is it possible for the adaptive equalizer to compensate symbol timing error? Why or why not? *7 points*.

Problem 2.4. *Potpourri. 18 points.*

Please determine whether the following claims are true or false and **support each answer with a brief justification**. A true or false answer without any justification will not earn any points.

(a) An increase in thermal noise power always causes a decrease in the signal-to-noise ratio (SNR) in the receiver after the analog-to-digital (A/D) converter. *3 points*.

(b) For an M-level pulse amplitude modulation (PAM) transmitter using a raised cosine pulse shape and a B-bit digital-to-analog (D/A) converter, setting $B = \log_2 M$ will avoid any clipping in amplitude of the input of the D/A converter. *3 points.*

(c) PAM and QAM transmission using the same constellation size and symbol rate will always have the same bit rate. *3 points*.

(d) The number of constellation points for a PAM system must always be a power of 2. *3 points*.

(e) In a QAM system, the only way to reduce the symbol error rate is to reduce the symbol rate. *3 points*.

(f) In a communication system, the sampling rate used in the receiver must always be equal to the sampling rate used in the transmitter. *3 points*