The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Date: December 5, 2022 Course: EE 445S Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except your instructor, Prof. Evans, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	**Point Value**	**Your score**	**Topic**
1	22		Baseband PAM System
2	30		QAM Communication Performance
3	24		Channel Equalization
4	24		Potpourri
Total	100		

Problem 2.1. *Baseband PAM System. 22 points.*

Consider a binary phase shift keying (BPSK) system, a.k.a. a two-level pulse amplitude modulation (2-PAM) system.

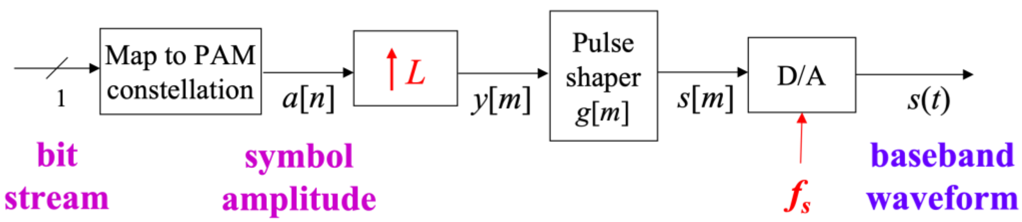
The system parameters are described on the right:

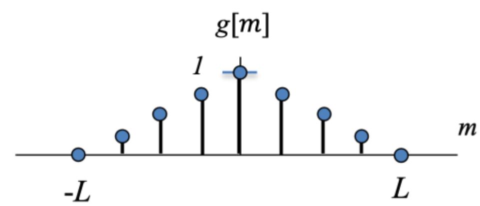
- J = 1 bit/symbol
- L = 4 samples per symbol period
- Pulse shape $g[m]$ is a **triangular** pulse of 8 samples in duration plotted below on the right.
- A bit of value 0 is mapped to symbol amplitude *-1*, and a bit of value 1 is mapped to symbol amplitude *1*.

PAM System Parameters

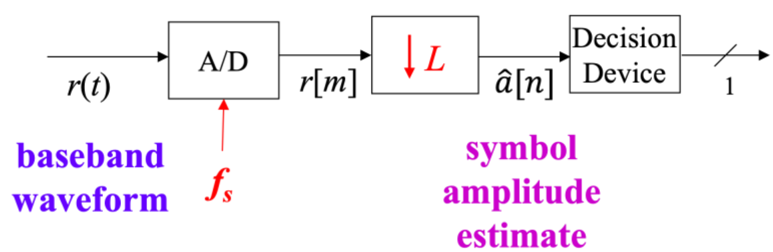
$a[n]$	symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index

(a) For the **BPSK transmitter** below, the input bit stream is 101. Plot the discrete-time signals $a[n]$, $y[m]$ and $s[m]$. *12 points*.



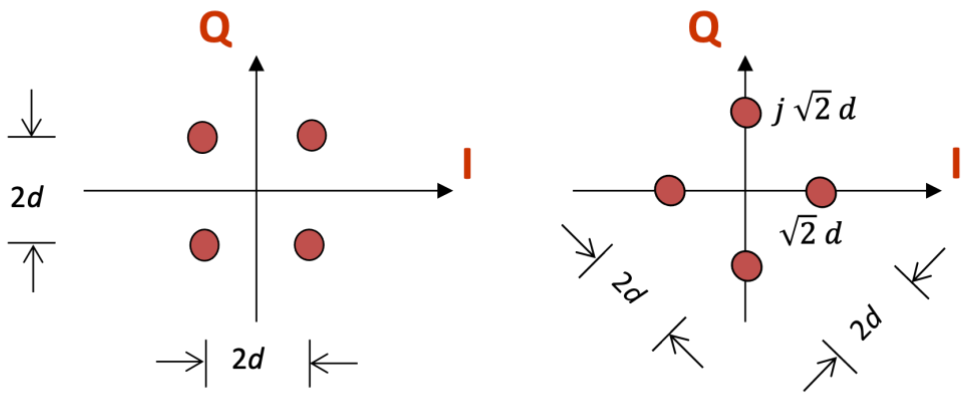


(b) For the **BPSK receiver** to the right, assume there is no channel distortion or additive noise and assume that $r[m] = s[m]$. **There is no matched filter.** Plot the discrete-time signal $\hat{a}[n]$ and give the received bit stream based on the BPSK transmitter in (a). *10 points*.



Problem 2.2 *QAM Communication Performance. 30 points.*

In transmitting synchronization sequences from its low-earth orbit satellites to ground terminals, the SpaceX Starlink system uses both 4-QAM constellations below [1]. The constellation on the right is the constellation on the left rotated by 90 degrees. Constellation spacing for both constellations is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s.

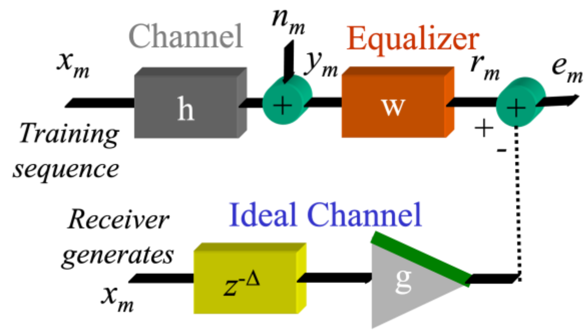
Each part below is worth 3 points. **Please fully justify your answers. Show intermediate steps.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$2d^2$	
(b) Average transmit power	$2d^2$	
(c) Peak-to-average power ratio	$\frac{2d^2}{2d^2} = 1$	
(d) For the left constellation, the decision regions are the four quadrants and the IQ axes are the boundaries. For the right constellation, draw rotated type I, II and/or III decision regions ***that will minimize the probability of symbol error using such decision regions.***		
(e) Number of type I QAM regions	0	
(f) Number of type II QAM regions	0	
(g) Number of type III QAM regions	4	
(h) Probability of symbol error in presence of additive Gaussian noise with zero mean & variance σ^2.	$P_e = 2Q\left(\frac{d}{\sigma}\right) - Q^2\left(\frac{d}{\sigma}\right)$	
(i) Express the argument of the *Q* function as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{2d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{SNR}{2}}$	

(j) Give a Gray coding for right constellation or show that one does not exist. *3 points*.

[1] T.E. Humphreys, P.A Iannucci, Z. Komodromos and A.M. Graff,"Signal Structure of the Starlink Ku-Band Downlink", submitted Oct. 20, 2022. *The primary and secondary synchronization sequences use the 4-QAM constellation on the right, and the coda synchronization sequence uses the 4-QAM constellation on the left. The primary synchronization sequence can also use 16-QAM in high SNR scenarios.*

Problem 2.3. *Channel Equalization*. *24 points*.



In the discrete-time system on the right, the equalizer operates at the sampling rate. The equalizer is a finite impulse response (FIR) fitter with N real coefficients $w_0, w_1, \ldots w_{N-1}$:

$$r[m] = w_0\, y[m] + w_1\, y[m-1] + \ldots + w_{N-1}\, y[m - (N-1)]$$

Channel model is an FIR filter with impulse response $h[m]$ in cascade with additive noise $n[m]$.

When using a least mean squares (LMS) cost function

$$J_{LMS}(e[m]) = \frac{1}{2} e^2[m]$$

the adaptive update equation using a steepest descent algorithm becomes

$$\vec{w}[m+1] = \vec{w}[m] - \mu\, e[m]\, \vec{y}[m]$$

$$\text{where } \vec{w}[m] = [\, w_0[m] \quad w_1[m] \ \cdots \ w_{N-1}[m] \,]$$

$$\text{and } \vec{y}[m] = [\, y[m] \quad y[m-1] \ \cdots \ y[m-(N-1)]\,]$$

With channels that have deep nulls in their frequency response, and a high SNR in the received signal, the minimization of the LMS cost function results in a large spike in the equalizer frequency response to compensate for the nulls. The large spikes also amplify additive noise.

To inhibit large values of FIR filter coefficients w_i needed to create a frequency response with bands of high gain, consider a cost function that penalizes the sum of the squares of the FIR coefficients:

$$J_{LMSnorm}(e[m]) = \frac{1}{2} e^2[m] + \frac{1}{2} \lambda \sum_{i=0}^{N-1} w_i^2[m]$$

where λ is a positive constant.

(a) What training sequence for $x[m]$ would you use? Why? *3 points.*

(b) How you would estimate the delay parameter Δ in the ideal channel model. *3 points.*

(c) Derive the adaptive update equation for the FIR equalizer coefficients using the $J_{LMSnorm}(e[m])$ cost function. *15 points.*

(d) What values of the step size (learning rate) μ would you use? *3 points.*

Problem 2.4. *Potpourri*. *24 points*.

(a) ***Duplexing***. A cellular communication system is deployed in one of two ways at present:

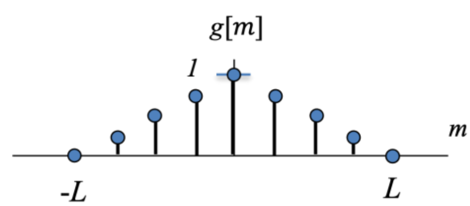
- *Frequency division duplexing* means the basestation sends data to the smart phone on one frequency band and the smart phone sends data to the basestation on another frequency band.
- *Time division duplexing* means that in a timeslot, the basestation sends data to the smart phone in the first 40% of the timeslot and the smart phone sends data to the basestation in the last 40% of the timeslot. The 20% gap is needed to toggle each side between transmit and receive.

In a full duplex system, the basestation is sending/receiving data to/from the smart phone at the same time and in the same frequency band.

i. What is the maximum spectral efficiency improvement of full duplex vs. frequency division duplexing? Why? *6 points.*

ii. What is the maximum spectral efficiency improvement of full duplex vs. time division duplexing? Why? *6 points.*

(b) For the baseband PAM receiver in problem 2.1(b), we'll include additive noise in the channel model and add a matched filter in the receiver to compensate for additive noise. The pulse shape in the transmitter (where L = 4 samples) is shown on the right:



i. Give a general formula for the impulse response $h[m]$ of the matched filter that optimizes the SNR in the symbol amplitude estimate $\hat{a}[n]$ and that is expressed in terms of $g[m]$, L, and possibly other system parameters. *4 points.*

ii. Plot the impulse response $h[m]$ of the matched filter that optimizes the SNR in $\hat{a}[n]$. *4 points*.

iii. What is the reduction in the symbol error rate by using the matched filter? *4 points.*