# The University of Texas at Austin Dept. of Electrical and Computer Engineering Midterm #2

### Prof. Brian L. Evans

Date: May 5, 2017

Course: EE 445S

Name:

Last,

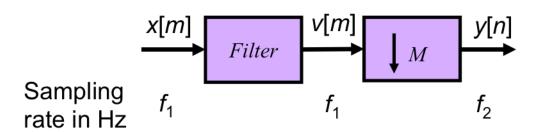
First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all smart phones and other personal communication devices.
- Please remove headphones.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- Fully justify your answers unless instructed otherwise. When justifying your answers, you may refer to the Johnson, Sethares & Klein (JSK) textbook, the Welch, Wright and Morrow (WWM) lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content in your justification.

Problem	Point Value	Your score	Торіс
1	21		Decimation
2	27		QAM Communication Performance
3	28		Blind Channel Equalization
4	24		Channel Equalization With Training
Total	100		

## Problem 2.1. Decimation. 21 points.

Decimation can change the sampling rate of discrete-time signal x[n] through discrete-time operations of filtering and then downsampling by M.

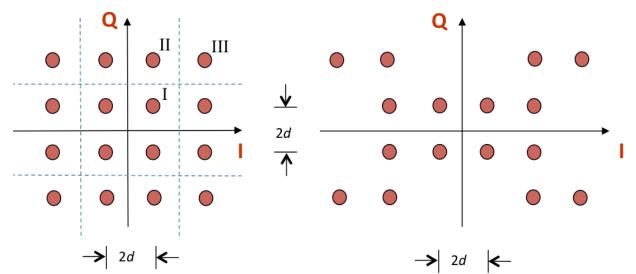


- (a) Give a formula for *y*[*n*] in terms of *v*[]. *3 points*.
- (b) Give a formula for  $f_2$  in terms of  $f_1$ . 3 points.
- (c) Specify the filter's passband frequency  $\omega_{\text{pass}}$  and stopband frequency  $\omega_{\text{stop}}$  in rad/sample to pass as many frequencies in x[m] as possible and reduce as many artifacts due to downsampling in y[n] as possible. *6 points*.

- (d) In converting an audio signal sampled at 48 kHz to a speech signal sampled at 8 kHz,
  - i. What is the value of M? 3 points.
  - ii. Would you use a finite impulse response filter or an infinite impulse response filter. Why? 6 points

# Problem 2.2 QAM Communication Performance. 27 points.

Consider the two 16-QAM constellations below. Constellation spacing is 2d.

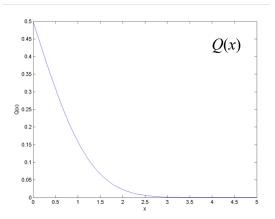


Energy in the pulse shape is 1. Symbol time  $T_{sym}$  is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

Each part below is worth 3 points. Please fully justify your answers.

	Left Constellation	<b>Right Constellation</b>		
(a) Peak transmit power	$18d^{2}$			
(b) Average transmit power	$10d^{2}$			
(c) Draw the decision regions for the right constellation on top of the right constellation.				
(d) Number of type I regions	4			
(e) Number of type II regions	8			
(f) Number of type III regions	4			
(g) Probability of symbol error	$3O(d) - {}^{9}O^{2}(d)$			
for additive Gaussian noise	$3Q\left(\frac{d}{\sigma}\right) - \frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$			
with zero mean & variance $\sigma^2$				

(h) Which constellation has a lower probability of symbol error vs. signal-to-noise ratio? Why? *6 points*.



Problem 2.3. Blind Channel Equalization. 28 points.

Blind channel equalization occurs without a training sequence, as shown below.

When the transmitted sequence x[k] is binary phase shift keying (BPSK), i.e. is +1 or -1, an adaptive method can be based on the fact that  $x^{2}[k] = 1$ .

Assume a two-tap finite impulse response (FIR) equalizer with its first coefficient fixed at one:

$$w[k] = \delta[k] + w_1 \,\delta[k\text{-}1]$$

(a) Using the objective function

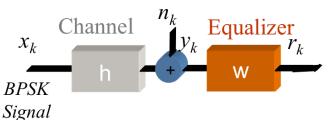
$$J(k) = \frac{1}{4} \left(1 - r^2[k]\right)^2$$

derive the adaptive update equation for  $w_1$ . 16 points

(b) Give an initial value for  $w_1$ . Why did you choose that value? 3 points

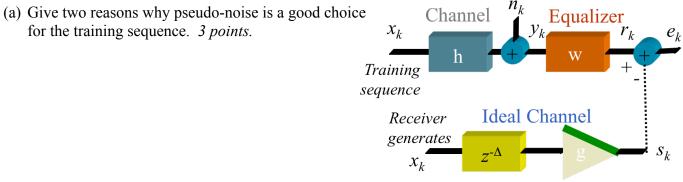
(c) What range of values would you use for the step size  $\mu$ ? Why? 3 points

(d) How would you adjust the objective function for 4-level pulse amplitude modulation? 6 points



### Problem 2.4. Channel Equalization With Training. 24 points

For the finite impulse response (FIR) channel equalizer on the right:



(b) Here is the update equation for an adaptive least mean squares FIR filter with N coefficients w:

 $\mathbf{w}[k+1] = \mathbf{w}[k] - \mu \, e[k] \, \mathbf{y}[k]$ 

where  $\mathbf{y}[k] = [y[k] \ y[k-1] \ \dots \ y[k-(N-1)]$  and  $e[k] = r[k] - g \ x[k - \Delta]$  and  $r[k] = FIR\{y[k]\}$ 

i. How many multiplications are needed per iteration? How does this compare with an FIR filter? *6 points* 

- ii. How many words of memory are needed the adaptive FIR filter? How does this compare with an FIR filter? 6 points
- iii. What range of values would you use for the step size  $\mu$ ? Why? 3 points
- (c) For a training sequence of length 2*N*, would you advocate using a least squares equalizer or an adaptive least mean squares equalizer? 6 points