

The University of Texas at Austin  
Dept. of Electrical and Computer Engineering  
Midterm #2

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Date: May 4, 2018

Course: EE 445S

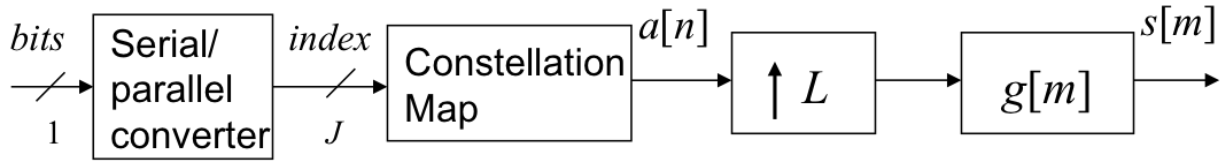
Name: \_\_\_\_\_  
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system.**
- Please turn off all smart phones and other personal communication devices.
- Please remove headphones.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise.** When justifying your answers, you may refer to the Johnson, Sethares & Klein (JSK) textbook, the Welch, Wright and Morrow (WWM) lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content in your justification.

Problem	Point Value	Your score	Topic
1	24		Baseband Pulse Amplitude Modulation
2	28		QAM Communication Performance
3	24		QAM Constellation Derotation
4	24		Potpourri
Total	100		

**Problem 2.1. Baseband Pulse Amplitude Modulation. 24 points.**

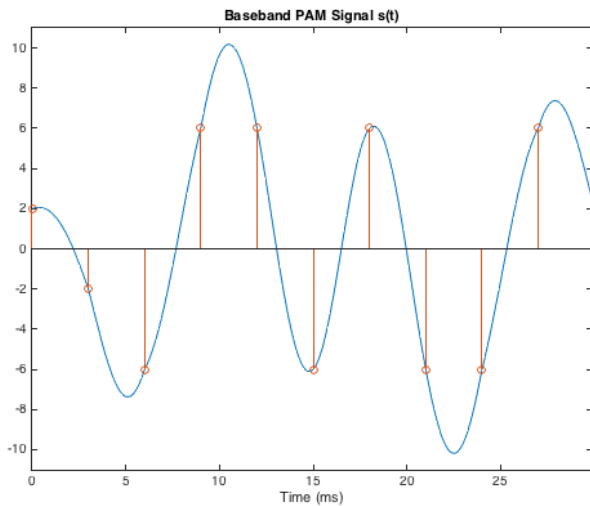
A baseband pulse amplitude modulation transmitter is described as



where

$a[n]$  symbol amplitude       $f_s$  sampling rate       $f_{sym}$  symbol rate       $g[m]$  pulse shape  
 $J$  bits/symbol       $L$  samples/symbol       $N_g$  symbol periods in a pulse shape

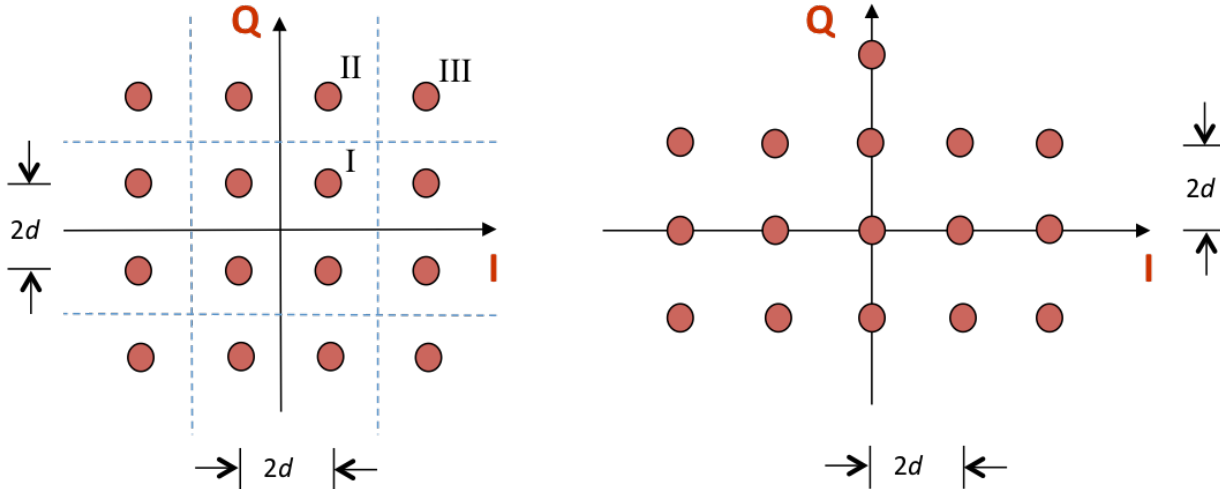
For  $N_g = 4$  and  $L = 20$ , a plot is shown below for 10 symbol periods over 0 to 30 ms of  $s[m]$  after it had passed through a digital-to-analog converter, and the symbol amplitudes  $a[n]$  are shown as a stem plot:



- (a) What is the value of  $J$ , the number of bits per symbol? Why? 3 points.
- (b) If the spacing between constellation points is  $2d$ , what is the value of  $d$ ? Why? 3 points.
- (c) Draw a constellation map with Gray coding. 3 points.
- (d) Accurately compute the symbol time,  $T_{sym}$ , in milliseconds. 3 points.
- (e) Give a formula for the pulse shape,  $g[m]$ . How many samples are in  $g[m]$ ? 6 points.
- (f) Infer an upper bound on the amplitude of  $s[m]$  as a function of  $d$ ,  $J$  and  $N_g$ . 6 points.

**Problem 2.2 QAM Communication Performance.** 28 points.

Consider the two 16-QAM constellations below. Constellation spacing is  $2d$ .



Energy in the pulse shape is 1. Symbol time  $T_{\text{sym}}$  is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

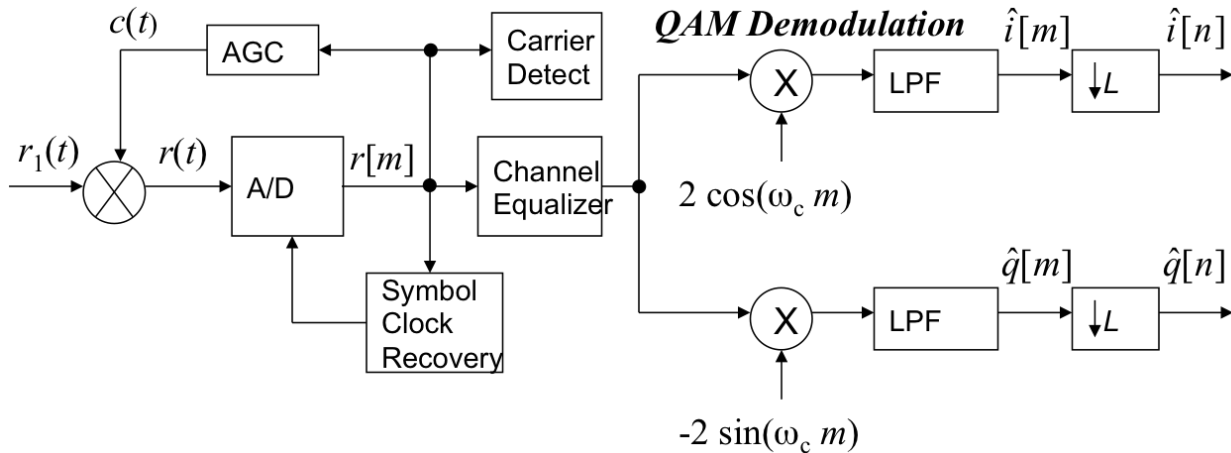
Each part below is worth 3 points. **Please fully justify your answers.**

	Left Constellation	Right Constellation
(a) Peak transmit power	$18d^2$	
(b) Average transmit power	$10d^2$	
(c) Draw the decision regions for the right constellation on top of the right constellation.		
(d) Number of type I regions	4	
(e) Number of type II regions	8	
(f) Number of type III regions	4	
(g) Probability of symbol error for additive Gaussian noise with zero mean & variance $\sigma^2$	$3Q\left(\frac{d}{\sigma}\right) - \frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$	

(h) Consider using the constellations in upconverted QAM. In the QAM receiver, how would the Costas loop for the phase locked loop perform for the right constellation vs. the left constellation? 7 points.

**Problem 2.3. QAM Constellation Derotation.** 24 points.

A baseband Quadrature Amplitude Modulation (QAM) receiver is given below



where  $\hat{i}[n]$  and  $\hat{q}[n]$  are the received in-phase and quadrature symbol amplitudes at symbol index  $n$ .

At the receiver, the QAM constellation may rotate due to a mismatch in the carrier frequencies.

A phase locked loop running at the sampling rate could track the time-varying phase that is due to the carrier frequency mismatch.

An alternative is to derotate the constellation at the symbol rate by multiplying the complex symbol  $\hat{i}[n] + j \hat{q}[n]$  by  $e^{j\theta}$ , i.e.  $i[n] + j q[n] = (\hat{i}[n] + j \hat{q}[n])e^{j\theta} = (\hat{i}[n] + j \hat{q}[n])(\cos \theta + j \sin \theta)$ :

$$i[n] = \hat{i}[n] \cos \theta - \hat{q}[n] \sin \theta \quad \text{and} \quad q[n] = \hat{q}[n] \cos \theta + \hat{i}[n] \sin \theta$$

We will adapt the phase offset  $\theta$  based on the error vector magnitude  $e[n]$  in the decision device, i.e.

$$e^2[n] = (i[n] - \hat{i}[n])^2 + (q[n] - \hat{q}[n])^2$$

(a) Give an objective function  $J(e[n])$ . 6 points.

(b) Derive the update equation for  $\theta_{k+1}$ , where  $k$  is a symbol index. 9 points.

(c) What range of values would you recommend for  $\mu$ ? 3 points.

(d) This method can work with or without a training sequence. If you were to use a training sequence, which one would you use? Why? 6 points.

