The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Prof. Brian L. Evans

Date: May 8, 2019 Course: EE 445S

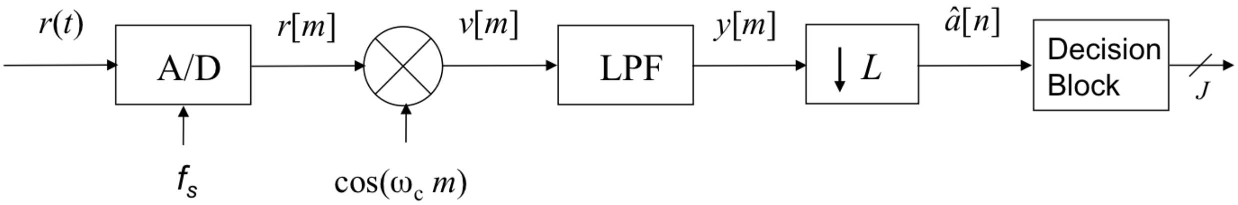
Name: ____
Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all smart phones and other personal communication devices.
- Please remove headphones.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein (JSK) textbook, the Welch, Wright and Morrow (WWM) lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content in your justification.

Problem	Point Value	Your score	Topic
1	24		Bandpass PAM Receiver Tradeoffs
2	30		QAM Communication Performance
3	28		QAM Receiver Design
4	18		Bandpass PAM Receiver Decisions
Total	100		

Problem 2.1. *Bandpass Pulse Amplitude Modulation Receiver Tradeoffs. 24 points.*

A bandpass pulse amplitude modulation (PAM) receiver is described as



where m is the sampling index and n is the symbol index, and has **system parameters**

$\hat{a}[n]$ received symbol amplitude	f_s sampling rate	f_{sym} symbol rate
$g[m]$ raised cosine pulse	J bits/symbol	L samples/symbol
M number of levels, i.e. $M = 2^J$	ω_c carrier frequency in rad/sample	

The only impairment being considered is additive thermal noise $w(t)$.

Hence, $r(t) = s(t) + w(t)$ where $s(t)$ is the transmitted bandpass PAM signal.

(a) For the additive thermal noise $w(t)$,

i. What is the probability distribution used to model the amplitude values of $w(t)$? *3 points.*

ii. What is the justification for using that probability distribution? *3 points.*

(b) If an optimal matched filter is used for the LPF,

i. Which signal in the receiver is being optimized? *3 points*

ii. By what measure is the signal in part (b)i optimal? *3 points.*

(c) Give formulas for communication signal quality measures below in terms of system parameters:

i. Bit rate. *3 points*

ii. Probability of symbol error. *3 points*

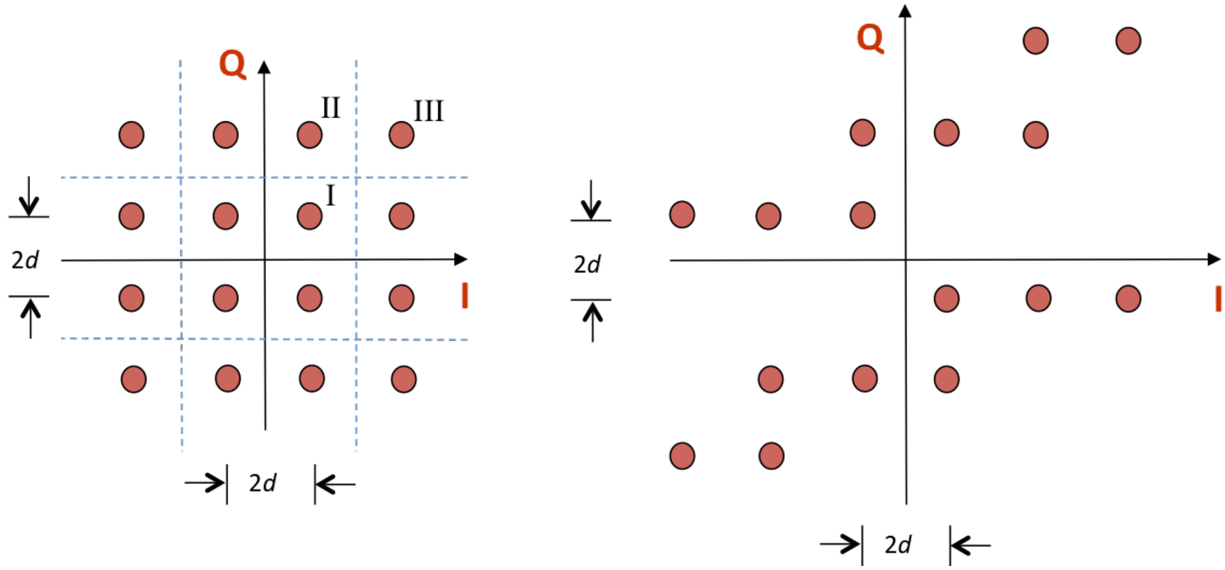
(d) Based on the formulas in (c), what's the impact on bit rate and probability of symbol error if

i. Transmit power is increased. *3 points*

ii. Number of samples/symbol, L, is increased. *3 points*

Problem 2.2 *QAM Communication Performance. 30 points.*

Consider the two 16-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

Each part below is worth 3 points. **Please fully justify your answers.**

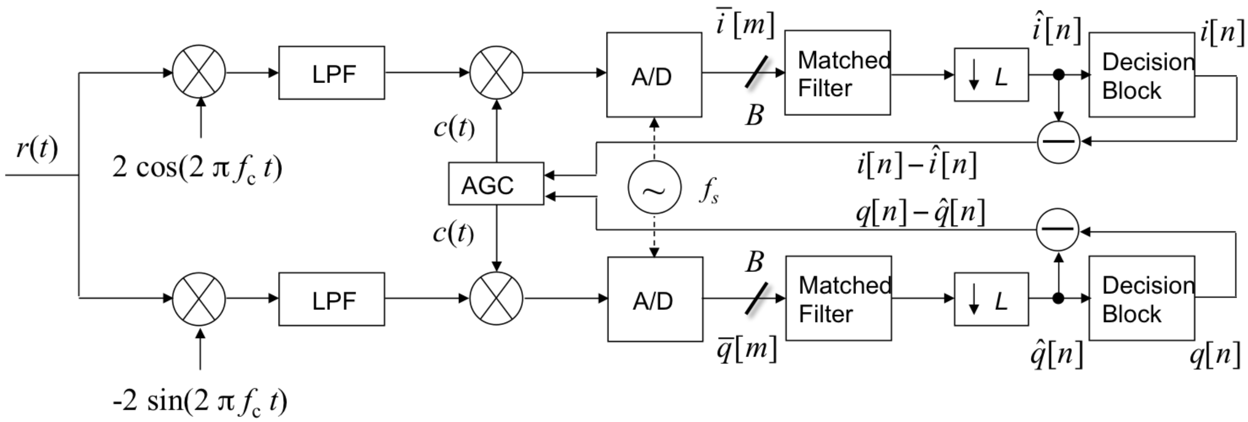
	Left Constellation	**Right Constellation**
(a) Peak transmit power	$18d^2$	
(b) Average transmit power	$10d^2$	
(c) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***		
(d) Number of type I regions	4	
(e) Number of type II regions	8	
(f) Number of type III regions	4	
(g) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$3Q\left(\frac{d}{\sigma}\right)-\frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$	
(h) Express d/σ as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR}=\frac{10d^2}{\sigma^2}$ $\frac{d}{\sigma}=\sqrt{\frac{\text{SNR}}{10}}$	

(i) In a 16-QAM receiver for the right constellation, an estimated symbol amplitude $-3d - j\,0.5\,d$. What is the decoded transmitted constellation point using

- Your constellation regions given above. *3 points*
- Smallest Euclidean distance. *3 points*

Problem 2.3. *Quadrature Amplitude Modulation (QAM) Receiver Design. 28 points.*

Some QAM receivers have a separate analog-to-digital (A/D) converter for the in-phase component and the quadrature component, as shown below.



System parameters: B bits at A/D output, $2d$ constellation spacing, f_s sampling rate, f_{sym} symbol rate, J bits/symbol, L samples/symbol, and M constellation points (i.e. $M = 2^J$). B is much greater than J.

Assume a rectangular, uniformly spaced, QAM constellation.

(a) If the signal-to-noise ratio (SNR) due to thermal noise in the system increases by 6 dB, and the system is matching the SNR due to thermal noise with the SNR due to quantization noise,

i. How many additional bits are possible for each A/D converter? *3 points.*

ii. What is the overall dB/bit increase in the system? *4 points.*

(b) What is the largest value of d that prevents clipping in the A/D converter? *3 points.*

(c) Receiver supports up to 16-QAM. For a 4-QAM training signal, develop an adaptive automatic gain control (AGC) algorithm. Gain $c(t)$ will be applied to the in-phase and quadrature channels. The gain sampled at the symbol time, $c[n] = c(n\, T_{sym})$, will be adapted every symbol period.

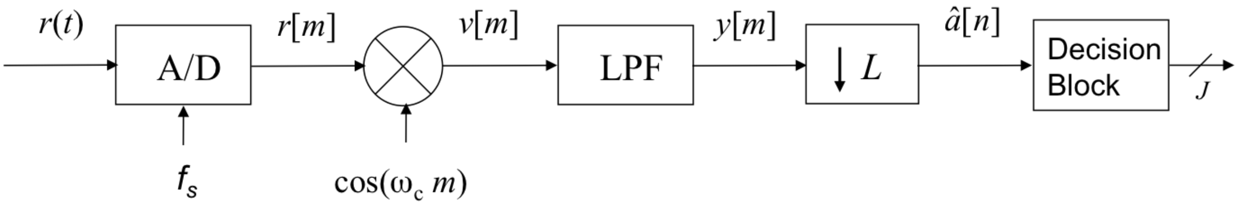
i. Give an objective function $J(n)$. *6 points.*

ii. Derive an update equation for gain $c[n]$. Compute all derivatives. Simplify result. *9 points*

iii. What range of values would you recommend for the step size μ? Why? *3 points.*

Problem 2.4. *Bandpass Pulse Amplitude Modulation Receiver Decisions. 18 points*

A bandpass pulse amplitude modulation (PAM) receiver is described as



where m is the sampling index and n is the symbol index, and has system parameters

$\hat{a}[n]$ received symbol amplitude	f_s sampling rate	f_{sym} symbol rate
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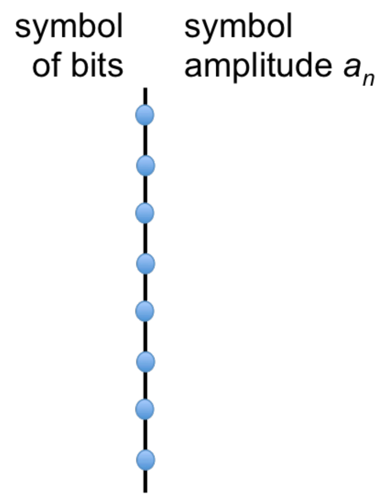
The only impairment being considered is additive thermal noise $w(t)$.

Hence, $r(t) = s(t) + w(t)$ where $s(t)$ is the transmitted bandpass PAM signal.

(a) Consider an 8-PAM bandpass transmitter.

i. Draw an 8-PAM constellation map with Gray coding on the right. *6 points*

ii. Explain how you would build a lookup table for the constellation map in part (a)i. *3 points*



(b) Consider an M-PAM bandpass receiver. The decision block quantizes the estimated symbol amplitude $\hat{a}[n]$ for M-PAM into a symbol of bits. Give formulas for the computational complexity as a function of M for each decision block quantization algorithm below. *9 points.*

i. Compare $\hat{a}[n]$ against each constellation point in the transmitter constellation map.

ii. Divide-and-conquer to discard half of the candidate constellation points each comparison.

iii. Determine the index of the closest constellation point using round $\left(\frac{\hat{a}[n]-d}{2d}\right)$, limit the unsigned index to a value between 0 and M-1 inclusive, and then use the unsigned index to find the symbol of bits in the lookup table for the constellation map.