The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2 **Take-Home Exam**

Prof. Brian L. Evans

Date: May 6, 2020 Course: EE 445S

Name: ____
Last, First

Please sign your name below to certify that you did not receive any help, directly or indirectly, on this test from another human other your instructor, Prof. Brian L. Evans, and to certify that you did not provide help, directly or indirectly, to another student taking this exam.

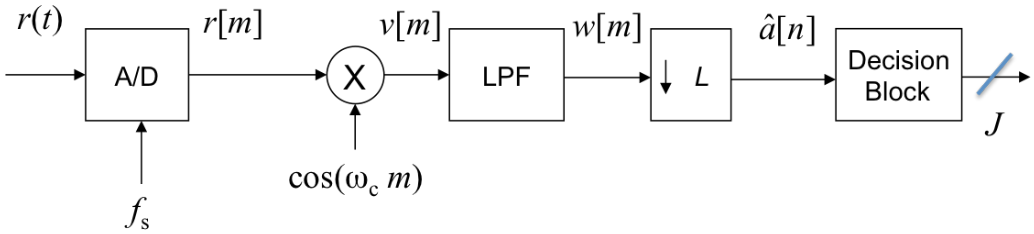
(please sign here) ________________________________

- **Take-home exam** is scheduled for Wednesday, May 6, 2020, from noon to 11:59pm.
  - The exam will be available on the course Canvas page at noon on May 6, 2020.
  - Please upload your solution to the course Canvas page by 11:59pm on May 6, 2020.
- **Perform all work on test.** All work should be performed on the exam. If more space is needed, then use the backs of the pages or scan in the extra page(s) with each problem.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification.
- **Internet access.** Yes, you may fully access the Internet when answering exam questions provided that you comply with the other instructions on this page.
- **Academic integrity.** You shall not receive help directly or indirectly on this test from another human except your instructor, Prof. Evans. You shall not provide help, directly or indirectly, to another student taking this exam.
- **Send questions to Prof. Evans.** You may send any questions or concerns about midterm #2 to Prof. Evans by e-mail at bevans@ece.utexas.edu.
- **Contact by Prof. Evans.** Prof. Evans might contact all students in the class during the exam through Canvas announcements. Please periodically monitor those announcements.

Problem	Point Value	Your score	Topic
1	24		Bandpass PAM Receiver Tradeoffs
2	30		QAM Communication Performance
3	28		Nonlinear Channel Equalization
4	18		Potpourri
Total	100		

Problem 2.1. *Bandpass Pulse Amplitude Modulation Receiver Tradeoffs*. *24 points*.

A bandpass pulse amplitude modulation (PAM) receiver is described as



where m is the sampling index and n is the symbol index, and has **system parameters**

$a[n]$ transmitted symbol amplitude	$\hat{a}[n]$ received symbol amplitude	
$2d$ constellation spacing	f_s sampling rate	f_{sym} symbol rate
$g[m]$ raised cosine pulse with rolloff α	J bits/symbol	L samples/symbol
M number of levels, i.e. $M = 2^J$	N_g symbol periods in $g[m]$	ω_c carrier freq. in rad/sample

The only impairment is additive thermal noise $w(t)$ modeled as zero-mean Gaussian with variance σ^2.

Hence, $r(t) = s(t) + w(t)$ where $s(t)$ is the transmitted bandpass PAM signal.

Give a formula for each quantity below in terms of the symbol rate f_{sym} and describe how much the quantity changes when the symbol rate increases.

(a) Bit rate in bits/s. *4 points*

(b) Transmission bandwidth in Hz. *4 points.*

(c) Sampling rate f_s. *4 points*.

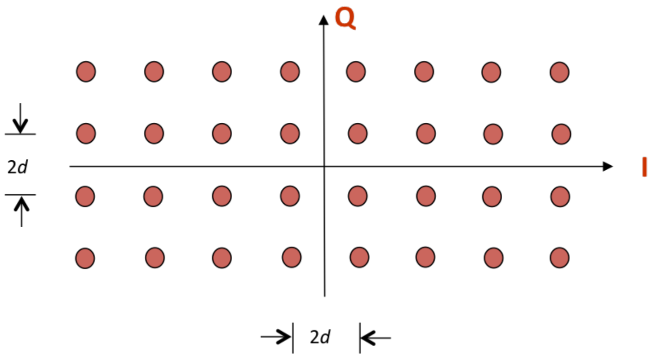
(d) Probability of symbol error. *4 points*

(e) Implementation complexity in multiplications per second. *8 points*

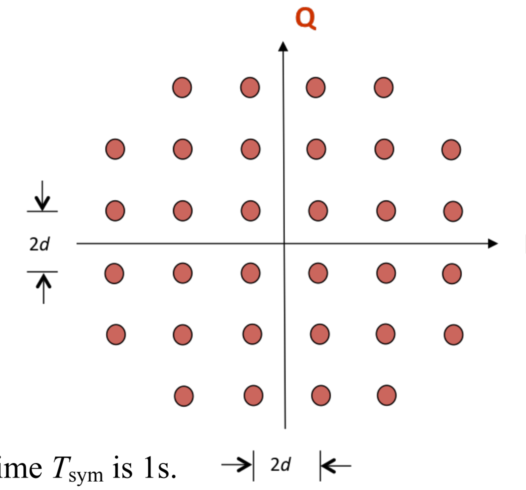
Problem 2.2 *QAM Communication Performance. 30 points. Problem continues onto the next page.*

Part I. Consider choosing a 32-QAM constellation.

Constellation #1: 8x4 rectangular



Constellation #2: Power Efficient



Constellation spacing is $2d$. Pulse shape energy is 1. Symbol time T_{sym} is 1s.

Compute the peak and average power for constellation #2. *3 points each.*

	Constellation #1	**Constellation #2**	
(a) Peak transmit power	$58d^2$		*3 points*
(b) Average transmit power	$26d^2$		*3 points*

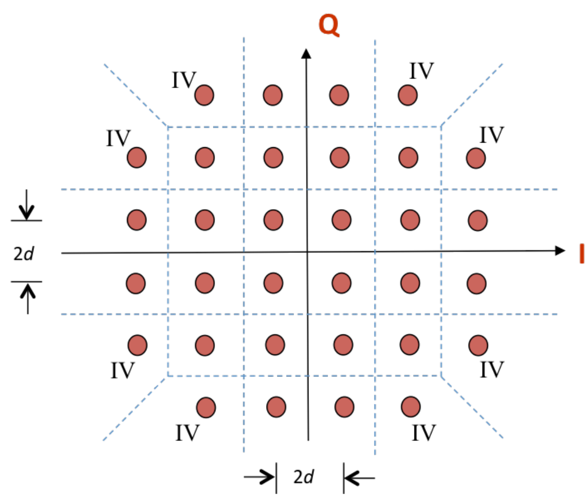
Constellation #2 has the lowest peak and average power possible, and is commonly used in practice.

Part II. For constellation #2, we're going to introduce a type IV constellation region. We'll also use it in Part III.

On the right, decision region boundaries are shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

Type IV region has a diagonal line separating two nearest neighbors on a corner. It's a union of a type II region (finite in one dimension and infinite in the other) and half of a type III region (quarter plane).

We now have eight type IV regions instead of four type III regions (quarter planes at corner points).

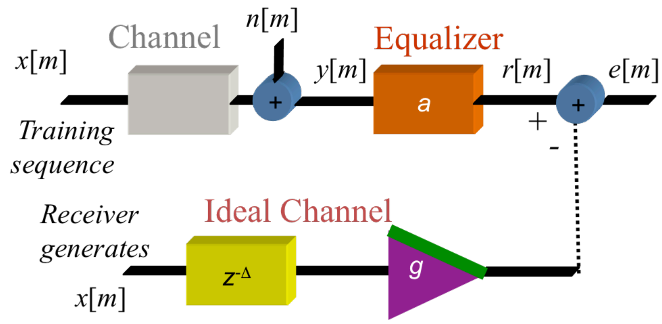


	Constellation #1	**Constellation #2**	
(c) Number of type I regions	12		*3 points*
(d) Number of type II regions	16		*3 points*
(e) Number of type III regions	4	**0**	
(f) Number of type IV regions	0	**8**	
(g) Symbol error probability for additive Gaussian noise, zero mean & variance σ^2	$\frac{13}{4} Q\left(\frac{d}{\sigma}\right) - \frac{21}{8} Q^2\left(\frac{d}{\sigma}\right)$		*6 points*
(h) Express d/σ as a function of Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = 26\left(\frac{d^2}{\sigma^2}\right)$ $\frac{d}{\sigma} = \sqrt{\frac{SNR}{26}} \approx 0.196\sqrt{\text{SNR}}$		*3 points*

Part III. In the receiver, finding the nearest constellation point to the received QAM symbol amplitude using Euclidean distance provides high-accuracy in the symbol detection. Complexity is proportional to the number of levels $M = 2^J = 32$ where $J=5$ is the number of bits in a symbol: 64 multiplications, 96 additions, and 64 memory reads in words for each (I, Q) symbol amplitude.

We introduced the type IV region in part II to unlock a low-complexity divide-and-conquer method that is just as accurate as using Euclidean distance but only needs to use comparison operations. Describe the method and compare its complexity with the Euclidean distance method. *9 points.*

Problem 2.3. *Nonlinear Channel Equalization*. *28 points*.



In the discrete-time system on the right, the equalizer operates at the sampling rate.

The channel has significant **nonlinear distortion**.

We're going to use a **nonlinear equalizer** of the form

$r[m] = a_0 + a_1\, y[m] + a_2\, y^2[m] + \ldots + a_N\, y^N[m]$

where $a_0, a_1, a_2, \ldots a_N$ are real-valued coefficients.

The channel model includes additive noise $n[m]$ that has a Gaussian distribution with zero mean and variance σ^2.

(a) Give a training sequence for $x[m]$ that you would use? Why? *3 points.*

(b) For one of the training sequences in part (a), describe how you would estimate the transmission delay parameter Δ in the ideal channel model. *3 points.*

(c) What objective function would you use? Why? *6 points.*

(d) For an adaptive nonlinear equalizer, **derive the update equation** for the vector of coefficients $\vec{a}$ for the objective function in part (c). Here, $\vec{a} = [\, a_0 \;\; a_1 \;\; a_2 \;\; \cdots \;\; a_N \,]$. *12 points.*

(e) For your answer in (c), what values of the step size (learning rate) μ would you use? *4 points.*

Problem 2.4. *Potpourri*. *18 points*

(a) A handheld garage door opener transmits a binary request to an automatic garage door control unit. The binary request is to open the garage door if closed, and close the garage door if open.

Please describe the signals based on PN sequences that the garage door opener would transmit to indicate that it is sending its binary request to the automatic garage door control unit (receiver)?

The transmission/reception would have to be

- Reliable -- a very high probability of correct detection
- Secure -- nearly impossible for an opener meant for another unit to work on your garage door.

(b) Steepest descent algorithm to minimize an objective (cost) function.

i. Draw an objective function that has at least one global minimum value and at least one local minimum that is not a global minimum. *3 points.*

ii. Give a way to determine if a steepest descent algorithm has converged to an answer. *3 points.*

iii. How would you use multiple steepest descent algorithms running in parallel to reach a solution with a lower objective (cost) function? *3 points*.

iv. If you could only run one steepest descent algorithm, how could you modify it to get out of a possible local minimum? *3 points.*