The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Date: May 4, 2022 Course: EE 445S Evans

Name: ____
Last, First

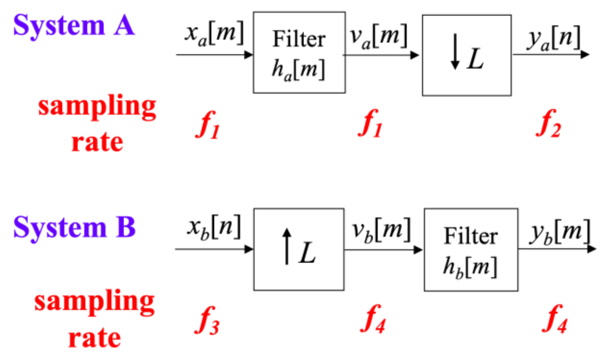
- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except your instructor, Prof. Evans, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	Point Value	Your score	Topic
1	19		Changing Sampling Rates
2	33		QAM Communication Performance
3	24		Adaptive Spatial Filter
4	24		Potpourri
Total	100		

Problem 2.1. *Changing Sampling Rates. 19 points.*

Consider the two systems to change the sampling rate:

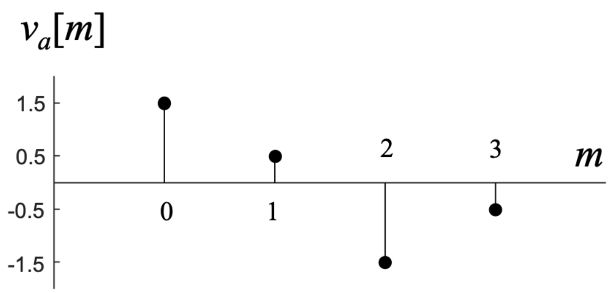
- System A consists of linear time-invariant (LTI) filtering followed by downsampling by L.
- System B consists of upsampling by L followed by LTI filtering.



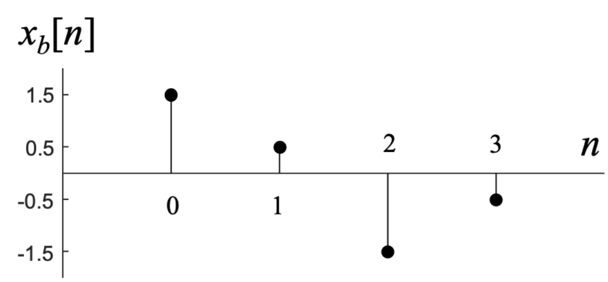
(a) Give a formula for f_2 in terms of f_1. *2 points.*

(b) Give a formula for f_4 in terms of f_3. *2 points.*

(c) Assuming $L = 2$, draw $y_a[n]$ corresponding to the input $v_a[m]$ shown below. *4 points.*



(d) Assuming that $L = 2$, draw $v_b[m]$ corresponding to the input $x_b[n]$ shown below. *4 points.*



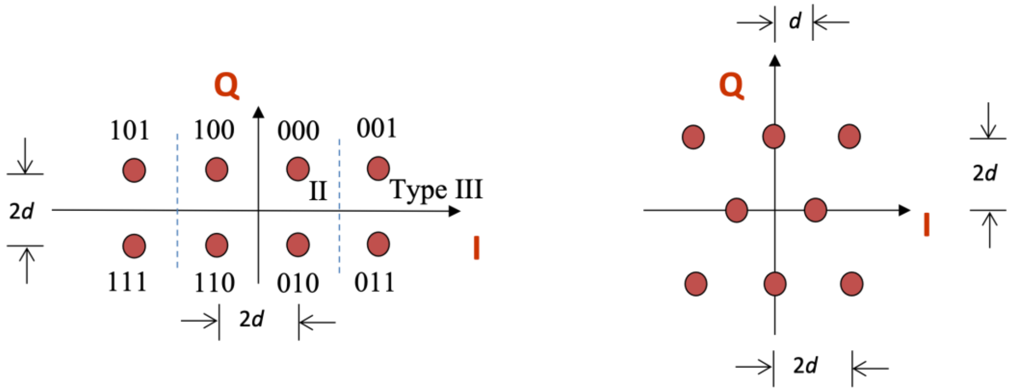
(e) Assume the filter in System B is a finite impulse response (FIR) filter with N coefficients.

1. How many multiplication operations per second does System B use in the block diagram above? *3 points.*

2. How many multiplication operations per second would System B use if implemented as a polyphase filter bank? *4 points.*

Problem 2.2 *QAM Communication Performance. 33 points.*

Consider the two 8-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s.

Each part below is worth 3 points. **Please fully justify your answers.**

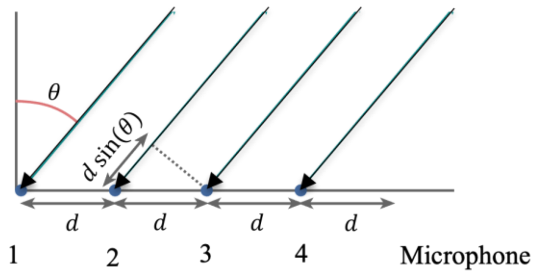
	Left Constellation	**Right Constellation**
(a) Peak transmit power	$10d^2$	
(b) Average transmit power	$6d^2$	
(c) Peak-to-average power ratio	$\frac{10d^2}{6d^2} = \frac{5}{3} \approx 1.67$	
(d) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***		
(e) Number of type I QAM regions	0	
(f) Number of type II QAM regions	4	
(g) Number of type III QAM regions	4	
(h) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2.	$P_e = \frac{5}{2} Q\left(\frac{d}{\sigma}\right) - \frac{3}{2} Q^2\left(\frac{d}{\sigma}\right)$	
(i) Express the argument of the Q function as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\mathrm{SNR} = \frac{6d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{\mathrm{SNR}}{6}}$	

(j) Give one advantage of the left constellation vs. the right constellation. *3 points.*

(k) Give one advantage of the right constellation vs. the left constellation. *3 points.*

Problem 2.3 *Adaptive Spatial Filter. 24 points.*

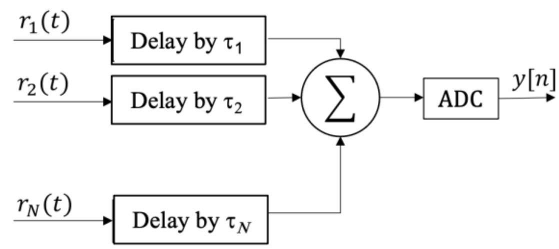
A single sound source is recorded by several microphones simultaneously as shown on the right.



The microphones are arranged in a line and separated by distance d. Sound arrives at an unknown angle θ.

Sound arrives at the ith microphone with a different delay t_i based on the distance to the source. The source is far enough away for the propagation to be a plane wave.

The signal recorded by the ith microphone $r_i(t)$ is delayed by τ seconds, and all the signals are added together before being sampled by an analog-to-digital converter:



We would like to design an adaptive spatial filter that amplifies a signal located at an unknown angle θ by adapting the delays $\tau_1, \tau_2, \dots, \tau_N$.

A known training signal $x[n]$ is sent by the source so that we can find the best values for $\tau_1, \tau_2, \dots, \tau_N$. Hence $r_i(t) = x(t - t_i)$ where $x(t)$ is the continuous-time version of the training signal $x[n]$.

(a) **Training**. What training signal $x[n]$ would you send? Why? Describe its parameters. *6 points.*

(b) **Propagation Delay**. Consider the signal received by the ith microphone $r_i(t)$. Propose and explain an algorithm to find the delay t_i from the source to the ith microphone. *6 points.*

(c) **Adaptive Spatial Filter**. Develop a discrete-time adaptive algorithm to apply to $y[n]$ to determine the best set of delays

$$\vec{\tau} = [\tau_1 \quad \tau_2 \quad \cdots \quad \tau_N]$$

for the microphone array to amplify the sound coming from the source at an unknown angle θ.

1. Give an objective function and explain why you have chosen it. *3 points.*

2. Give an adaptive steepest descent/ascent algorithm for $\vec{\tau}[i+1]$ in terms of $\vec{\tau}[i]$. *6 points.*

3. What values of the step size would you use? Why? *3 points.*

Problem 2.4. *Potpourri*. *24 points*.

(a) Consider 16-QAM system transmitting at a 1200 bps (bits per second) using a 12-sample discrete-time raised cosine pulse shaping filter. What are the possible sampling rates in Hz? *12 points*.

(b) Consider a wireless communication system that uses two transmit antennas and two receive antennas. This allows two signals $x_1[n]$ and $x_2[n]$ to be sent at the same time and over the same frequency band as shown below: *12 points*

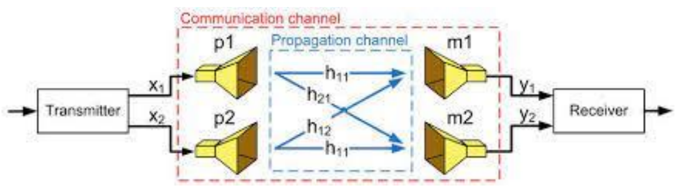


Figure from Lars Reichardt, Juan Pontes, Yoke Leen Sit, and Thomas Zwick, "Antenna Optimization for Time-Variant MIMO Systems", EuCap, 2011.

Each antenna at the receiver receives both transmitted signals.

The communication channel has a **complex-valued** scalar gain between the *i*th transmit antenna and *j*th receive antenna. No other impairments are being modeled.

The received signal is

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

which can be written as

$$\vec{y} = \boldsymbol{H} \, \vec{x}$$

In the receiver, assume $\boldsymbol{H}$ is known.

Find **two possible values** for matrix $\boldsymbol{G}$ in terms of $\boldsymbol{H}$ that can allow us to estimate $\vec{x}$ from $\vec{y}$ via

$$\vec{x}_{estimated} = \boldsymbol{G} \, \vec{y}$$

Hints: One way could equalize (invert) the channel. Other could use a matched filtering approach.