EE 313

Linear Systems & Signals

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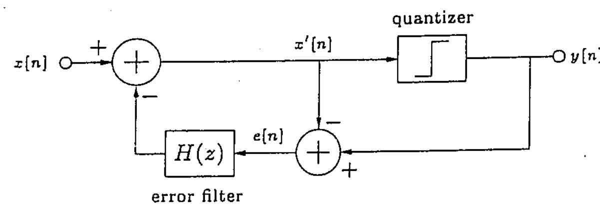
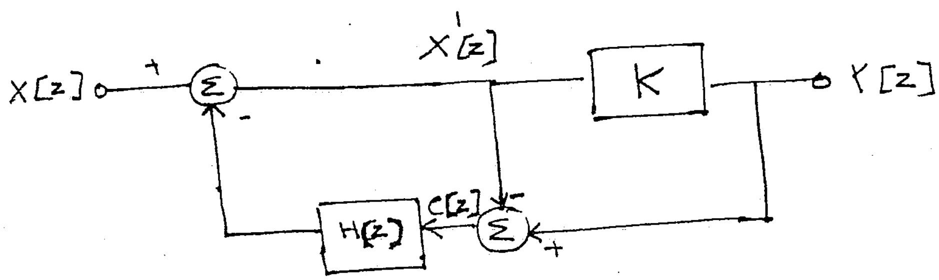
Problem 2.4 Sigma-Delta Modulation. 20 points.

Shown below is a type of sigma-delta modulator called a noise-shaping feedback coder.



We can approximate the effect of the quantizer as a gain K, which would make the overall system linear and time-invariant. Replace the quantizer with a gain of K and derive the transfer function from input $x[n]$ to output $y[n]$.



$$Y[z] = K x'[z]$$

$$x'[z] = X[z] - H(z)e[z]$$

$$e[z] = Y[z] - x'[z]$$

$$x'[z] = x[z] - H(z)\left[Y[z] - x'[z]\right]$$

$$x'[z] = \frac{X[z] - H(z)Y[z]}{1 - H(z)}$$

$$Y[z] = K\left[\frac{X[z] - H(z)Y[z]}{1 - H(z)}\right]$$

$$Y[z]\left[1 + \frac{KH(z)}{1 - H(z)}\right] = \frac{KX[z]}{1 - H(z)}$$

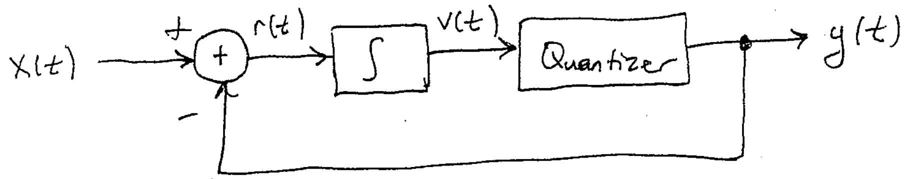
$$Y[z] = \frac{\dfrac{KX[z]}{(1 - H(z))}}{\dfrac{(1 - H(z)) + KH(z)}{(1 - H(z))}}$$

$$Y[z] = \frac{k\,X[z]}{1 + H(z)(k-1)}$$

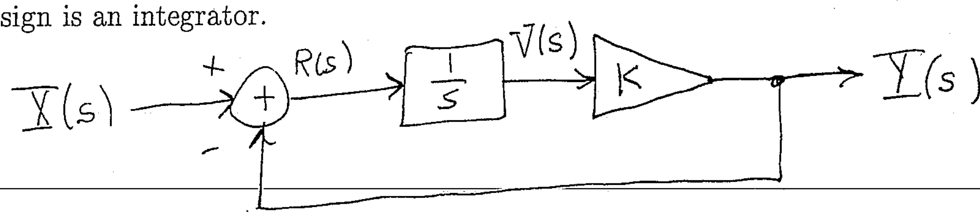
Therefore $$\boxed{\frac{Y[z]}{X[z]} = \frac{K}{1 + (k-1)H(z)}}$$

Problem 2.4 Sigma-Delta Modulation. 15 points.

Shown below is a type of continuous-time sigma-delta modulator:



We can approximate the effect of the quantizer as a gain K, which would make the overall system linear and time-invariant. Replace the quantizer with a gain of K and derive the transfer function from input $x(t)$ to output $y(t)$. The LTI system shown graphically as an integral sign is an integrator.



$$R(s) = X(s) - Y(s)$$

$$V(s) = \frac{1}{s} R(s)$$

$$Y(s) = K\, V(s)$$

Combining these three equations:

$$Y(s) = K \cdot \frac{1}{s} \cdot \left(X(s) - Y(s) \right)$$

$$\left(1 + \frac{K}{s}\right) Y(s) = \frac{K}{s} X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{\frac{K}{s}}{1 + \frac{K}{s}} = \frac{K}{s + K}$$