Pulse Amplitude Modulation (PAM)

- Communications:
  1. Analog communications: transmit and receive analog waveform \((e.g., \text{AM, PM, FM})\).
  2. Digital communications: transmit and receive discrete information \((e.g., \text{PAM, PSK, QAM, spread spectrum, \ldots})\).

- Pulse Modulation:
  Modulate \(M = 2^J\) discrete messages or \(J\) bits of information into the amplitude of a signal. If the amplitude mapping changes at a rate of \(f_s\), then the bit rate is \(Jf_s\).
  The conventional mapping of the discrete messages to amplitude is
  \[
  l_i = d(2i - 1), \quad i = -\frac{M}{2} + 1, \ldots, 0, \ldots, \frac{M}{2}.
  \]
  The PAM modulated signal can be expressed as
  \[
  s^*(t) = \sum_{k=-\infty}^{\infty} a_k \delta(t - kT). 
  \]

- Pulse Shaping:
  The signal \(s^*(t)\) has infinite bandwidth and cannot be sent by a real transmitter. Thus, we need to limit its bandwidth by a pulse shaping filter whose impulse response is \(g_T(t)\). Then the transmit signal is
  \[
  s(t) = \sum_{k=-\infty}^{\infty} a_k g_T(t - kT).
  \]

- Eye Diagram and Intersymbol Interference
  Figure 6 shows an eye diagram. The wider the “eye” opens, the better the signal quality is. An eye diagram is a empirical measure of the quality of the received signal.
  \[
  x(n) = \sum_{k=-\infty}^{\infty} a_k g(nT - kT) = g(0) \left( a_n + \sum_{k=-\infty}^{\infty} a_k \frac{g(nT - kT)}{g(0)} \right)_{\text{ISI}}.
  \]
  The intersymbol interference (ISI):
  \[
  D \leq (M - 1)d \sum_{k=-\infty, k\neq n}^{\infty} \left| \frac{g(nT - kT)}{g(0)} \right| = (M - 1)d \sum_{k=-\infty, k\neq 0}^{\infty} \left| \frac{g(kT)}{g(0)} \right|.
  \]
  The ISI is the interference to the current symbol by preceding symbols. The raised-cosine filter has zero ISI at the correct sample point.

- Symbol Clock Recovery
In typical digital communication system, it is critical to sample a correct time instance to have the maximum signal power and minimum ISI. Since the transmitter and receiver normally have different crystal oscillators, a digital receiver should try its best to synchronize with the transmitter clock. In other words, the receiver must extract the clock information from the received signal and then adjust its A/D timing.

Let \[ g(t) \] is the impulse response of the composite channel including pulse shaping, real communication media, and the receive low-pass filter. Then the received signal \( q(t) \) becomes

\[
q(t) = s^*(t) \odot g(t) = \sum_{k=-\infty}^{\infty} a_k g_1(t - kT).
\]

Then

\[
p(t) = q^2(t) = \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_k a_m g_1(t - kT)g_1(t - mT).
\]

and

\[
E\{p(t)\} = \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} E\{a_k a_m\} - a^2 \delta(k - m)g_1(t - kT)g_1(t - mT) = a^2 \sum_{k=-\infty}^{\infty} g_1^2(t - kT).
\]

Since we can easily show that \( E\{p(t + T)\} = E\{p(t)\}, \) \( E\{p(t)\} \) is a periodic function, i.e.,

\[
E\{p(t)\} = \sum_{k=-\infty}^{\infty} p_k e^{jk\omega_p t}.
\]
where

\[ p_k = \frac{1}{T} \int_0^T E\{p(t)\} e^{-jk\omega_s t} dt. \]

Clearly, if we let \( E\{p(t)\} \) passes through a low-pass filter that covers the fundamental frequency \( \omega_s \) and filters out the harmonics, \( 2\omega_s, 3\omega_s, \ldots \). We can extract a sin wave that has the same frequency as the symbol rate. This signal can be used as a reference signal to a digital phase locked loop or reshaped as a sampling clock.
Figure 3: Implementation of Pulse Shaping via a Filter Bank: Filter No. 2

Figure 4: Implementation of Pulse Shaping via a Filter Bank: Filter No. 3
Figure 5: Implementation of Pulse Shaping via a Filter Bank: Filter No. 4

Figure 6: An Example of Eye Pattern